Exam

Power Electronics

Winter 2024/25

First name:

Last name:

Matriculation number:

Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 90 minutes.

Evaluation:

Task	1	2	3	4	$\sum$
Maximum score	10	10	10	12	42
Achieved score					

Task 1: Step-down converter **[10 Points]**

In industrial control systems, a 50 V DC power supply is commonly used to power various components. Additionally, several sensors and servo motors require a stable 12 V DC power supply. For this purpose an efficient step-down (buck) converter is required to provide high currents, especially when multiple servo motors or actuators are operating simultaneously.

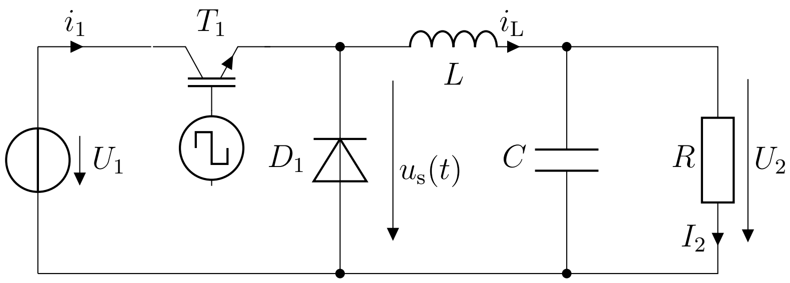


Figure 1: Circuit with one transistor, filter and one load resistor.

General parameters:		**IGBT:**	
Input voltage:	$U_1 = 50$ V	Collector-emitter voltage:	$u_{on,CE} = 2$ V
Output voltage:	$U_2 = 12$ V	Switch-on losses:	$E_{on,D} = 20$ µJ
Output current:	$I_2 = 70$ A	Switch-off losses:	$E_{off,D} = 40$ µJ
Switching frequency:	$f_s = 100$ kHz		
Inductance:	$L = 20$ µH		
The diode is considered as ideal and the filter capacitor is $C \to \infty$.			

Table 1: Parameters of the circuit.

1.1 At what duty cycle D should the step-down converter be operated? Calculate and sketch the voltage $u_L(t)$ and current $i_L(t)$ over 2 periods. [4 Points]

Hint: The voltage drop across the transistor must be taken into account.

Answer:

The duty cycle corresponds to

$$D = \frac{U_{out}}{U_{in}} = \frac{U_2}{U_1 - u_{on,CE}} = \frac{12\ \text{V}}{50\ \text{V} - 2\ \text{V}} = 0.25.$$

If the transistor conducts, the voltage u_s results in

$$u_{s,on} = U_1 - u_{on,CE} = 50\ \text{V} - 2\ \text{V} = 48\ \text{V}.$$

If the transistor does not conduct, the diode conducts and u_s yielding

$$u_\mathrm{s,off} = 0\ \mathrm{V}.$$

This leads to the result

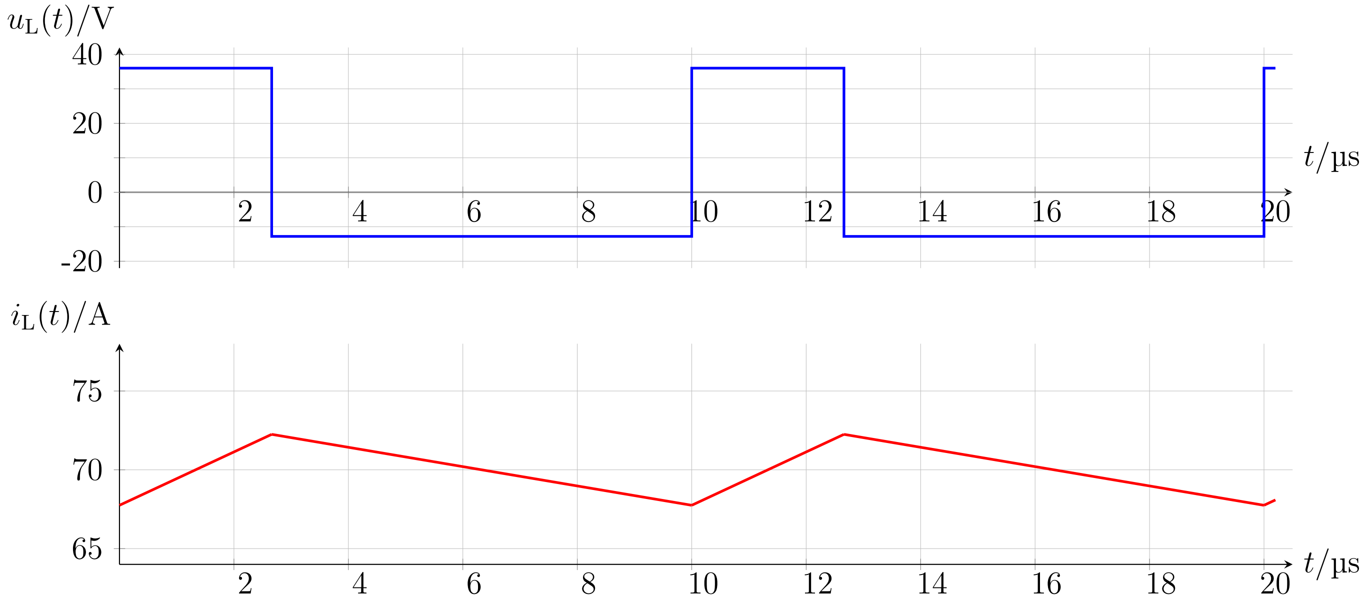
$$u_\mathrm{L} = u_\mathrm{s} - U_2 \begin{cases} u_\mathrm{L,on} = 48\ \mathrm{V} - 12\ \mathrm{V} = 36\ \mathrm{V}, & \text{if } T_1 \text{ conducts,} \\ u_\mathrm{L,off} = -12\ \mathrm{V}, & \text{if } T_1 \text{ does not conduct.} \end{cases}$$

Using the voltage at the inductor while the transistor conducts, the current ripple yields:

$$i_\mathrm{L,ripple} = \frac{u_\mathrm{L,on}}{L} \cdot T_\mathrm{on} = \frac{u_\mathrm{L,on} \cdot D}{L f_\mathrm{s}} = \frac{36\ \mathrm{V} \cdot 0.25}{20\ \mathrm{\mu H} \cdot 100\ \mathrm{kHz}} = 4.5\ \mathrm{A}.$$

The maximum and minimum inductor current result in:

$$i_\mathrm{L,max} = I_2 + 0.5 \cdot i_\mathrm{L,ripple} = 70\ \mathrm{A} + 2.25\ \mathrm{A} = 72.25\ \mathrm{A}$$
$$i_\mathrm{L,min} = I_2 - 0.5 \cdot i_\mathrm{L,ripple} = 70\ \mathrm{A} - 2.25\ \mathrm{A} = 67.75\ \mathrm{A}.$$



Solution Figure 1: Relevant voltage and current signals.

1.2 How large is the power demand of the load, if the step-down converter operates in boundary conduction mode (BCM)? [2 Points]

Answer:

The current at boundary conduction mode depends on the current ripple according

$$I_\mathrm{2,BCM} = \frac{i_\mathrm{L,ripple}}{2} = \frac{4.5\ \mathrm{A}}{2} = 2.25\ \mathrm{A}.$$

Using the current at boundary conduction mode, the power of the load is obtained by

$$P_{\text{load,BCM}} = U_2 \cdot I_2 = 12\ \text{V} \cdot 2.25\ \text{A} = 27\ \text{W}.$$

1.3 In which case the step-down converter operates in discontinous conduction mode (DCM) and what is the effect and the potential risk of this mode? [1 Point]

Answer:
If the power demand is less than $P_{\text{load,BCM}}$ the output voltage increases. The step-down converter operated in DCM-mode, if the power consumption is less than $P_{\text{load,BCM}}$. In this case the current through the inductor is 0 A for a certain time at the end of a switching period. Risk: compared to CCM, this leads to a voltage increase, which could lead to the damage of overvoltage sensitive components.

1.4 Calculate the switching power loss and the total power loss of the IGBT. [1 Point]

Answer:
The power loss of the transistor consists of the power loss while conducting the current and the switching losses.

$$P_{\text{T}_1} = u_{\text{on,CE}} I_2 D + (E_{\text{on}} + E_{\text{off}}) \cdot f_{\text{s}} = 2\ \text{V} \cdot 70\ \text{A} \cdot 0.25 + (20\ \mu\text{J} + 40\ \mu\text{J}) \cdot 100\ \text{kHz} = 41\ \text{W}.$$

1.5 Calculate the efficiency η of the step-down converter. [2 Points]

Answer:
The efficiency is calculated by the power demand of the load divided by the total input power. The power demand of the load is

$$P_{\text{load}} = U_2 \cdot I_2 = 12\ \text{V} \cdot 70\ \text{A} = 840\ \text{W}.$$

Taking into account the power demand, the efficiency is calculated by:

$$\eta = \frac{P_{\text{load}}}{P_{\text{total}}} = \frac{P_{\text{load}}}{P_{\text{load}} + P_{\text{T}_1}} = \frac{840\ \text{W}}{840\ \text{W} + 41\ \text{W}} = 0.95.$$

Task 2: Four-quadrant converter with pulse width modulation [10 Points]

The components of the four quadrant converter according Fig. 2 are considered as ideal. The converter's data is displayed in Tab. 2. The inner load voltage is constant: $u_{2i}(t) = U_{2i}$.

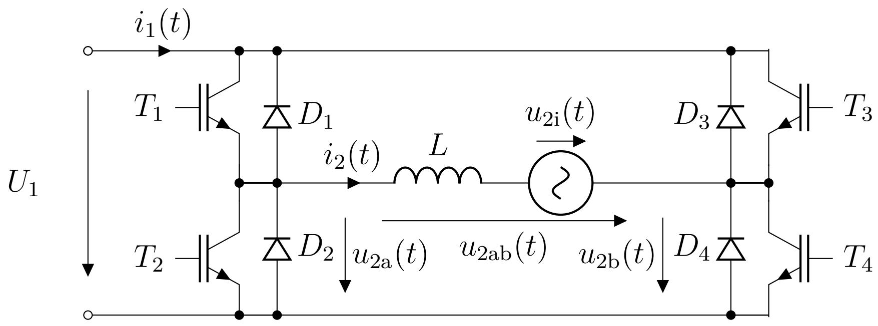


Figure 2: Four-quadrant converter.

Input voltage:	$U_1 = 450$ V	Current at $t = 0$ s:	$i_2(0) = 30$ A
Inner voltage:	$U_{2i} = 150$ V	Inductance:	$L = 60$ µH

Table 2: Parameters of the four-quadrant converter.

The four quadrant converter is controlled by a PWM with interleaved switching. T_1 is connected to the output of the PWM, which uses the non-inverted reference voltage. On the other hand, T_3 is connected to the output of the PWM, which uses the inverted reference voltage. The triangular carrier voltage and the reference voltage are displayed in the the solution diagram. Note that the reference signal changes at $t = 24$ µs and $t = 36$ µs.

Der Vier-Quadranten-Steller wird durch eine PWM mit phasenversetztem Schalten gesteuert. Der Ausgang der Steuerung, der die nicht invertierte Referenzspannung verwendet, ist mit T_1 verbunden, während der Ausgang, der die invertierte Referenzspannung verwendet, mit T_3 verbunden ist. Der Dreiecksträger und die Referenzspannung werden im Lösungsdiagramm dargestellt. Beachten Sie, dass das Referenzsignal bei $t = 24$ µs and $t = 36$ µs wechselt.

2.1 Add the voltage signals of $u_{\mathrm{a}(t)}$, $u_{\mathrm{b}(t)}$ and $u_{\mathrm{ab}(t)}$ to the diagram in Fig. 3 and and complete the axis scaling of the ordinates. [3 Points]

2.2 Calculate the current signals $i_2(t)$ and $i_1(t)$ and add them to template diagram. [4 Points]
Hint: If you are not able to exactly calculate the current signals, you can qualitatively add them to the template diagram for partial points.

Answer:

The voltage at the inductor L is calculated by

$$u_{\mathrm{L}}(t) = u_{2\mathrm{ab}}(t) - U_{2\mathrm{i}}.$$

The slope of the current is determined by the voltage at the inductance.

$$\frac{\mathrm{d}}{\mathrm{d}t} i_2(t) = \frac{1}{L} u_{\mathrm{L}}(t).$$

Using the two equations above for the slope of $i_2(t)$ at the three different voltages, we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t} i_2(t) = \begin{cases} u_{2\mathrm{ab}}(t) = 450\ \mathrm{V}: & \frac{\mathrm{d}}{\mathrm{d}t} i_2(t) = \frac{1}{60\ \mathrm{\mu H}}(450\ \mathrm{V} - 150\ \mathrm{V}) = 5\ \frac{\mathrm{A}}{\mathrm{\mu s}}, \\ u_{2\mathrm{ab}}(t) = 0\ \mathrm{V}: & \frac{\mathrm{d}}{\mathrm{d}t} i_2(t) = \frac{1}{60\ \mathrm{\mu H}}(-150\ \mathrm{V}) = -2.5\ \frac{\mathrm{A}}{\mathrm{\mu s}}, \\ u_{2\mathrm{ab}}(t) = -450\ \mathrm{V}: & \frac{\mathrm{d}}{\mathrm{d}t} i_2(t) = \frac{1}{60\ \mathrm{\mu H}}(-450\ \mathrm{V} - 150\ \mathrm{V}) = -10\ \frac{\mathrm{A}}{\mathrm{\mu s}}. \end{cases}$$

The voltage at the inductor L is constant for discrete time intervals. So $i_2(t)$ is calculated by

$$i_2(t) = i_2(\tau) + \frac{\mathrm{d}}{\mathrm{d}t} i_2(\tau) \cdot (t - \tau)$$

with τ is start of a constant time interval.

The current $i_1(t)$ corresponds to:

$$i_1(t) = \begin{cases} 0\ \mathrm{A} & u_{2\mathrm{ab}}(t) = 0\ \mathrm{V} \\ i_2(t) & u_{2\mathrm{ab}}(t) \neq 0\ \mathrm{V}. \end{cases}$$

These results are used to sketch $i_2(t)$ and $i_1(t)$ within the solution sheet.

2.3 Mark which semiconductors carry the current $i_2(t)$ in the template diagram. For example, if T_1 and D_4 conduct during the range 10 µs and 14 µs, the range must be marked with 2 vertical lines and T_1/D_4 entered there. [3 Points]

Answer:

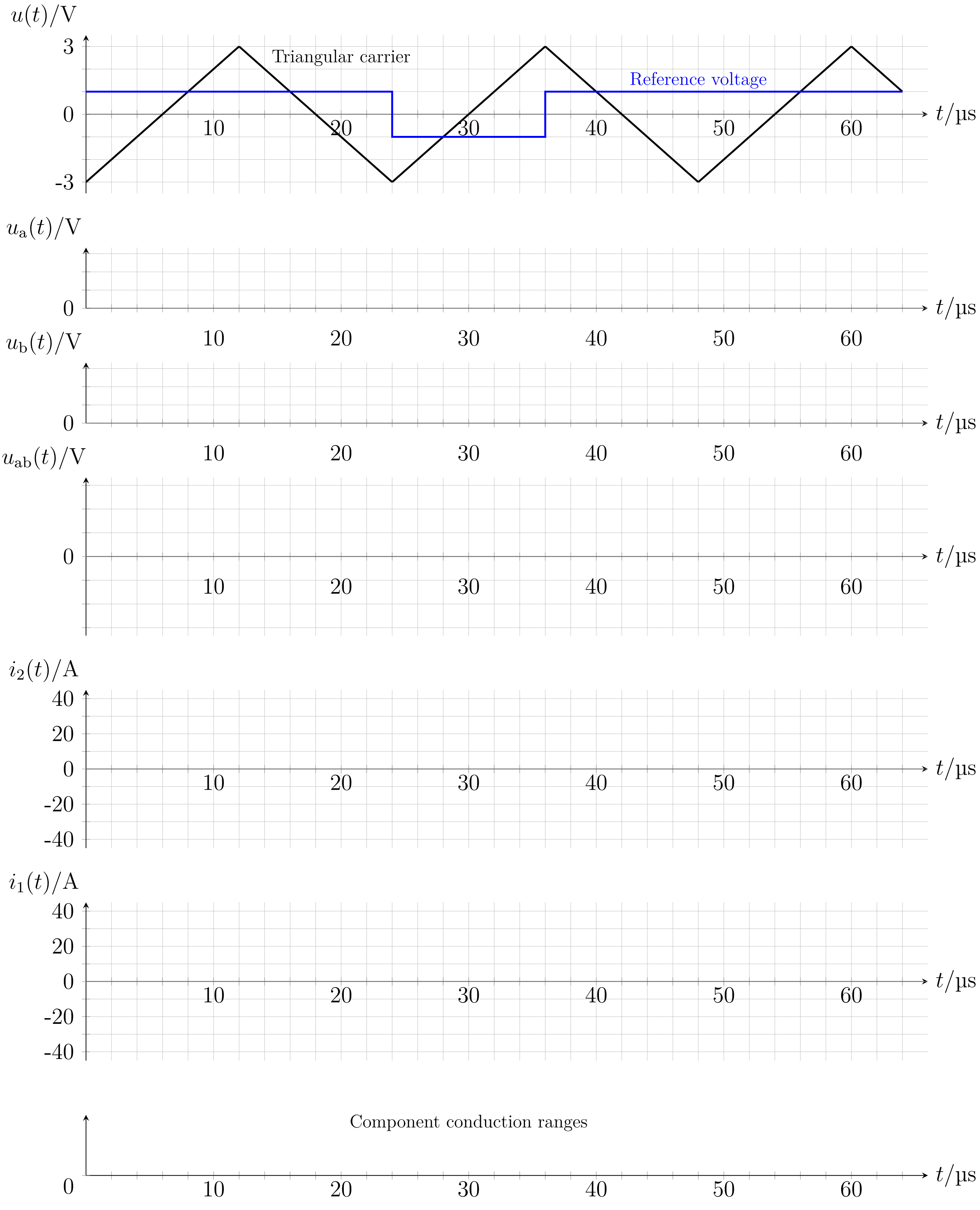
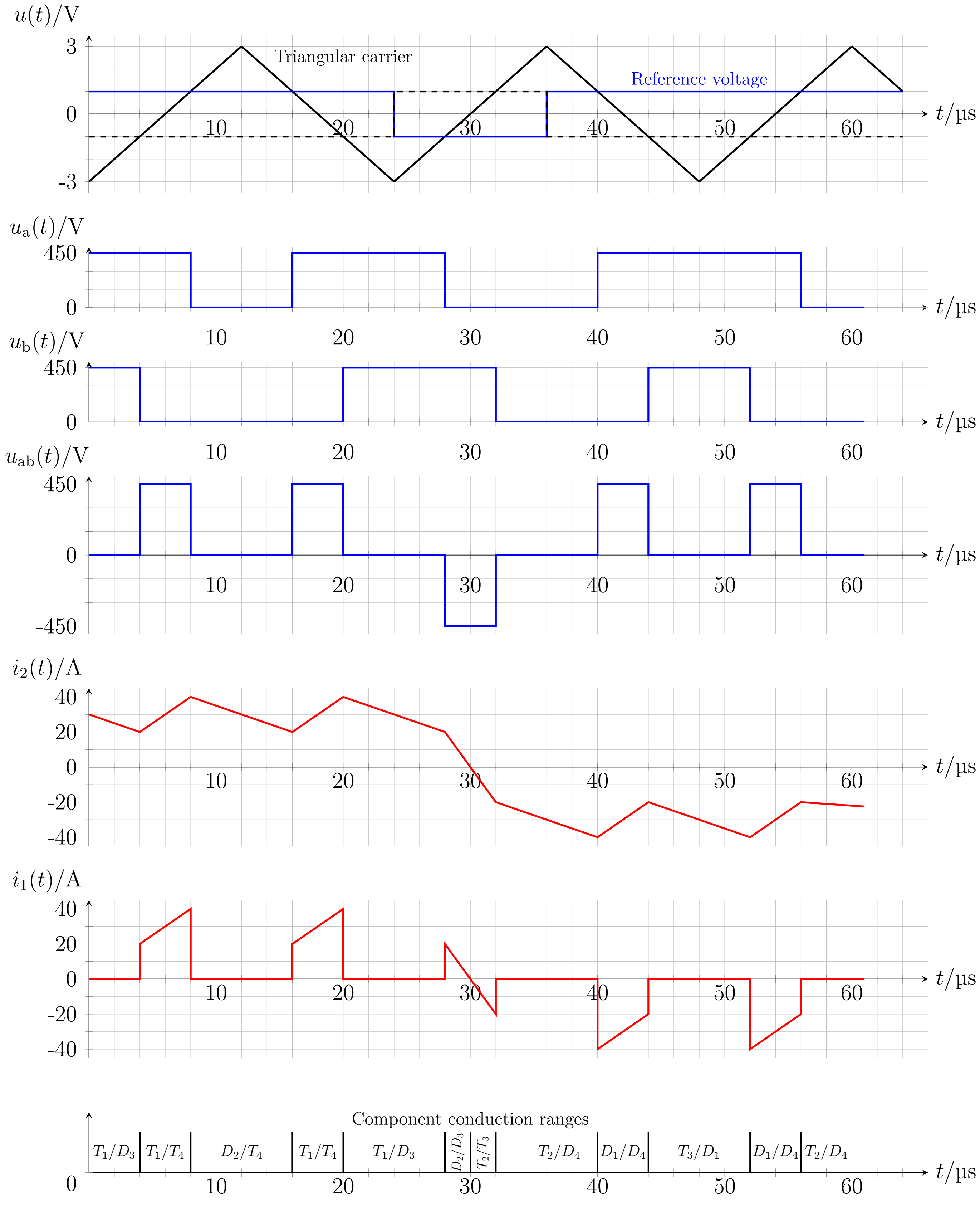


Figure 3: Relevant voltage and current signals of the four-quadrant converter.



Solution Figure 2: Relevant voltage and current signals of the four-quadrant converter.

Task 3: Line-commuted three-phase rectifier **[10 Points]**

A controlled three-phase midpoint rectifier (M3C) charges a battery of an electric motorbike, with $R = 0.1\ \Omega$ modeling the internal resistance of the battery, and $U_\mathrm{batt} = 125$ V as the battery voltage. An inductor filter L is used to smooth the output current I_2, with an inductance that is, initially, assumed to be infinitely large. The ideal transformer in the converter is connected to a symmetrical three-phase grid with an effective phase voltage $U_\mathrm{N} = 230$ V and line-to-line voltage of $U_\mathrm{N,LL} = 400$ V. The phase voltage on the secondary side of the transformer has effective value of $U_\mathrm{1,i} = 230$ V, $\forall i = \mathrm{a, b, c}$. The switching components are assumed to be ideal.

Ein gesteuerter Dreiphasen-Mittelpunkt-Gleichrichter (M3C) lädt die Batterie eines Elektromotorrads, wobei $R = 0,1\ \Omega$ den Innenwiderstand der Batterie modelliert und $U_\mathrm{batt} = 125$ V die Batteriespannung ist. Ein Induktionsfilter L wird zur Glättung des Ausgangsstroms I_2 verwendet, wobei die Induktivität zunächst als unendlich groß angenommen wird. Der ideale Transformator im Konverter ist an ein symmetrisches dreiphasiges Netz mit einer effektiven Phasenspannung $U_\mathrm{N} = 230$ V und einer Leiter-Leiter-Spannung von $U_\mathrm{N,LL} = 400$ V angeschlossen. Die Phasenspannung auf der Sekundärseite des Transformators hat einen Effektivwert von $U_\mathrm{1,i} = 230$ V, $\forall i = \mathrm{a, b, c}$. Die Schaltkomponenten sind als ideal anzunehmen.

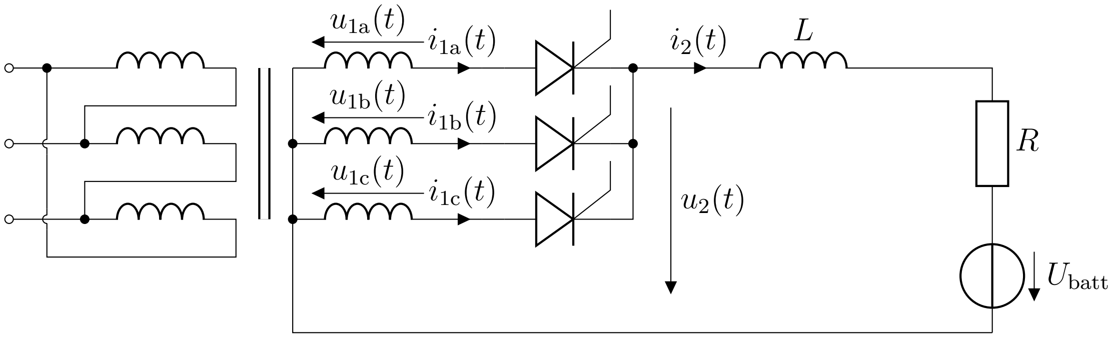


Figure 4: M3C rectifier used for battery charging.

3.1 Draw the output voltage signal $u_2(t)$ for the control angle $\alpha = \frac{\pi}{3}$ into Fig. 5. [2 Points]

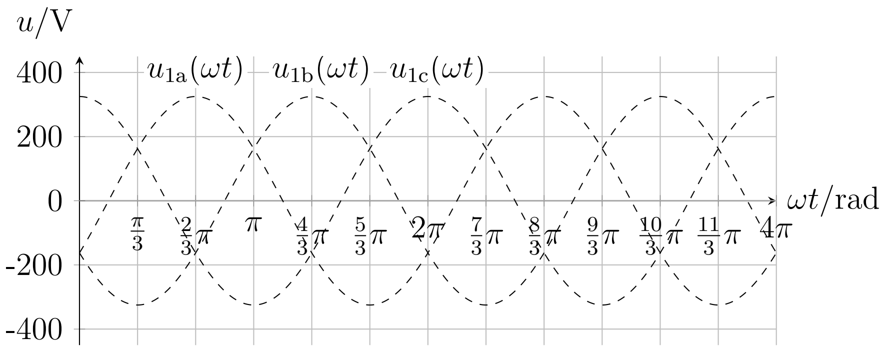
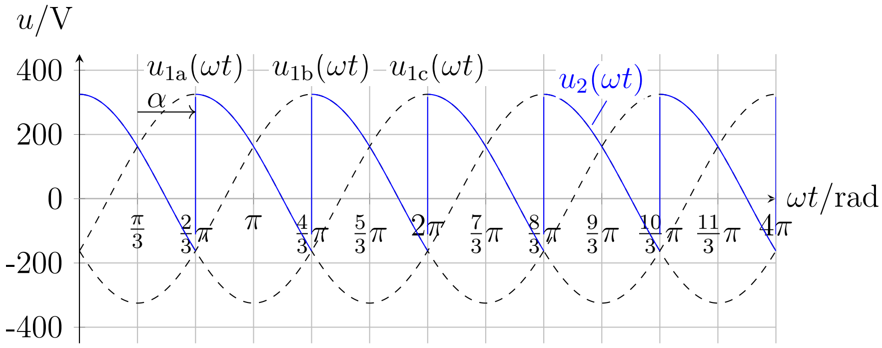


Figure 5: Output voltage $u_2(t)$ for $\alpha = \frac{\pi}{3}$.

Answer:



Solution Figure 3: Output voltage $u_2(t)$ for $\alpha = \frac{\pi}{3}$.

3.2 For the same α, calculate the average output voltage $\bar{u}_2$. [2 Points]

Answer:

Recalling the average output voltage equation in CCM:

$$\bar{u}_2(\alpha) = \frac{3\sqrt{3}}{2\pi}\hat{u}_1 \cos(\alpha),$$

where the peak value $\hat{u}_1$ can be calculated as:

$$\hat{u}_1 = \sqrt{2}U_1 = \sqrt{2} \cdot 230\ \text{V} \approx 325.27\ \text{V}.$$

Hence, the average output voltage is:

$$\bar{u}_2(\alpha = \frac{\pi}{3}) = \frac{3\sqrt{3}}{2\pi} \cdot 325.27\ \text{V} \cdot \cos(\frac{\pi}{3}) \approx 134.5\ \text{V}.$$

3.3 Calculate the corresponding average load current I_2. [1 Point]

Answer:

The average output voltage can be written as:

$$\bar{u}_2(\alpha) = I_2 R + U_{\text{batt}}.$$

Accordingly, the average output current can be calculated as:

$$I_2 = \frac{\bar{u}_2(\alpha) - U_{\text{batt}}}{R} = \frac{134.5\ \text{V} - 125\ \text{V}}{0.1\ \Omega} = 95\ \text{A}.$$

3.4 Calculate the power loss in the resistor and the power delivered to the battery U_{batt}. [2 Points]

Answer:

The power dissipated by the resistor can be calculated by:

$$P_{\text{resistor}} = I_2^2 \cdot R = (95\ \text{A})^2 \cdot 0.1\ \Omega = 902.5\ \text{W},$$

or using the voltage across the resistor by:

$$P_{\text{resistor}} = (\bar{u}_2(\alpha = \frac{\pi}{3}) - U_{\text{batt}}) \cdot I_2 = (134.5\ \text{V} - 125\ \text{V}) \cdot 95\ \text{A} = 902.5\ \text{W},$$

while the power delivered to U_{batt} is:

$$P_{\text{batt}} = U_{\text{batt}} \cdot I_2 = 125\ \text{V} \cdot 95\ \text{A} = 11.875\ \text{kW}.$$

3.5 Consider the case where the inductance L is finite, such that the converter operates in DCM. For an output voltage u_2 where $\alpha = \frac{\pi}{3}$ and $\beta = \frac{\pi}{2}$, as shown in Fig. 6, calculate the average output voltage $\bar{u}_2$. [3 Points]

Hint: In this question, the DCM is directly influenced by the presence of the battery voltage, which is different from having a capacitive filter at the output.

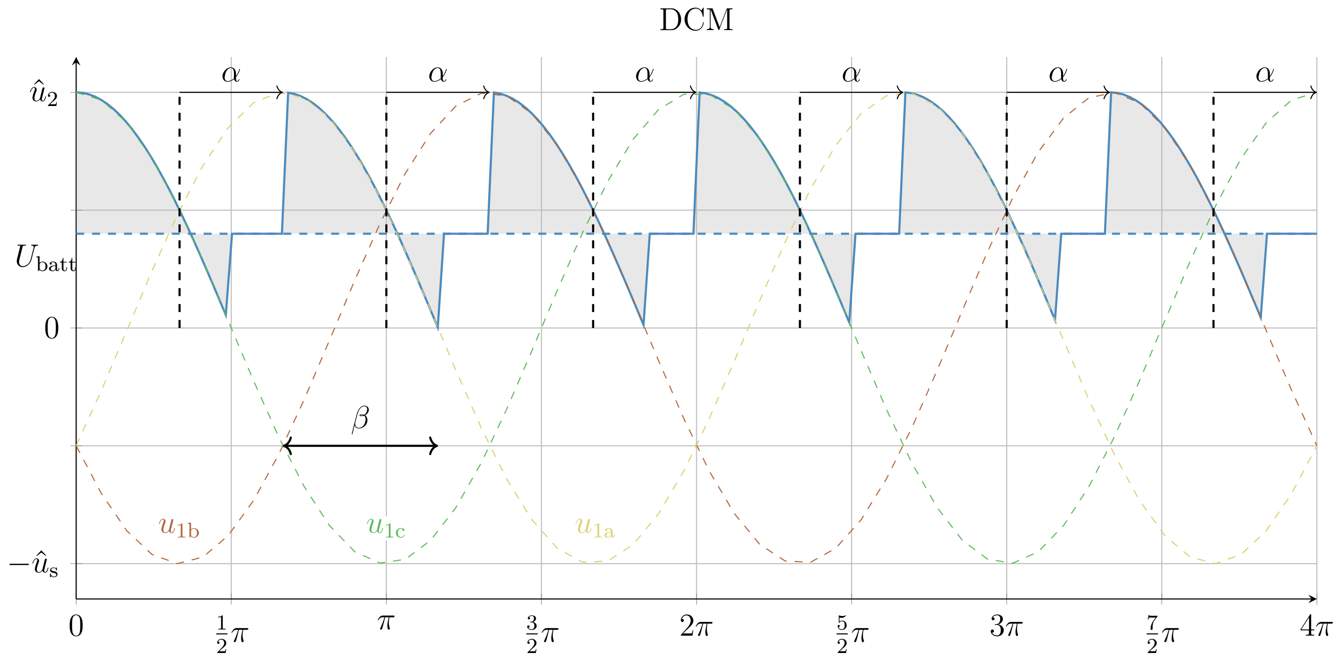


Figure 6: M3C voltages in DCM.

Answer:

In DCM mode, the inductor's current falls below the thyristors holding current, resulting in all thyristors stop conducting during certain intervals within the cycle. Since the output voltage during the non-conducting intervals is $u_2(t) = U_{\text{batt}}$, the average output voltage can be calculated as:

$$\bar{u}_2(\beta) = \frac{3}{2\pi}\left[\int_0^{\beta} \hat{u}_1 \cos(\omega t)\mathrm{d}\omega t + \int_{\beta}^{\frac{2\pi}{3}} U_{\text{batt}}\mathrm{d}\omega t\right].$$

After integration, we get

$$\bar{u}_2(\beta) = \frac{3}{2\pi}\left[\hat{u}_1(\sin(\beta) - \sin(0)) + U_{\mathrm{batt}}(\frac{2\pi}{3} - \beta)\right].$$

So, for the given $\beta = \frac{\pi}{2}$, the average output voltage is

$$\bar{u}_2(\beta) = \frac{3}{2\pi}\left[325.27\ \mathrm{V}\cdot(\sin(\frac{\pi}{2}) - \sin(0)) + 125\ \mathrm{V}\cdot(\frac{2\pi}{3} - \frac{\pi}{2})\right] \approx 186.55\ \mathrm{V}.$$

Task 4: Single-phase active front end rectifier **[12 Points]**

The circuit shown in Fig. 7 is a single-phase DC converter. It supplies the DC link of a locomotive, from which the traction motors are fed from the mains side. The converter uses PWM for modulation. All components are ideal, the voltage U_1 is constant.

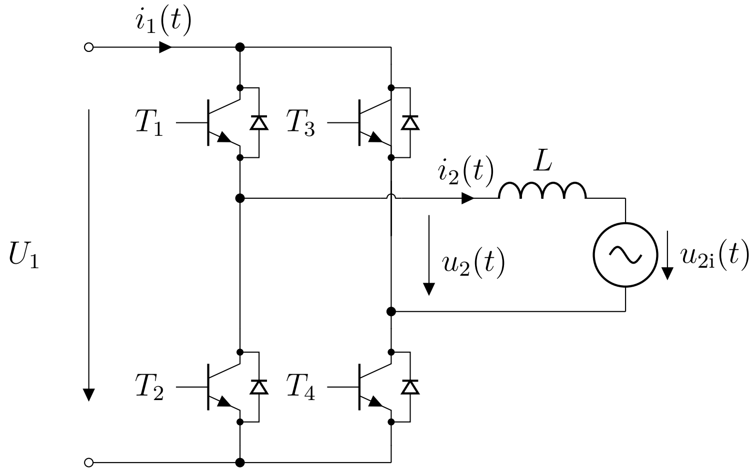


Figure 7: Single-phase AC-DC converter.

Table 3: Parameters of the single-phase AC-DC converter.

DC-link voltage:	$U_1 = 1400$ V	Grid voltage:	$u_{2\mathrm{i}} = 1200\ \mathrm{V} \cdot \sin(\omega t)$
Grid frequency:	$f = 16\frac{2}{3}$ Hz	Line filter:	$L = 2.7$ mH

4.1 Qualitatively add into Fig. 8 the fundamental components $u_2^{(1)}(t)$, $u_\mathrm{L}^{(1)}(t)$, and $i_2^{(1)}(t)$ for different operating modes of the locomotive: [3 Points]

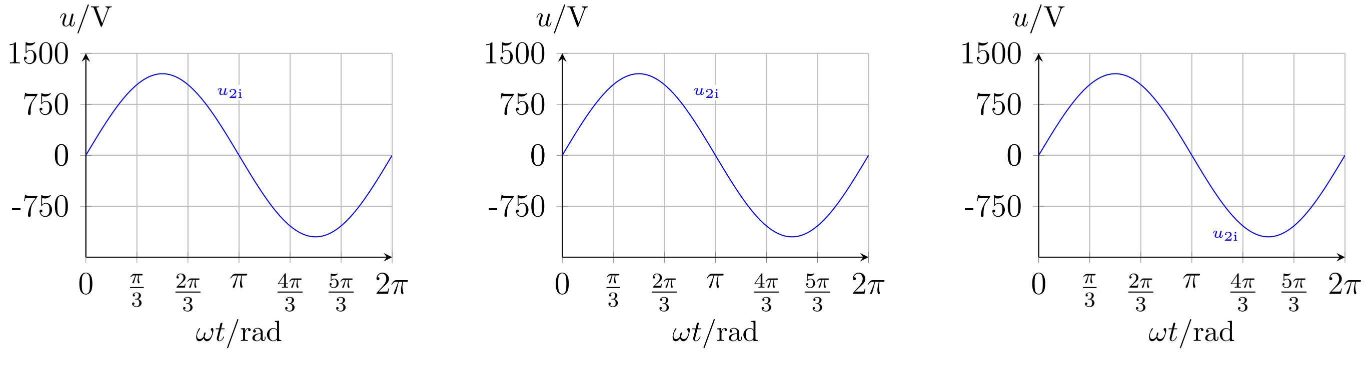
- starting (locomotive draws pure active power from the grid),
- rolling (locomotive draws neither active nor reactive power),
- and braking (locomotive delivers pure active power).

Answer:

In starting case, since only active power is drawn current $i_2^{(1)}(t)$ will be in phase with $u_{2\mathrm{i}}$. Moreover, as inductors oppose changes in current (Lenz's law), the voltage across the inductor leads the current by $\frac{\pi}{2}$ rad. Applying KVL at the inductor side leads to:

$$u_2^{(1)} = u_\mathrm{L}^{(1)} + u_{2\mathrm{i}}.$$

Consequently, the resulting $u_2^{(1)}(t)$ will be leading $u_{2\mathrm{i}}$ by angle $0 < \varphi_2 < \frac{\pi}{2}$ rad, as shown in Fig. 5a.



(a) Starting (b) Rolling (c) Braking

Figure 8: Signals $u_2^{(1)}(t)$, $u_{\mathrm{L}}^{(1)}(t)$, $i_2^{(1)}(t)$ in different operating modes.

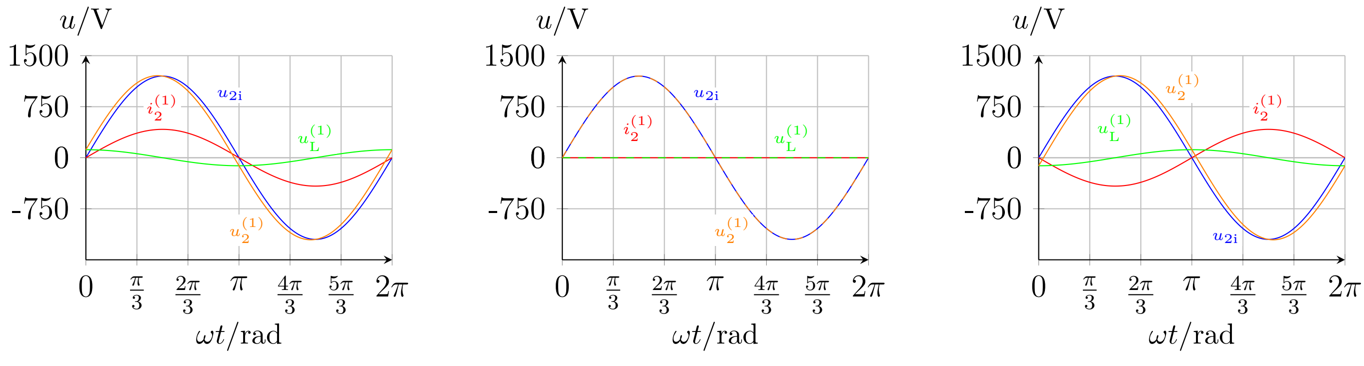
During rolling neither active nor reactive power is drawn by the locomotive. As a result, no current flows through the inductor $i_2^{(1)}(t) = 0$. From the inductor current-voltage equation

$$u_{\mathrm{L}}^{(1)} = L\frac{\mathrm{d}i_2^{(1)}(t)}{\mathrm{d}t},$$

we can see that the voltage $u_{\mathrm{L}}^{(1)}$ will be zero as well. Observing $u_2^{(1)} = u_{\mathrm{L}}^{(1)} + u_{2\mathrm{i}}$, this would lead to $u_2^{(1)} = 0 + u_{2\mathrm{i}} = u_{2\mathrm{i}}$.

Lastly, while braking the locomotive delivers pure active power. This means current direction is reversed, hence, shifted from $u_{2\mathrm{i}}$ by angle π rad. From Lenz's law, we can say that $u_{\mathrm{L}}^{(1)}$ will be leading by $\frac{\pi}{2}$ rad. As a result, $u_2^{(1)}(t)$ will be lagging by angle $0 < \varphi_2 < \frac{\pi}{2}$ rad, see Fig. 5c.

The corresponding time-domain plots are shown in Fig. 4



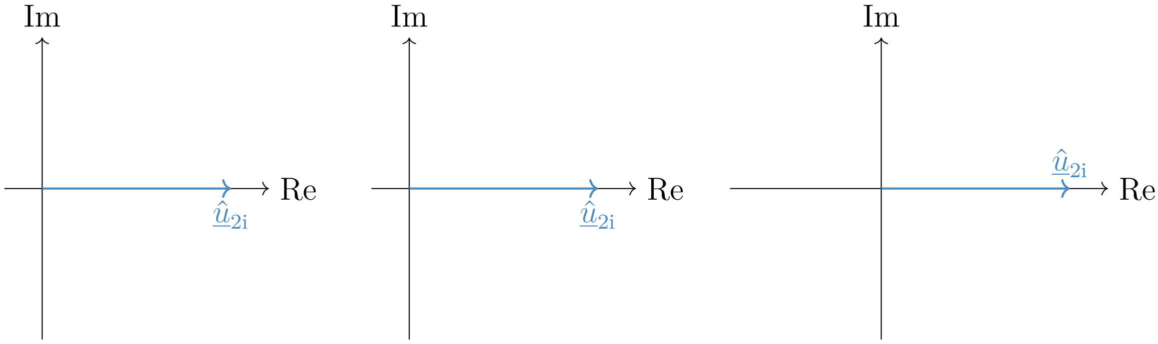
(a) Starting (b) Rolling (c) Braking

Solution Figure 4: Signals $u_2^{(1)}(t)$, $u_{\mathrm{L}}^{(1)}(t)$, $i_2^{(1)}(t)$ in different operating modes.

4.2 Draw the corresponding complex phasors for the same quantities in Fig. 9. [3 Points]

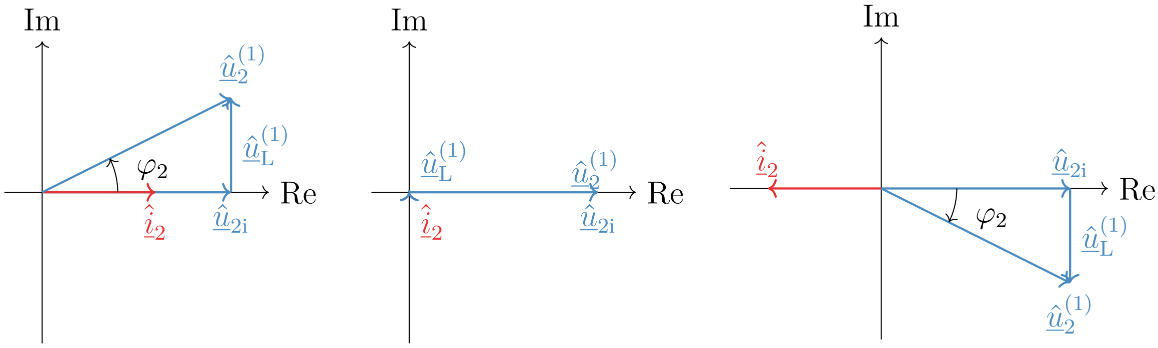
Answer:

The solution is shown in Sol.-Fig. 5.



(a) Starting (b) Rolling (c) Braking

Figure 9: Steady-state phasor diagrams for $\underline{u}_2^{(1)}$, $\underline{u}_L^{(1)}$, $\underline{i}_2^{(1)}$ in different operating modes.



(a) Starting (b) Rolling (c) Braking

Solution Figure 5: Steady-state phasor diagrams.

4.3 How large must the amplitude of the mains fundamental current $\hat{i}_2^{(1)}$ be if pure fundamental active power of 250 kW is drawn from the grid? [2 Points]

Answer:

The fundamental active power can be calculated from the effective values of the mains voltage U_{2i} as well as fundamental current $I_2^{(1)}$ using:

$$P^{(1)} = U_{2i} \cdot I_2^{(1)} = \frac{\hat{u}_{2i} \cdot \hat{i}_2^{(1)}}{2}.$$

Solving with respect to $\hat{i}_2^{(1)}$ results in:

$$\hat{i}_2^{(1)} = \frac{2P^{(1)}}{\hat{u}_{2i}} = \frac{2 \cdot 250\ \mathrm{kW}}{1200\ \mathrm{V}} \approx 417\ \mathrm{A}.$$

4.4 How large must the fundamental amplitude $\hat{u}_2^{(1)}$ of the inverter output voltage be in the same load case of 250 kW? Calculate the corresponding modulation index m. [2 Points]

Answer:

Recalling:

$$u_2^{(1)} = u_L^{(1)} + u_{2i}.$$

The amplitude of the inductor fundamental voltage $\hat{u}_{\mathrm{L}}^{(1)}$ can be calculated from $\hat{i}_2^{(1)}$ as:

$$\hat{u}_{\mathrm{L}}^{(1)} = \hat{i}_2^{(1)}\omega L = 417\ \mathrm{A} \cdot (2\pi 16\frac{2}{3})\ \mathrm{rad/s} \cdot 2.7\ \mathrm{mH} \approx 118\ \mathrm{V}.$$

Hence, the corresponding fundamental amplitude of the output voltage is:

$$\hat{u}_2^{(1)} = \sqrt{(\hat{u}_{\mathrm{L}}^{(1)})^2 + (\hat{u}_{2\mathrm{i}})^2} = \sqrt{(118\ \mathrm{V})^2 + (1200\ \mathrm{V})}^2 \approx 1205.8\ \mathrm{V}.$$

While the corresponding modulation index is

$$m = \frac{\hat{u}_2^{(1)}}{U_1} = \frac{1205.8\ \mathrm{V}}{1400\ \mathrm{V}} \approx 0.861.$$

4.5 Due to a semiconductor defect, there is a short circuit in the inverter (all transistors conduct). What is the active and reactive power drawn from the grid in this case? [2 Points]

Answer:

$$P = 0\ \mathrm{W}, \quad Q = \frac{U_{2\mathrm{i}}^2}{\omega L} = \frac{(\frac{1200}{\sqrt{2}}\ \mathrm{V})^2}{(2\pi \cdot 16\frac{2}{3})\ \mathrm{rad/s} \cdot 2.7\ \mathrm{mH}} \approx 2.546\ \mathrm{MVA}.$$