Exam

Power Electronics

Summer 2025

First name: Last name:

Matriculation number: Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 90 minutes.

Evaluation:

Task	1	2	3	4	$\sum$
Maximum score	7	6	15	14	42
Achieved score					

Task 1: Step-up converter **[7 Points]**

In an industrial control system, a stable voltage of 24 V is available. However, some devices require 48 V. Hence, a step-up converter shall be designed for a load current according Tab. 1.

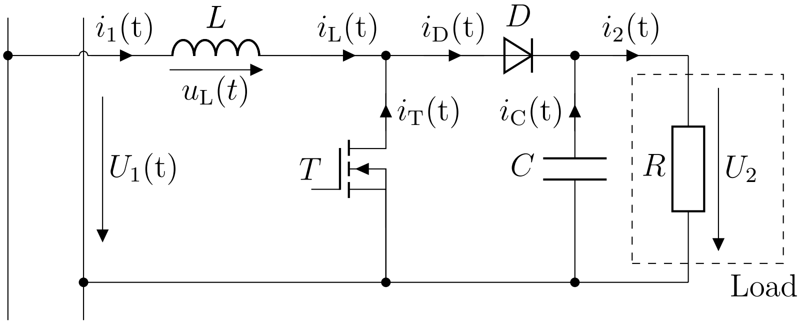


Fig. 1: Step-up converter with filter capacitor.

General parameters:		**IGBT and diode:**	
Input voltage:	$U_1 = 24$ V	Drain-source voltage:	$U_{\mathrm{on,DS}} = 1.2$ V
Output voltage:	$U_2 = 48$ V	Output current range:	$I_2 = 2$ A … 20 A
Switching frequency:	$f_s = 100$ kHz	Diode forward voltage:	$U_{\mathrm{D,f}} = 0.81$ V

The switching losses of the semiconductors are negligible.
The filter capacitor C is to be considered only for subtask 1.2.

Tab. 1: Parameters of the circuit.

1.1 Calculate the duty cycle and the minimal inductance in case of ideal components (no voltage drop over transistor and diode), when the converter operates in BCM at minimum load. [2 Points]

Answer:

The duty cycle corresponds to

$$\frac{U_2}{U_1} = \frac{1}{1-D}.$$

Solving this equation with respect to D yields

$$D = \frac{U_2 - U_1}{U_2} = \frac{48\text{ V} - 24\text{ V}}{48\text{ V}} = 0.5.$$

The average current $\bar{i}_{1,\mathrm{min}}$ is calculated by

$$\bar{i}_{1,\mathrm{min}} = \frac{\bar{I}_{2,\mathrm{min}}}{1-D} = \frac{2\text{ A}}{0.5} = 4\text{ A}.$$

The ripple current Δi_L yields

$$\Delta i_L = \frac{U_1 \cdot T_{\mathrm{on}}}{L} = \frac{U_1 \cdot D}{L \cdot f_s}.$$

In case of boundary conduction mode following equation is to apply:

$$\Delta i_\mathrm{L} = 2 \cdot \bar{i}_{1,\mathrm{on}} = 2 \cdot\ 4\ \mathrm{A} = 8\ \mathrm{A}.$$

The value of the inductance is to be calculated with

$$\Delta i_\mathrm{L} = \frac{U_1}{L} \cdot T_\mathrm{on} = \frac{U_1}{L} \cdot \frac{D}{f_\mathrm{s}}.$$

Solving this equation with respect to L yields

$$L = \frac{U_1}{\Delta i_\mathrm{L}} \cdot \frac{D}{f_\mathrm{s}} = \frac{24\ \mathrm{V}}{8\ \mathrm{A}} \cdot \frac{0.5}{100\ \mathrm{kHz}} = 15\ \mathrm{\mu H}.$$

1.2 Some devices within the 48 V group tolerate an overvoltage of 5 %. Calculate the capacity of the smoothing capacitor C to keep this limit assuming a constant load current I_2. What is the maximum voltage that can occur at the capacitor? [2 Points]

Answer:
The maximum allowed voltage ripple results in:

$$\Delta u_\mathrm{c} = 0.05 \cdot U_2 = 0.05 \cdot 48\ \mathrm{V} = 2.4\ \mathrm{V}.$$

The voltage drop of the capacitor corresponds to the integral of the capacitor current and the time, when the transistor is active:

$$\Delta u_\mathrm{c} = \int_0^{T_\mathrm{on}} \frac{i_\mathrm{c}(t)}{C}\,\mathrm{d}t = \frac{I_2 D}{C f_\mathrm{s}}.$$

The highest load current of the capacitor corresponds to the highest negative output current, when the transistor is active. This leads to:

$$\Delta u_\mathrm{c} = \frac{I_{2,\mathrm{max}} D}{C f_\mathrm{s}}.$$

Solving the previous equation to C yields

$$C = \frac{I_{2,\mathrm{max}} D}{\Delta u_\mathrm{c} f_\mathrm{s}} = \frac{20\ \mathrm{A} \cdot 0.5}{2.4\ \mathrm{V} \cdot 100\ \mathrm{kHz}} = 41\ \mathrm{\mu F}.$$

The maximal voltage is calculated by

$$u_\mathrm{c,max} = U_2 + \frac{\Delta u_\mathrm{c}}{2} = 48\ \mathrm{V} + \frac{2.4\ \mathrm{V}}{2} = 49.2\ \mathrm{V}.$$

1.3 Calculate the duty cycle and the minimal inductance in case of ideal components, but consider the voltage drop over the transistor and the diode. The converter shall operates in BCM at minimum load. [2 Points]

Answer:

If the transistor is active the voltage at the inductance is calculated by

$$U_{\mathrm{L,on}} = U_1 - U_{\mathrm{on,DS}} = 24\ \mathrm{V} - 2\ \mathrm{V} = 22.8\ \mathrm{V}.$$

If the transistor is blocking the voltage at the inductance yields

$$U_{\mathrm{L,off}} = U_1 - U_{\mathrm{D,f}} - U_2 = 24\ \mathrm{V} - 0.81\ \mathrm{V} - 48\ \mathrm{V} = -24.81\ \mathrm{V}.$$

In steady state, the inductor's voltage-time integral must be zero over one switching period:

$$\frac{U_{\mathrm{L,on}}T_{\mathrm{on}} + U_{\mathrm{L,off}}T_{\mathrm{off}}}{L} = 0.$$

The substitution of T_{on} and T_{off} by the switching frequency and the duty cycle leads to

$$U_{\mathrm{L,on}}\frac{D}{f_{\mathrm{s}}} + U_{\mathrm{L,off}}\frac{1-D}{f_{\mathrm{s}}} = 0.$$

Solving this equation with respect to D yields

$$D = \frac{-U_{\mathrm{L,off}}}{U_{\mathrm{L,on}} - U_{\mathrm{L,off}}} = \frac{-24.81\ \mathrm{V}}{-24.81\ \mathrm{V} - 22.8\ \mathrm{V}} = 0.52.$$

The average current $\bar{i}_1$ is expressed by

$$\bar{i}_{1,\mathrm{min}} = \frac{\bar{i}_{2,\mathrm{min}}}{1-D} = \frac{2\ \mathrm{A}}{1-0.52} = 4.18\ \mathrm{A}.$$

In case of boundary conduction mode following equation is to apply:

$$\Delta i_{\mathrm{L}} = 2 \cdot \bar{i}_{1,\mathrm{min}} = 2 \cdot\ 4.18\ \mathrm{A} = 8.35\ \mathrm{A}.$$

The ripple current Δi_{L} yields

$$\Delta i_{\mathrm{L}} = \frac{U_{\mathrm{L,on}} \cdot T_{\mathrm{on}}}{L} = \frac{U_{\mathrm{L,on}} \cdot D}{L \cdot f_{\mathrm{s}}}.$$

Solving this equation with respect to L yields

$$L = \frac{U_{\mathrm{L,on}} \cdot D}{\Delta i_{\mathrm{L}} \cdot f_{\mathrm{s}}} = \frac{22.8\ \mathrm{V}}{8.35\ \mathrm{A}} \cdot \frac{0.52}{100\ \mathrm{kHz}} = 14.2\ \mathrm{\mu H}.$$

1.4 Calculate the efficiency of the step-up converter at maximum load current considering the transistor and diode forward losses. Moreover, sketch the curve of the voltage drop $u_{\mathrm{L}}(t)$ and the curves of the currents $i_{\mathrm{D}}(t)$ and $i_{\mathrm{L}}(t)$ in the diagrams below for this operating point and add the y-labels (assume U_2 as constant). [1 Point]

Fig. 2: Relevant voltage and current signals.

Answer:

The input current

$$\bar{i}_{1,\mathrm{max}} = \frac{\bar{i}_{2,\mathrm{max}}}{1-D} = \frac{20\ \mathrm{A}}{1-0.52} = 41.8\ \mathrm{A}.$$

The efficiency of the step-up converter is calculated by

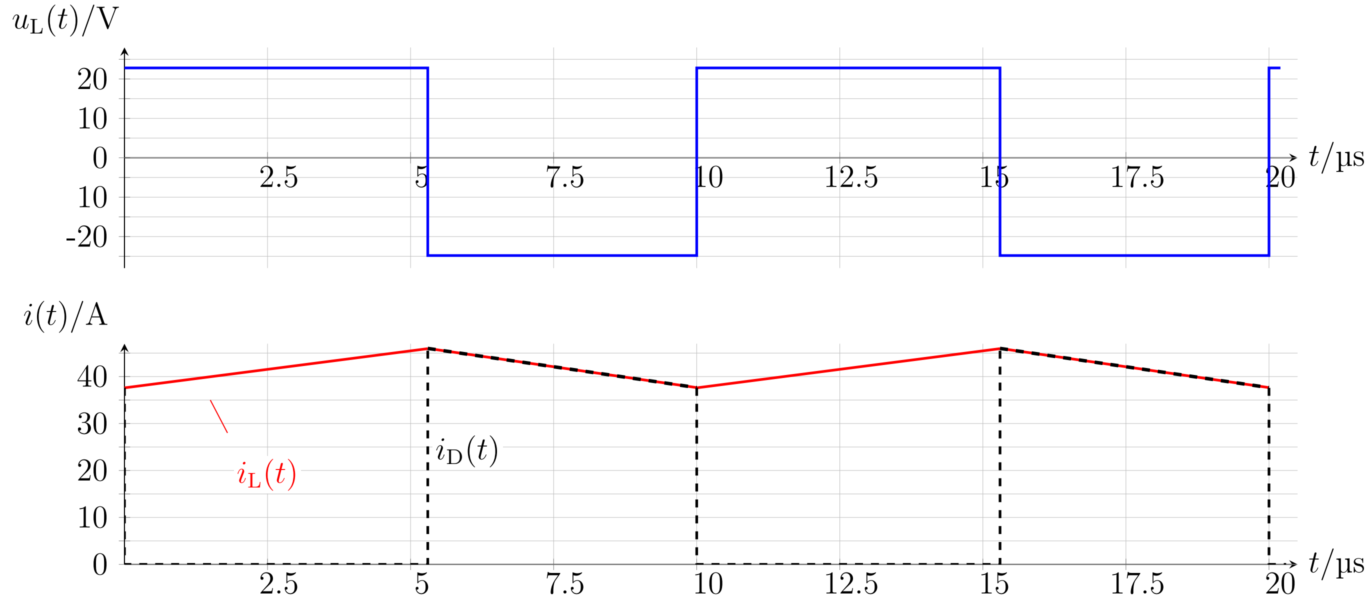
$$\eta = \frac{P_2}{P_1} = \frac{U_2 I_2}{U_1 \bar{i}_\mathrm{L}} = \frac{48\ \mathrm{V}\cdot 20\ \mathrm{A}}{24\ \mathrm{V}\cdot 41.8\ \mathrm{A}} = 0.96.$$

The values for $u_\mathrm{L}(t)$ are taken from subtask 1.3 ($U_\mathrm{L,on}$ and $U_\mathrm{L,off}$). The maximum and minimum current of the inductance results in

$$\bar{i}_\mathrm{L,max} = \bar{i}_\mathrm{1,max} + \frac{\Delta i_\mathrm{L}}{2} = 41.8\ \mathrm{A} + \frac{8.35\ \mathrm{A}}{2} = 46\ \mathrm{A}$$

and

$$\bar{i}_\mathrm{L,max} = \bar{i}_\mathrm{1,max} - \frac{\Delta i_\mathrm{L}}{2} = 41.8\ \mathrm{A} - \frac{8.35\ \mathrm{A}}{2} = 37.6\ \mathrm{A}.$$



Solution Fig. 1: Relevant voltage and current signals.

Task 2: Multi-port (flyback) converter for the production area [6 Points]

In a modern industrial automation system, a central 12 V DC power supply is used to power distributed sensor units and actuators throughout a production area. While many actuators and some programmable logic controllers (PLC modules) operate directly on 12 V, some sensors (e.g., proximity switches, temperature sensors, camera modules) require a stable voltage of 5 V or 3.3 V. Therefore, a multi-port (flyback) converter with the parameters according Tab. 2 is used.

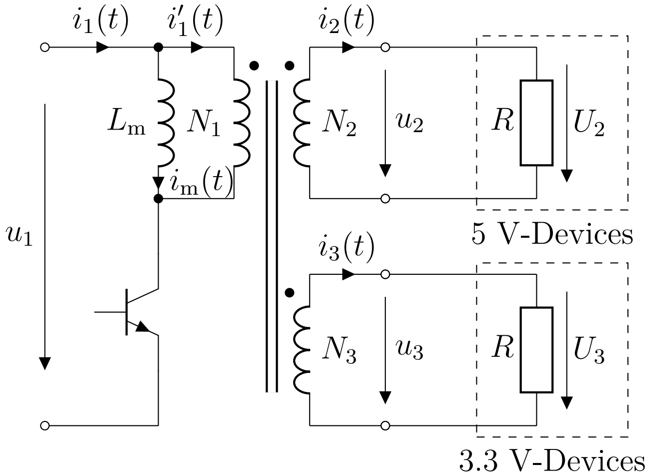


Fig. 3: Multi-port (flyback) converter.

Input voltage:	$U_1 = 12$ V	Switching frequency:	$f_\mathrm{s} = 100$ kHz
Output voltage:	$U_2 = 5$ V	Min. output current:	$I_{2,\mathrm{min}} = 2$ A
Output voltage:	$U_3 = 3.3$ V	Min. output current:	$I_{3,\mathrm{min}} = 3$ A
Duty cycle:	0.2	Primary turns	$N_1 = 2040$
All components are ideal. No losses need to be considered.			

Tab. 2: Parameters of the multi-port flyback converter.

2.1 Calculate the number of turns N_2 and N_3 for the two outputs. [2 Points]

Answer:

The voltage ratio depends on the duty cycle and turn ratio

$$\frac{U_2}{U_1} = \frac{N_2}{N_1}\frac{D}{1-D}.$$

Solving the equation with respect to N_2 yields

$$N_2 = \frac{U_2}{U_1}N_1\frac{1-D}{D} = \frac{5\ \mathrm{V}}{12\ \mathrm{V}}2040\frac{0.2}{1-0.2} = 3400.$$

In the same N_3 yields

$$N_3 = \frac{U_3}{U_1}N_1\frac{1-D}{D} = \frac{3.3\ \mathrm{V}}{12\ \mathrm{V}}2040\frac{1-0.2}{0.2} = 2244.$$

2.2 Calculate the required magnetizing inductance L_m so that the multi-port flyback converter is in BCM at minimum current consumption. [3 Points]
Hint: the input current i_1 is composed of the transformed current components from the 5 V output and from the 3.3 V output according to $i_1 = i_{1,\mathrm{port1}} + i_{1,\mathrm{port2}}$.

Answer:
At boundary conduction mode the average of magnetizing current i_Lm corresponds to the half of its delta and and the magnetic current can be calculated with help of the input current. The average input current is calculated by the sum of both output currents, which are to be transformed by the turn ratio and the duty cycle. For boundary conduction mode the minimum output current of each port is to apply:

$$\bar{i}_{1,\mathrm{min}} = \frac{N_2}{N_1}\frac{1-D}{D}\bar{i}_{2,\mathrm{min}} + \frac{N_3}{N_1}\frac{1-D}{D}\bar{i}_{3,\mathrm{min}} = \frac{3400}{2040}\frac{1-0.2}{0.2}2\ \mathrm{A} + \frac{2244}{2040}\frac{1-0.2}{0.2}3\ \mathrm{A} = 1.68\ \mathrm{A}.$$

The relation between average of the magnetizing current $\bar{i}_\mathrm{Lm}$ and the average of the input current yields

$$\bar{i}_\mathrm{L} \cdot T_\mathrm{on} = \bar{i}_1 \cdot T_\mathrm{s}.$$

The average of the magnetizing current i_Lm is calculated by

$$\bar{i}_\mathrm{L} = \frac{\bar{i}_1}{D} = \frac{1.68\ \mathrm{A}}{0.2} = 8.38\ \mathrm{A}.$$

At boundary conduction mode the ripple of magnetizing current i_Lm results in

$$\Delta i_\mathrm{Lm} = 2 \cdot \bar{i}_\mathrm{Lm} = 2 \cdot 8.38\ \mathrm{A} = 16.75\ \mathrm{A}.$$

The value of the inductance is calculate by

$$\Delta i_\mathrm{Lm} = \frac{U_1}{L} \cdot T_\mathrm{on} = \frac{U_1}{L_\mathrm{m}} \cdot \frac{D}{f_\mathrm{s}}.$$

Solving this equation with respect to L_m yields

$$L_\mathrm{m} = \frac{U_1}{\Delta i_\mathrm{Lm}} \cdot \frac{D}{f_\mathrm{s}} = \frac{12\ \mathrm{V}}{16.75\ \mathrm{A}} \cdot \frac{0.2}{100\ \mathrm{kHz}} = 1.43\ \mathrm{\mu H}.$$

2.3 Calculate the output voltage U_2 for the case, that all loads of the 3.3 V-output are disconnected, while minimum current $I_{2,\mathrm{min}}$ at 5 V port. What is the risk in this case? [1 Point]

Answer:
The voltage in DCM-mode for an flyback converter is calculated by

$$U_2 = U_1^2\frac{D^2 T_\mathrm{s}}{2L_\mathrm{m}\bar{i}_{2,\mathrm{min}}} = U_1^2\frac{D^2}{2L_\mathrm{m}\bar{i}_{2,\mathrm{min}}f_\mathrm{s}} = 12\ \mathrm{V}^2\frac{0.2^2}{2 \cdot 1.43\ \mathrm{\mu H} \cdot 2\ \mathrm{A} \cdot 100\ \mathrm{kHz}} = 10.05\ \mathrm{V}.$$

The voltage increase may leads to the damage of the 5 V devices.

Task 3: Line-commuted three-phase rectifier [15 Points]

An industrial conveyor system is powered by a DC motor that is supplied via a controlled three-phase midpoint rectifier (M3C). The motor draws a steady current I_2 and operates at a nominal induced motor voltage $U_{\mathrm{mot,ind}}$. The motor has an internal armature resistance R. To smooth the output current, a large inductor ($L \rightarrow \infty$) is placed such that CCM is maintained. The input to the rectifier is a symmetrical three-phase grid connected via an ideal transformer. On the secondary side, each phase has an effective voltage of $U_{1,\mathrm{i}} = 200$ V, $\forall i = \mathrm{a,b,c}$. All components (switches, transformer, etc.) are ideal.

Ein industrielles Fördersystem wird von einem Gleichstrommotor angetrieben, der über einen geregelten Dreiphasen-Mittelpunktgleichrichter (M3C) versorgt wird. Der Motor zieht einen konstanten Strom I_2 und weist eine induzierte Spannung $U_{\mathrm{mot,ind}}$ im Nennpunkt auf. Der Motor hat einen internen Ankerwiderstand R. Um den Ausgangsstrom zu glätten, wird eine große Induktivität ($L \rightarrow \infty$) so platziert, dass der CCM aufrechterhalten bleibt. Der Eingang des Gleichrichters ist ein symmetrisches Dreiphasennetz, das über einen idealen Transformator angeschlossen ist. Auf der Sekundärseite hat jede Phase eine effektive Spannung von $U_{1,\mathrm{i}} = 200$ V, $\forall i = \mathrm{a,b,c}$. Alle Komponenten (Schalter, Transformator usw.) sind ideal.

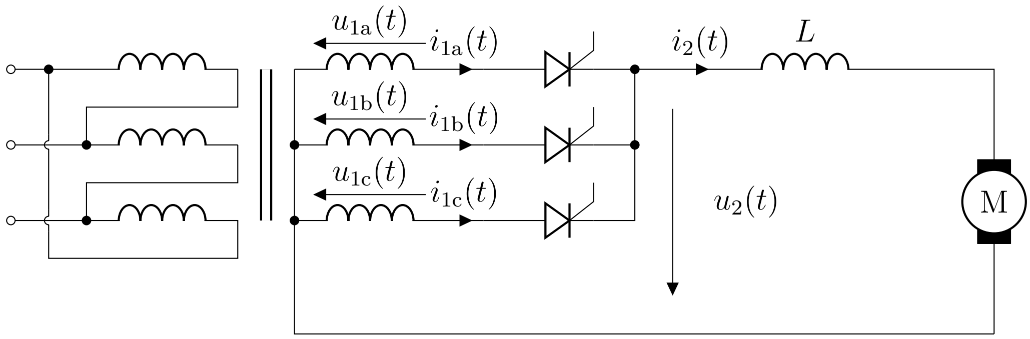


Fig. 4: M3C rectifier used for driving a DC motor.

Input voltages ($i = \mathrm{a,b,c}$):	$U_{1,i} = 200$ V (phase voltage)
	$U_{1,\mathrm{LL},i} = 346$ V (line-to-line voltage)
Nom. motor current:	$I_2 = 50$ A
Nom. motor induced voltage:	$U_{\mathrm{mot,ind}} = 150$ V
Motor internal resistance:	$R_{\mathrm{mot}} = 0.2\ \Omega$
Grid frequency:	$f = 50$ Hz

Tab. 3: Drive parameters

3.1 Calculate the required firing angle α to deliver the specified motor terminal voltage (induced + ohmic). Draw the output voltage signal $u_2(t)$ for this angle into Fig. 5. [3 Points]

Answer:

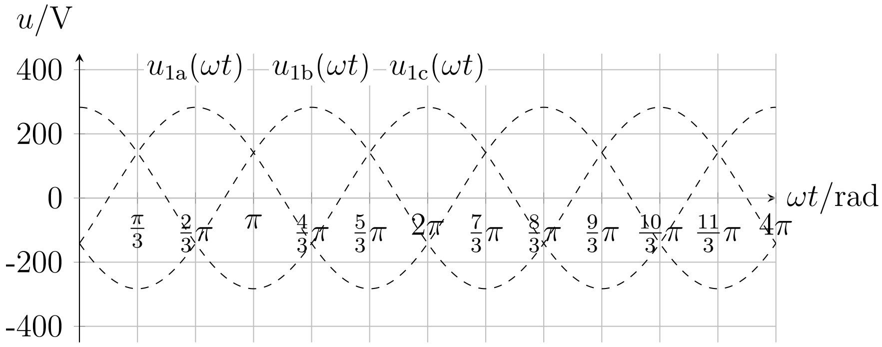


Fig. 5: Output voltage $u_2(t)$ for α.

First, the voltage drop across the internal resistance can be calculated with

$$U_\mathrm{R} = I_2 \cdot R = 50\ \mathrm{A} \cdot 0.2\ \Omega = 10\ \mathrm{V}.$$

Hence, the required rectifier output voltage is

$$\bar{u}_2 = U_\mathrm{mot,term} = U_\mathrm{mot,ind} + U_\mathrm{R} = 150\ \mathrm{V} + 10\ \mathrm{V} = 160\ \mathrm{V}.$$

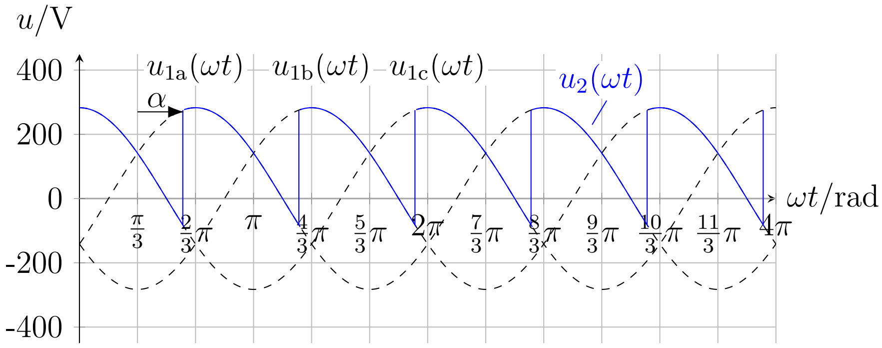
Also, the peak phase voltage is:

$$\hat{u}_1 = \sqrt{2} \cdot U_\mathrm{1,i} = \sqrt{2} \cdot 200\ \mathrm{V} \approx 282.84\ \mathrm{V}.$$

So, using the average voltage equation of B6C, α can be calculated as

$$160\ \mathrm{V} = \frac{3\sqrt{3}}{2\pi} \cdot 282.84\ \mathrm{V} \cdot \cos(\alpha) \Rightarrow \cos(\alpha) = \frac{160\ \mathrm{V} \cdot 2\pi}{3\sqrt{3} \cdot 282.84\ \mathrm{V}} \approx 0.684,$$

$$\alpha = \cos^{-1}(0.684) \approx 46.84°.$$



Solution Fig. 2: Output voltage $u_2(t)$ for $\alpha = 46.84$.

3.2 Explain how adjusting α affects the speed of the motor, especially $\alpha = \frac{\pi}{2}$ and $\alpha = 0$. [3 Points]

Hint: the motors' speed is proportional to the back EMF ($\omega \propto U_{\mathrm{mot,ind}}$).

Answer:

The average voltage applied to the motor is controlled by the firing angle α. The motor speed ω (assuming constant flux and neglecting armature reaction) is approximately proportional to the back EMF $U_{\mathrm{mot,ind}}$, which is itself proportional to the applied voltage minus internal resistance drop:

$$U_{\mathrm{mot,ind}} = \bar{u}_2 - I_2 R_{\mathrm{internal}}.$$

Since:

$$\omega \propto U_{\mathrm{mot,ind}} \Rightarrow \omega \propto \bar{u}_2 - I_2 R_{\mathrm{internal}},$$

and:

$$\bar{u}_2 = \frac{3\sqrt{2}}{\pi} \hat{u}_1 \cos(\alpha),$$

we conclude:

$$\omega \propto \cos(\alpha).$$

So increasing α reduces $\cos(\alpha)$, leading to lower $\bar{u}_2$ and thus lower motor speed. Conversely, reducing α increases speed. **Constraints on α:**

- α should not be too small (close to 0°), as this results in maximum $\bar{u}_2$ and could lead to overvoltage and overspeed conditions in the motor.
- α must be kept below 90°. At $\alpha = 90°$, $\cos(\alpha) = 0$, which gives zero output voltage.

3.3 Calculate the output power delivered to the motor, the power dissipated in the internal resistance, and the total power drawn from the rectifier output. [3 Points]

Answer:

- Motor power output:

$$P_{\mathrm{mot,ind}} = U_{\mathrm{mot,ind}} \cdot I_2 = 150\ \mathrm{V} \cdot 50\ \mathrm{A} = 7500\ \mathrm{W}.$$

- Power loss in internal resistance:

$$P_{\mathrm{R}} = I_2^2 \cdot R = 50^2\ \mathrm{A}^2 \cdot 0.2\ \Omega = 500\ \mathrm{W}.$$

- Total power drawn from rectifier:

$$P_{\mathrm{total}} = \bar{u}_2 \cdot I_2 = 160\ \mathrm{V} \cdot 50\ \mathrm{A} = 8000\ \mathrm{W}.$$

3.4 Sketch the input current $i_{1a}(\omega t)$ in Fig. 6, calculate the effective value of the fundamental component $I_{1a}^{(1)}$ and the corresponding total harmonic distortion (THD). [4 Points]
Hint: to calculate the THD, use

$$\mathrm{THD} = \sqrt{\left(\frac{I_{1a}^2}{(I_{1a}^{(1)})^2}\right) - 1},$$

where I_{1a} is the effective value of the phase current.

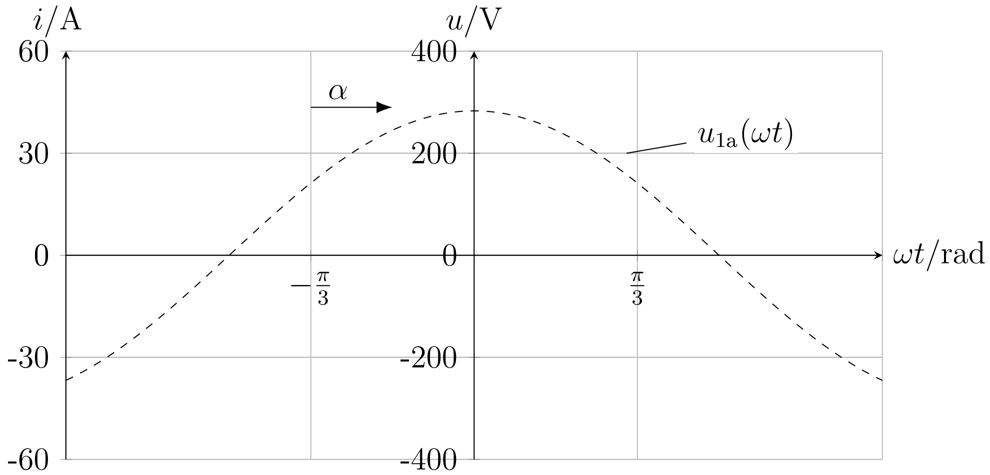
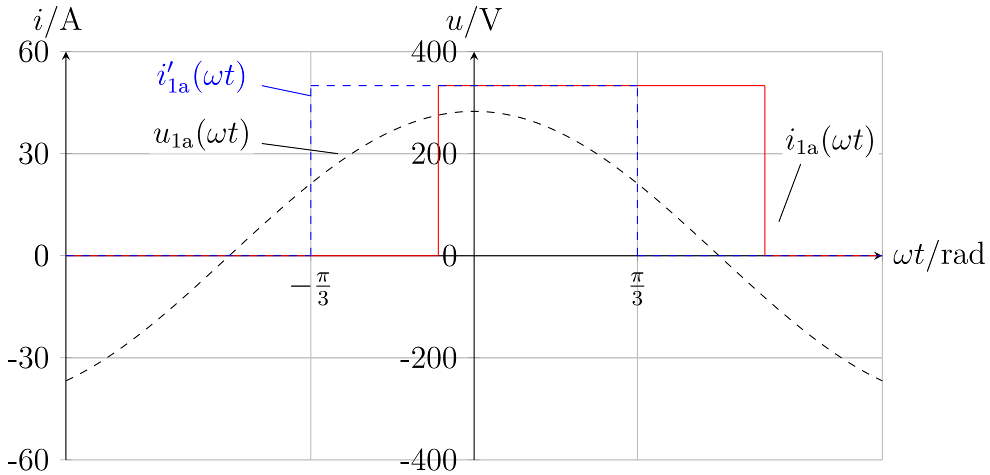


Fig. 6: Input current $i_{1a}(\omega t)$ and $u_{1a}(\omega t)$.

Answer:



Solution Fig. 3: Input current $i_{1a}(\omega t)$ and $u_{1a}(\omega t)$.

For simplification, we can position the y-xis in the middle of the conducting period of $i_{1a}(\omega t)$ signal and make use of the period's symmetry. This would simplify the calculations by integrating over one half of the period and then multiply by 2. One can see that the shifted signal $i'_{1a}(\omega t)$ is an even signal, hence, we expect that the fundamental component of the Fourier Series would contain only cosine component ($a_1 \neq 0$) and no sine component ($b_1 = 0$).

In this case the integration would be from 0 to $\frac{\pi}{3}$, as one conducting period is equal to $\frac{2\pi}{3}$. Thus, a_1 ($\hat{i}_{1a}^{(1)}$) can be calculated as:

$$a_1 = \hat{i}_{1a}^{(1)} = \frac{2}{\pi}\int_0^{\frac{\pi}{3}} I_2 \cos(\omega t)\mathrm{d}\omega t = \left[\frac{2}{\pi} I_2 \sin(\omega t)\right]_0^{\frac{\pi}{3}} = \frac{1}{\pi} I_2 \sqrt{3}.$$

Then the effective value $I_{1a}^{(1)}$ is

$$I_{1a}^{(1)} = \frac{\hat{i}_{1a}^{(1)}}{\sqrt{2}} \approx 19.49\ \mathrm{A}.$$

Moreover, the effective value of I_{1a} can be calculated as

$$I_{1a} = \sqrt{\frac{1}{2\pi}\int_0^{\frac{2\pi}{3}} I_2^2 \mathrm{d}\omega t} = \frac{I_2}{\sqrt{3}} \approx 28.87\ \mathrm{A}.$$

So, the THD is

$$\mathrm{THD} = \sqrt{\left(\frac{I_{1a}^2}{(I_{1a}^{(1)})^2}\right) - 1} = 109.3\ \%.$$

3.5 Suppose the inductance of the inductor is finite and was designed too low. How would that affect the output voltage and current? [2 Points]

Answer:
If the inductance is finite, it cannot sustain continuous current when the AC voltage drops below the DC output voltage. The current reaches zero before the next thyristor is triggered. This causes the load current to fall to zero during part of the cycle, discontinuity in load current.

Task 4: B6C converter at a motor load **[14 Points]**

An industrial DC motor is used in a rolling mill drive, powered through a B6C six-pulse controlled rectifier, shown in Fig. 7, from a three-phase 480 V, 50 Hz grid. The motor has a rated terminal voltage of 400 V and draws 50 A. A large smoothing inductor ($L \to \infty$) ensures constant output current. The converter is utilized to operate in motoring mode by adjusting the firing angle α.

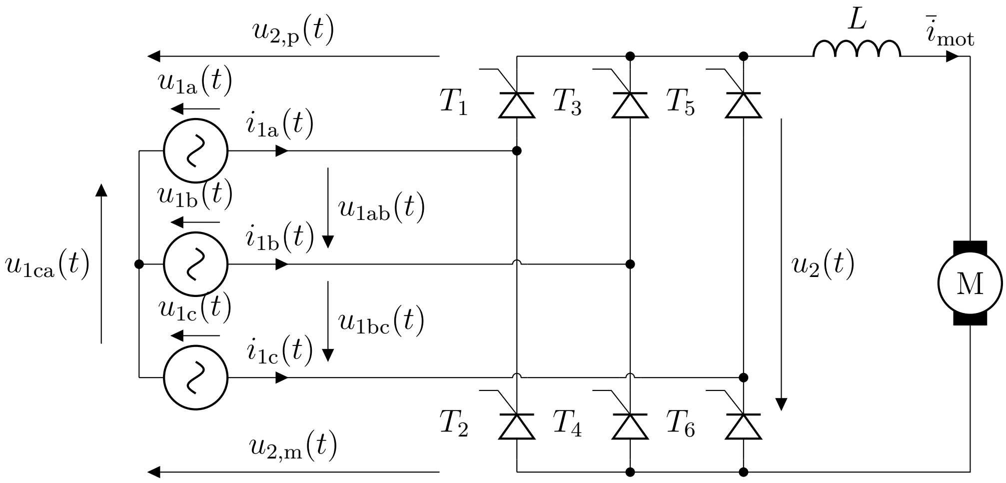


Fig. 7: B6C converter at a motor load.

Input voltages ($i = \text{a}, \text{b}, \text{c}$):	$U_{1,i} = 277$ V (phase voltage)
	$U_{1,\text{LL},i} = 480$ V (line-to-line voltage)
Nom. motor current:	$I_{\text{mot}} = 50$ A
Nom. motor terminal voltage:	$U_{\text{mot,term}} = 400$ V
Grid frequency:	$f = 50$ Hz

Tab. 4: Drive parameters.

4.1 Calculate the maximum average DC voltage the converter can deliver. In addition, calculate the firing angle α_{mot} to operate the motor at nominal speed. [3 Points]

Answer:

The maximum average voltage $\overline{u}_2$ is calculated at $\alpha = 0$ by

$$\overline{u}_2 = \hat{u}_{1,\text{LL}} \frac{p}{\pi} \sin\left(\frac{\pi}{p}\right) \cos(\alpha = 0).$$

In case of B6C-topology p is equal to 6. The line-to-line peak voltage $\hat{u}_{1,\text{LL}}$ is calculated by

$$\hat{u}_{1,\text{LL}} = \sqrt{2} \cdot U_{1,\text{LL}} = \sqrt{2} \cdot 480 \text{ V} = 678.82 \text{ V}.$$

The maximum output DC voltage of the converter:

$$\bar{u}_{2,max} = 678.82 \text{ V} \cdot \frac{6}{\pi} \cdot \sin\left(\frac{\pi}{6}\right) \cdot \cos(\alpha = 0) = 648.23 \text{ V}.$$

The voltage $\overline{u}_2$ corresponds to $U_{\mathrm{mot,term}}$. Solving the voltage equation with respect to α results in

$$\alpha = \arccos\left(\frac{\overline{u}_2 \cdot \pi}{\hat{u}_{1,\mathrm{LL}} \cdot p \cdot \sin(\frac{\pi}{p})}\right).$$

Substituting with values delivers the final result:

$$\alpha_{\mathrm{mot}} = \arccos\left(\frac{400\ \mathrm{V} \cdot \pi}{678.82\ \mathrm{V} \cdot 6 \cdot \sin(\frac{\pi}{6})}\right) = 51.9°.$$

4.2 Sketch the converter output voltage signal $u_2(t)$ at the firing angle α_{mot} in Fig. 8. [1 Point]
Hint: assume a firing angle $\alpha_{\mathrm{mot}} = 50°$ if, and only if, not calculated in the previous subtask.

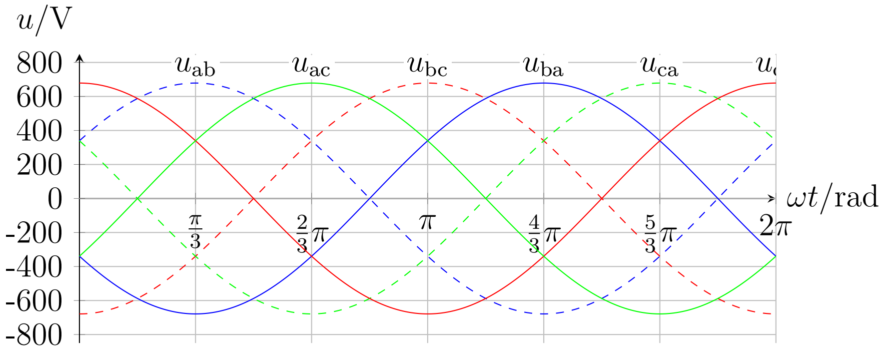
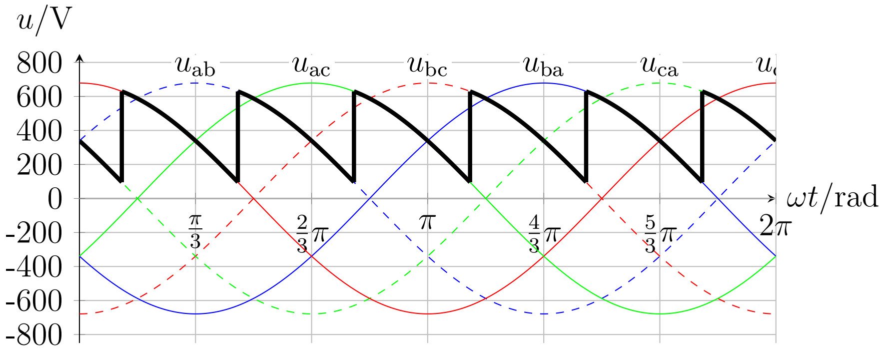


Fig. 8: Output voltage $u_2(t)$ for α_{mot}.

Answer:



Solution Fig. 4: Output voltage $u_2(t)$ for $\alpha = 51.9°$.

4.3 List the thyristor pairs that conduct during each interval over one grid period. For each interval, specify:

- Which thyristors conduct,
- which input line-to-line voltage appears at the converter output.

Interval (° electrical)	**Conducting Thyristor Pair**	**Load Voltage** $u_2(t)$
$30+\alpha$	T_1, T_4	u_{ab}

Tab. 5: Switching intervals, conducting thyristor pairs, and corresponding load voltage.

Add this information to Tab. 5, where already an example for the first interval is given. [3 Points]

Answer:

Interval (° electrical)	**Conducting Thyristor Pair**	**Load Voltage** $u_2(t)$
$30+\alpha$	T_1, T_4	u_{ab}
$90+\alpha$	T_1, T_6	u_{ac}
$150+\alpha$	T_3, T_6	u_{bc}
$210+\alpha$	T_3, T_2	u_{ba}
$270+\alpha$	T_5, T_2	u_{ca}
$330+\alpha$	T_5, T_4	u_{cb}

Tab. 6: Switching intervals, conducting thyristor pairs, and corresponding load voltage.

4.4 Given the fundamental input current amplitude $\hat{i}_{1\mathrm{a}}^{(1)} = 55.13$ A and nominal motor current I_{mot}, calculate the average active power supplied to the motor and the fundamental reactive power absorbed from the grid. [3 Points]

Answer:

$$P_{\mathrm{mot}} = U_{\mathrm{mot,term}} I_{\mathrm{mot}} = 400\ \mathrm{V} \cdot 50\ \mathrm{A} = 20\ \mathrm{kW}.$$

The input voltage $U_1^{(1)}$ corresponds to U_1, so that the apparent power results to

$$S_{\mathrm{mot}}^{(1)} = 3U_1 \frac{\hat{i}_{1\mathrm{a}}^{(1)}}{\sqrt{2}} = 3 \cdot 277\ \mathrm{V} \cdot \frac{55.13\ \mathrm{A}}{\sqrt{2}} = 32.40\ \mathrm{kVA}.$$

The reactive power is calculated by

$$Q_{\mathrm{mot}}^{(1)} = 3U_1 \frac{\hat{i}_{1\mathrm{a}}^{(1)}}{\sqrt{2}} \sin\left(\alpha_{\mathrm{mot}}\right) = 3 \cdot 277\ \mathrm{V} \cdot \frac{55.13\ \mathrm{A}}{\sqrt{2}} \sin\left(51.9\right) = 25.49\ \mathrm{kVA}.$$

4.5 Using the same value for $\hat{i}_{1\mathrm{a}}^{(1)}$ as in the previous subtask, calculate the THD of the phase current $i_{1\mathrm{a}}(\omega t)$. [2 Points]
Hint: use the same THD formula given in the previous task.

Answer:

The effective value of the fundamental current component is

$$I_{1a}^{(1)} = \frac{\hat{i}_{1a}^{(1)}}{\sqrt{2}} = 38.98\ \mathrm{A},$$

while the effective value of the input phase current i_{1a} is

$$I_{1a} = \sqrt{\frac{2}{2\pi}\left(\int_0^{\frac{\pi}{3}} I_2^2 \mathrm{d}\omega t + \int_{\frac{2\pi}{3}}^{\pi} I_2^2 \mathrm{d}\omega t\right)} = \frac{I_2 \cdot \sqrt{2}}{\sqrt{3}} \approx 40.82\ \mathrm{A}.$$

So, the THD is

$$\mathrm{THD} = \sqrt{\left(\frac{I_{1a}^2}{(I_{1a}^{(1)})^2}\right) - 1} = 31.086\ \%.$$

4.6 State the effect of changing the number of pulses p (by using other converter topologies) on the maximum achievable voltage and THD of the input current. [2 Points]

Answer:

Increasing the number of pulses p:

- Increases the maximum achievable DC output voltage.
- Reduces the THD of the input current.