Exercise 01: Step-down converter and power loss calculation

Task 1.1: Step-down converter without output filter

An ideal switching transistor is used for loss-free and stepless control of a car's rear window heating. By varying the duty cycle of the transistor the average value of the heating power can be adjusted. The voltage in the car's electrical system is assumed to be constant with $U_1 = 14$ V. The heater is dimensioned in such a way that at its nominal voltage $U_{2N} = 14$ V it consumes a power of $P_{LN} = 500$ W and can be modeled with an ohmic resistor. This circuit is shown in Fig. 1.1.1.

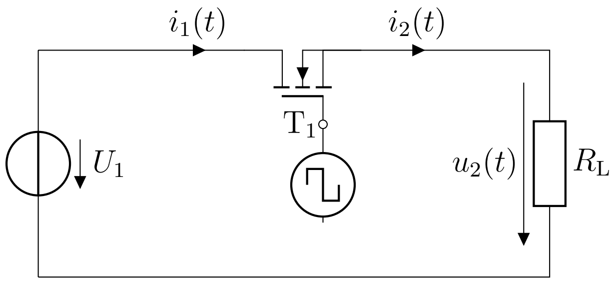
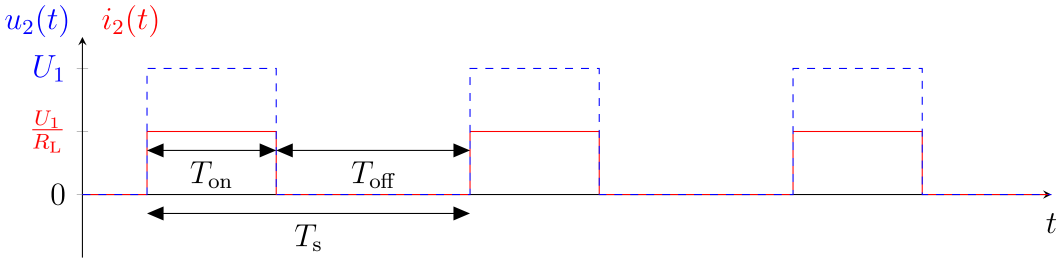


Figure 1.1.1: Circuit with one transistor and one load resistor.

1.1.1 Draw the qualitative current $i_2(t)$ and voltage $u_2(t)$ curves at the load resistor for some switching periods T_s.

Answer:

The graph Sol.-Fig. 1.1.1 shows the voltages and currents at the load resistor with the period T_s, the switch-on time T_{on} and the switch-off time T_{off}.

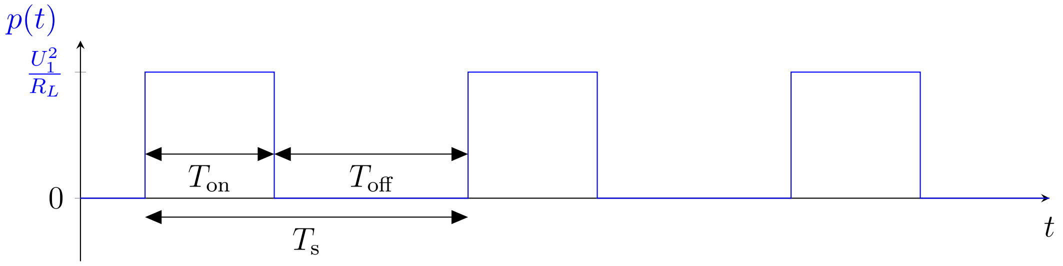


Solution Figure 1.1.1: Display of currents and voltages depending on the switching phases.

1.1.2 Draw the instantaneous power at the load resistor.

Answer:

The Sol.-Fig. 1.1.2 shows the time curve of the instantaneous power at the load resistor.



Solution Figure 1.1.2: Instantaneous power at the load resistor.

1.1.3 Derive the relationships for the mean voltage $\overline{u}_2$, the mean current $\overline{i}_2$ and the mean power $\overline{p}_2$.
Answer:
In order to calculate the average voltage $\overline{u}_2$, the voltage $u_2(t)$ must be integrated over the period T_{s}:

$$\begin{aligned}\overline{u}_2 &= \frac{1}{T_{\mathrm{s}}}\int_0^{T_{\mathrm{s}}} u_2(t)\,\mathrm{d}t = \frac{1}{T_{\mathrm{s}}}\int_0^{DT_{\mathrm{s}}} U_1\,\mathrm{d}t + \frac{1}{T_{\mathrm{s}}}\int_{DT_{\mathrm{s}}}^{T_{\mathrm{s}}} 0\,\mathrm{d}t \\ &= \frac{U_1}{T_{\mathrm{s}}} t\Big|_0^{DT_{\mathrm{s}}} + 0 = \frac{U_1 DT_{\mathrm{s}}}{T_{\mathrm{s}}} = U_1 D. \end{aligned} \tag{1.1.1}$$

For the mean value $\overline{i}_2$, the current $i_2(t)$ must be integrated over the period T_{s}:

$$\begin{aligned}\overline{i}_2 &= \frac{1}{T_{\mathrm{s}}}\int_0^{T_{\mathrm{s}}} i_2(t)\,\mathrm{d}t = \frac{1}{T_{\mathrm{s}}}\int_0^{DT_{\mathrm{s}}} \frac{U_1}{R_{\mathrm{L}}}\,\mathrm{d}t + \frac{1}{T_{\mathrm{s}}}\int_{DT_{\mathrm{s}}}^{T_{\mathrm{s}}} 0\,\mathrm{d}t \\ &= \frac{U_1}{R_{\mathrm{L}} T_{\mathrm{on}}} t\Big|_0^{DT_{\mathrm{s}}} = \frac{U_1 DT_{\mathrm{s}}}{R_{\mathrm{L}} T_{\mathrm{s}}} = \frac{U_1}{R_{\mathrm{L}}} D. \end{aligned} \tag{1.1.2}$$

To calculate the mean value $\overline{p}_2$, the power $p(t)$ must be integrated over the period T_{s}:

$$\overline{p}_2 = \frac{1}{T_{\mathrm{s}}}\int_0^{T_{\mathrm{s}}} p(t)\mathrm{d}t = \frac{1}{T_{\mathrm{s}}}\int_0^{DT_{\mathrm{s}}} \frac{U_1^2}{R_{\mathrm{L}}}\mathrm{d}t = \frac{U_1^2 DT_{\mathrm{s}}}{T_{\mathrm{s}} R_{\mathrm{L}}} = \frac{U_1^2 D}{R_{\mathrm{L}}}. \tag{1.1.3}$$

1.1.4 How large should the duty cycle D be selected so that an average voltage of $\overline{u}_2 = 8$ V is applied to the heater? What is the mean value of the current $\overline{i}_2$? What power $\overline{p}_2$ is converted into heat?
Answer:
To determine the duty cycle D, the average voltage $\overline{u}_2$ must be divided by the source voltage U_1

$$\overline{u}_2 = DU_1 \quad \Leftrightarrow \quad D = \frac{\overline{u}_2}{U_1} = \frac{8\ \mathrm{V}}{14\ \mathrm{V}} = 0.57. \tag{1.1.4}$$

In order to determine the average current $\overline{i}_2$, the load resistance R_{L} must be calculated first:

$$\overline{i}_2 = \frac{U_1}{R_{\mathrm{L}}} D. \tag{1.1.5}$$

The known power equation is used to determine the load resistance R_L

$$P_\mathrm{LN} = \frac{{U_\mathrm{2N}}^2}{R_\mathrm{L}} \quad \Leftrightarrow \quad R_\mathrm{L} = \frac{U_\mathrm{2N}^2}{P_\mathrm{LN}} = \frac{(14\ \mathrm{V})^2}{500\ \mathrm{W}} = 392\ \mathrm{m}\Omega. \tag{1.1.6}$$

Inserting into the current equation delivers :

$$\bar{i}_2 = \frac{14\ \mathrm{V}}{392\ \mathrm{m}\Omega} 0.57 = 20.36\ \mathrm{A}.$$

The average power $\overline{p}_2$ yields

$$\overline{p}_2 = \frac{U_1^2}{R_\mathrm{L}} D = \frac{(14\ \mathrm{V})^2}{392\ \mathrm{m}\Omega} 0.57 = 285\ \mathrm{W}. \tag{1.1.7}$$

1.1.5 When starting the engine, the heater may draw a maximum average current $\bar{i}_{2,\mathrm{s}} = 10$ A from the vehicle electrical system. With which duty cycle D should the transistor be switched in this case? What is the average voltage $\overline{u}_2$ at the heater? What power $\overline{p}_2$ is converted into heat?

Answer:

Due to the structure of the circuit, it is noticeable that the average current $\bar{i}_1$ is equal to the average current $\bar{i}_2$. The corresponding ratio for $\bar{i}_{2,\mathrm{s}}$ can be calculated from this.

$$\bar{i}_1 = \bar{i}_2 = \frac{U_1}{R_\mathrm{L}} D \overset{!}{\leq} \bar{i}_{2,\mathrm{s}} \quad \Leftrightarrow \quad D \leq \frac{\bar{i}_{2,\mathrm{s}} R_\mathrm{L}}{U_1} = \frac{10\ \mathrm{A} \cdot 392\ \mathrm{m}\Omega}{14\ \mathrm{V}} = 0.28 \tag{1.1.8}$$

To determine the average voltage $\overline{u}_2$ at the heater, the duty cycle D is offset against the source voltage U_1.

$$\overline{u}_2 = D U_1 = 0.28 \cdot 14\ \mathrm{V} = 3.92\ \mathrm{V} \tag{1.1.9}$$

To determine the power $\overline{p}_2$ that is converted into heat, the corresponding duty cycle D must be considered:

$$\overline{p}_2 = \frac{U_1^2 D}{R_\mathrm{L}} = \frac{(14\ \mathrm{V})^2 \cdot 0.28}{392\ \mathrm{m}\Omega} = 140\ \mathrm{W}. \tag{1.1.10}$$

1.1.6 During the journey, the heat output should be $\overline{p}_{2,\mathrm{f}} = 200$ W. How is the duty cycle D set? What are the mean values of the current $\bar{i}_2$ and the voltage $\overline{u}_2$?

Answer:

To determine the duty cycle D, the familiar power formula is applied:

$$\overline{p}_{2,\mathrm{f}} \overset{!}{=} \frac{U_1^2}{R_\mathrm{L}} D \quad \Leftrightarrow \quad D = \frac{R_\mathrm{L} \overline{p}_{2,\mathrm{f}}}{{U_1}^2} = \frac{392\ \mathrm{m}\Omega \cdot 200\ \mathrm{W}}{(14\ \mathrm{V})^2} = 0.4. \tag{1.1.11}$$

The known equations are used to determine the average current $\bar{i}_2$ and the average voltage $\overline{u}_2$:

$$\bar{i}_2 = \frac{U_1}{R_\mathrm{L}} D = \frac{14\ \mathrm{V}}{392\ \mathrm{m}\Omega} \cdot 0.4 = 14.29\ \mathrm{A}, \tag{1.1.12}$$

$$\overline{u}_2 = U_1 D = 14\ \text{V} \cdot 0.4 = 5.6\ \text{V}. \tag{1.1.13}$$

Task 1.2: Step-down converter with output filter

A step-down converter is used to charge a mobile phone from the vehicle electrical system with the vehicle electrical system voltage $U_1 = 13.5$ V. The input voltage of the mobile phone is $U_2 = 4.5$ V. Consider both voltages as constant, the inductance of the coil is $L = 10$ µH and the switching frequency is $f_s = 100$ kHz. All components are considered ideal.

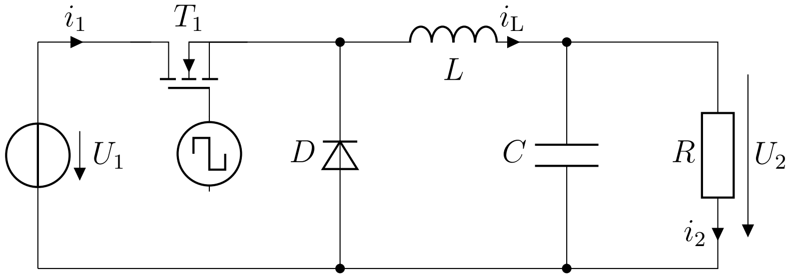
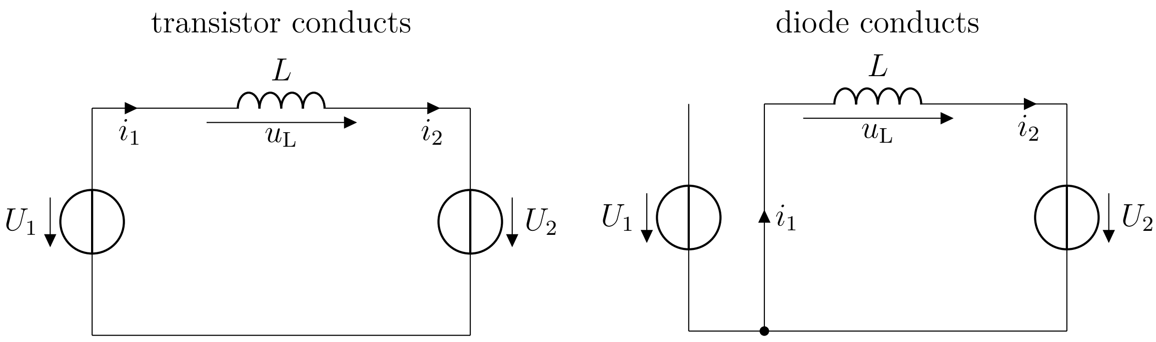


Figure 1.2.1: Circuit with one transistor, filter and one load resistor.

1.2.1 Draw the equivalent circuits for the two switching states.

Answer:



Solution Figure 1.2.1: Circuits for different switching states.

1.2.2 At what duty cycle D should the buck converter be operated?

Answer:

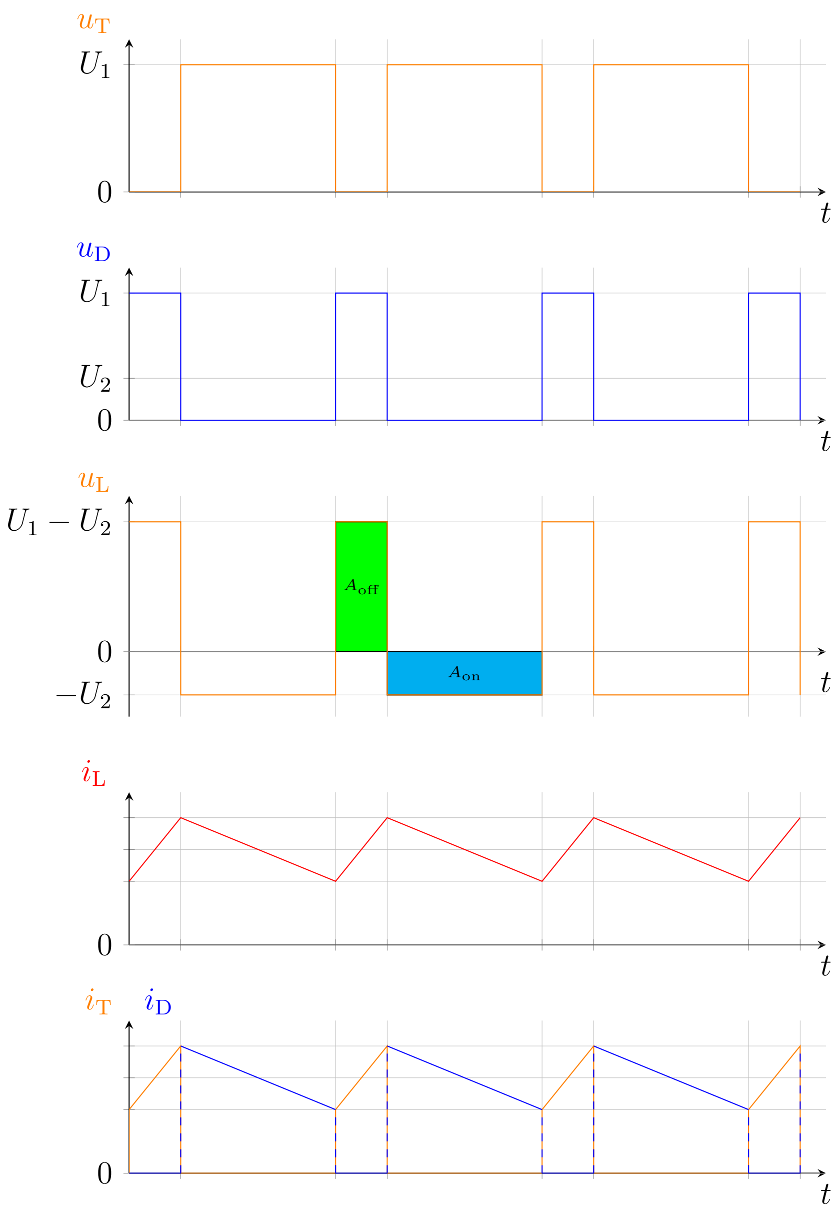
The duty cycle corresponds to the relation of output voltage to input voltage:

$$D = \frac{U_2}{U_1} = \frac{4.5\ \text{V}}{13.5\ \text{V}} = \frac{1}{3}. \tag{1.2.1}$$

1.2.3 Sketch the voltage and current signals in the components.

Answer:

The signals of u_T, u_D, u_L, i_L and i_T are displayed in Sol.-Fig. 1.2.2.



Solution Figure 1.2.2: Relevant voltage and current signals.

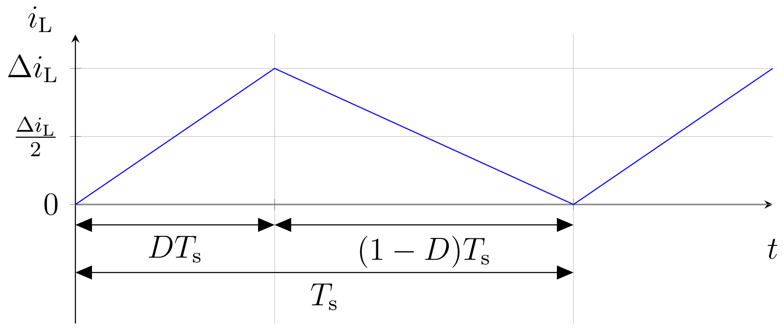
1.2.4 How large is the current ripple Δi_L of the coil current in boundary conduction mode (BCM) operation?

Answer:

The BCM operation is characterized by zero inductor current events at the beginning / end of a switching period, compare Sol.-Fig. 1.2.3.

The relevant switching time intervals are:

$$f_s = 100 \text{ kHz} \quad \Rightarrow \quad T_s = 10 \text{ µs} \quad \Rightarrow \quad T_{on} = D \cdot T_s = \frac{10 \text{ µs}}{3}. \tag{1.2.2}$$



Solution Figure 1.2.3: Qualitative inductor current during BCM

The (linear) inductor ordinary differential equations yields

$$U_L = U_1 - U_2 = L\frac{di_L}{dt} = L\frac{\Delta i_L}{\Delta t} = L\frac{\Delta i_L}{T_{on}} \qquad (1.2.3)$$

resulting to

$$\Delta i_L = \frac{U_1 - U_2}{L}T_{on} = \frac{13.5\text{ V} - 4.5\text{ V}}{10\text{ µH} \cdot \frac{10\text{ µs}}{3}} = 3\text{ A} \qquad (1.2.4)$$

and finally

$$I_{2,min} = \frac{\Delta i_L}{2} = 1.5\text{ A}. \qquad (1.2.5)$$

1.2.5 How large is the worst-case current ripple?

Answer:

The worst case ripple is at $D = 0.5$ leading to

$$U_2 = DU_1, \qquad (1.2.6)$$

$$\Delta i_{L,max} = \frac{U_1 - U_2}{L}0.5 \cdot T_s \Rightarrow \Delta i_{L,max} = \frac{13.5\text{ V} - 6.75\text{ V}}{13.5\text{ µH}}0.5 \cdot 10\text{ µs}, \qquad (1.2.7)$$

$$\Delta i_{L,min} = 3.38\text{ A}. \qquad (1.2.8)$$

1.2.6 When starting the engine, the input voltage drops to $U_{1,min} = 10$ V. The voltage regulator of the buck converter changes the duty cycle so that the output voltage $U_2 = 4.5$ V is kept stable. What duty cycle D is set?

Answer:

The new duty cycle results in:

$$D = \frac{U_2}{U_{1,min}} = \frac{4.5\text{ V}}{10\text{ V}} = 0.45. \qquad (1.2.9)$$

1.2.7 Calculate the average and ripple current in BCM mode considering $U_{1,min}$ at the input.

Answer:

The boundary of discontinuous conduction mode depends on the inductance, duty cycle, switching frequency and voltages:

$$\Delta i_{\mathrm{L}} = \frac{U_{1,\min} - U_2}{L} D T_{\mathrm{s}} = \frac{(10\ \mathrm{V} - 4.5\ \mathrm{V}) \cdot 0.45 \cdot 10\ \mathrm{\mu s}}{10\ \mathrm{\mu H}} = 2.48\ \mathrm{A}, \tag{1.2.10}$$

$$I_{\mathrm{L,min}} = \frac{\Delta i_{\mathrm{L}}}{2} = 1.24\ \mathrm{A}. \tag{1.2.11}$$

Task 1.3: Power losses within the step-down converter

The power loss of a buck converter is to be analyzed. The inductance is so large that the current ripple in the output current can be neglected, i.e., $i_2(t) = I_2 = \text{const}$. Furthermore, the component values are given in Tab. 1.3.1 and the currents and voltages of the switch-on and switch-off processes in Fig. 1.3.2 and Fig. 1.3.3.

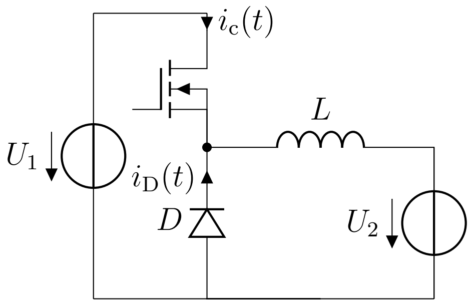


Figure 1.3.1: Buck converter with one transistor and one diode.

General parameters:		**Diode:**	
Input voltage:	$U_1 = 600$ V	Forward voltage:	$u_{\mathrm{F}} = 2.7$ V
Output current:	$I_2 = 30$ A	Switch-on losses:	$E_{\mathrm{on,D}} = 52$ µJ
Switching frequency:	$f_{\mathrm{s}} = 25$ kHz	Switch-off losses:	$E_{\mathrm{off,D}} = 240$ µJ
IGBT:		**Inductance:**	
Collector-emitter voltage:	$u_{\mathrm{on,CE}} = 2.5$ V	Copper resistance:	$R_{\mathrm{Cu}} = 45$ mΩ
		Iron losses:	$P_{\mathrm{l,Fe}} = 13$ W

Table 1.3.1: Parameters of the circuit.

1.3.1 Calculate the switch-on and switch-off loss work $E_{\mathrm{on,T}}$ and $E_{\mathrm{off,T}}$ of the IGBT.

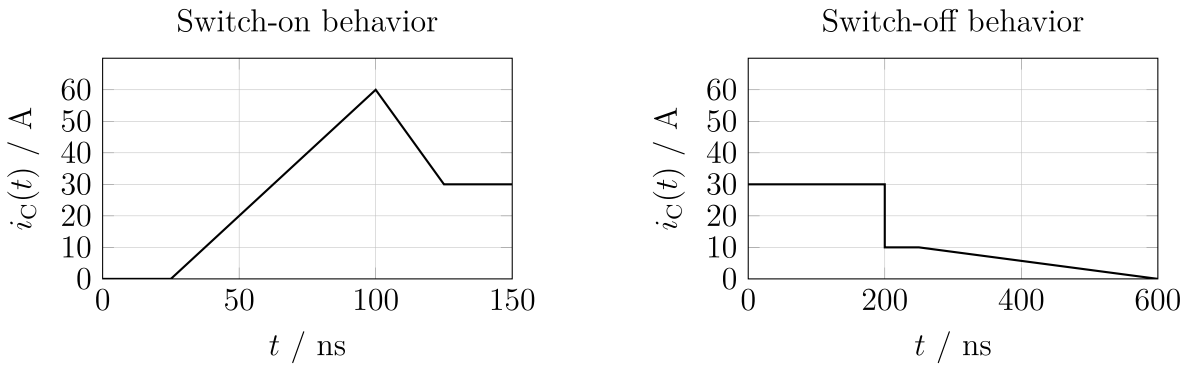


Figure 1.3.2: Switch-on behavior and switch-off behavior of $i_{\mathrm{C}}(t)$.

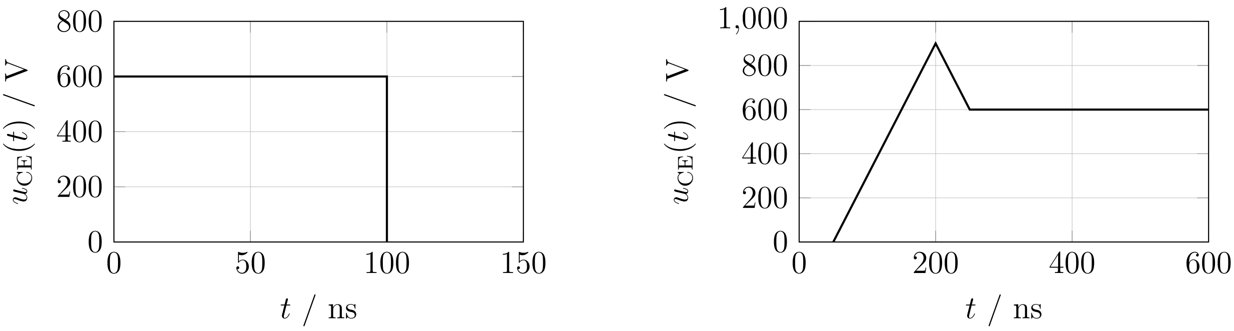


Figure 1.3.3: Switch-on behavior and switch-off behavior of $u_{\mathrm{CE}}(t)$.

Answer:

First, the power values for the switch-on process at the times $t = 25$ ns and $t = 100$ ns are calculated:

$$p(t = 25 \text{ ns}) = 600 \text{ V} \cdot 0 \text{ A} = 0, \tag{1.3.1}$$

$$p(t = 100 \text{ ns}) = 600 \text{ V} \cdot 60 \text{ A} = 36 \text{ kW}. \tag{1.3.2}$$

Sol.-Fig. 1.3.1a results from these power values. The switch-on losses of the IGBT can be calculated based on the triangular signal shapes of the provided signals:

$$E_{\mathrm{on,T}} = \frac{1}{2}\Delta P \Delta t = \frac{1}{2} 36 \text{ kW} \cdot 75 \text{ ns} = 1.35 \text{ mJ}. \tag{1.3.3}$$

The switch-off power can now be determined for the times $t = 200$ ns, $t = 200.1$ ns and $t = 250$ ns:

$$p(t = 200 \text{ ns}) = 900 \text{ V} \cdot 30 \text{ A} = 27 \text{ kW}, \tag{1.3.4}$$

$$p(t = 200.1 \text{ ns}) = 900 \text{ V} \cdot 10 \text{ A} = 9 \text{ kW}, \tag{1.3.5}$$

$$p(t = 250 \text{ ns}) = 600 \text{ V} \cdot 10 \text{ A} = 6 \text{ kW}. \tag{1.3.6}$$

Finally, Sol.-Fig. 1.3.1b results from these power values. The switch-off losses of the IGBT are calculated next. For this purpose, the power curve is divided into three parts:

$$E_{\mathrm{off,T}} = E_1 + E_2 + E_3 \tag{1.3.7}$$

with

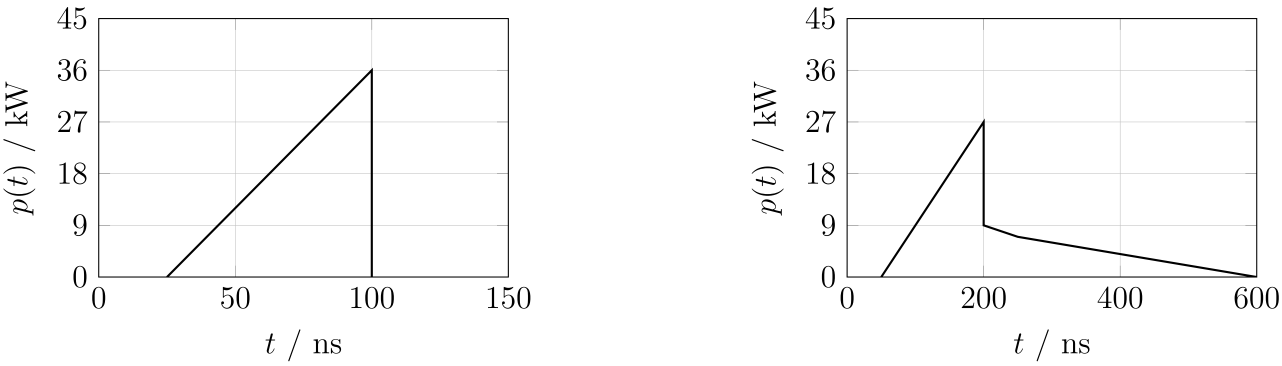
$$E_1 = \frac{1}{2} 27\ \mathrm{kW} \cdot 150\ \mathrm{ns} = 2.025\ \mathrm{mJ}, \tag{1.3.8}$$

$$E_2 = 6\ \mathrm{kW} \cdot 50\ \mathrm{ns} + \frac{1}{2} 3\ \mathrm{kW} \cdot 50\ \mathrm{ns} = 0.375\ \mathrm{mJ}, \tag{1.3.9}$$

$$E_3 = \frac{1}{2} 6\ \mathrm{kW} \cdot 350\ \mathrm{ns} = 1.05\ \mathrm{mJ}. \tag{1.3.10}$$

If all three loss work terms are added together, the result is the total switch-off loss work:

$$E_{\mathrm{off,T}} = 3.45\ \mathrm{mJ}. \tag{1.3.11}$$



(a) Power values for the switch-on process.

(b) Power values for the switch-off process.

Solution Figure 1.3.1: Switch-on behavior and switch-off behavior of $p(t)$.

1.3.2 Calculate the (average) switching power loss in the IGBT $P_{\mathrm{l,sw,T}}$ and in the diode $P_{\mathrm{l,sw,D}}$.
Answer:
The switching power loss of the IGBT is dependent on the switching frequency f_s in addition to the switch-on and switch-off power losses:

$$P_{\mathrm{l,sw,T}} = (E_{\mathrm{on,T}} + E_{\mathrm{off,T}}) \cdot f_\mathrm{s} = (1.35\ \mathrm{mJ} + 3.45\ \mathrm{mJ}) \cdot 25\ \mathrm{kHz} = 120\ \mathrm{W}. \tag{1.3.12}$$

Likewise, the switching power loss of the diode also depends on the switching frequency f_s:

$$P_{\mathrm{l,sw,D}} = (E_{\mathrm{on,D}} + E_{\mathrm{off,D}}) \cdot f_\mathrm{s} = (52\ \mathrm{\mu J} + 3.45\ \mathrm{\mu J}) \cdot 25\ \mathrm{kHz} = 7.3\ \mathrm{W}. \tag{1.3.13}$$

1.3.3 Calculate the (average) conduction losses in the IGBT $P_{\mathrm{l,cond,T}}(D)$ and in the diode $P_{\mathrm{l,cond,D}}(D)$ as a function of the duty cycle D.
Answer:

There are two operating states, one in which the IGBT is closed and another in which it is open. Therefore, the IGBT and diode never carry a current at the same time. This means that the conduction losses of the IGBT only occur when it is switched on:

$$P_{\mathrm{l,cond,T}}(D) = u_{\mathrm{CE,on}} I_2 D = 2.5\ \mathrm{V} \cdot 30\ \mathrm{A} \cdot D = 75\ \mathrm{W} \cdot D. \tag{1.3.14}$$

Furthermore, this means that the forward losses of the diode only occur when the IGBT is switched off:

$$P_{\mathrm{l,cond,D}}(D) = u_{\mathrm{f}} I_2 (1-D) = 2.7\ \mathrm{V} \cdot 30\ \mathrm{A} \cdot (1-D) = 81\ \mathrm{W} \cdot (1-D). \tag{1.3.15}$$

1.3.4 Calculate the efficiency $\eta(U_2)$ as a function of the output voltage.
Answer:
To determine the efficiency, the general equation $\eta = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}$ is used

$$\eta = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} = \frac{U_2 I_2}{U_2 I_2 + P_{\mathrm{l,total}}(D)} \tag{1.3.16}$$

where $P_{\mathrm{l,total}}(D)$ results from all losses:

$$P_{\mathrm{l,total}}(D) = P_{\mathrm{l,sw,T}} + P_{\mathrm{l,sw,D}} + P_{\mathrm{l,cond,T}}(D) + P_{\mathrm{l,cond,D}}(D) + P_{\mathrm{l,L}}. \tag{1.3.17}$$

The losses inside the inductor are:

$$P_{\mathrm{l,L}} = R_{\mathrm{Cu}}(I_2)^2 + P_{\mathrm{l,Fe}} = 45\ \mathrm{m\Omega} \cdot (30\ \mathrm{A})^2 + 13\ \mathrm{W} = 53.5\ \mathrm{W}, \tag{1.3.18}$$

where $P_{\mathrm{l,cond,T}}(D)$ and $P_{\mathrm{l,cond,D}}(D)$ can only be determined in general terms:

$$P_{\mathrm{l,cond,T}}(D) + P_{\mathrm{l,cond,D}}(D) = I_2(u_{\mathrm{CE,on}} D + u_{\mathrm{F}}(1-D)) = D I_2 (u_{\mathrm{CE,on}} - u_{\mathrm{F}}) + I_2 u_{\mathrm{F}}. \tag{1.3.19}$$

Summarizing all power loss components yields:

$$P_{\mathrm{l,total}}(D) = P_{\mathrm{l,sw,T}} + P_{\mathrm{l,sw,D}} + P_{\mathrm{l,L}} + D I_2 (u_{\mathrm{CE,on}} - u_{\mathrm{F}}) + I_2 u_{\mathrm{F}} = P_{\mathrm{l,const}} + D I_2 (u_{\mathrm{CE,on}} - u_{\mathrm{F}}) \tag{1.3.20}$$

The value $P_{\mathrm{l,const}}$ is considering all losses that could be calculated:

$$P_{\mathrm{l,const}} = 120\ \mathrm{W} + 7.3\ \mathrm{W} + 53.5\ \mathrm{W} + 2.7\ \mathrm{V} \cdot 30\ \mathrm{A}. \tag{1.3.21}$$

1.3.5 The circuit is to be evaluated at $U_2 = 300$ V. Calculate the total power loss in the IGBT $P_{\mathrm{l,T}}$ and in the diode $P_{\mathrm{l,D}}$.
Answer:

Firstly, the duty cycle must be determined:

$$D = \frac{U_2}{U_1} = \frac{300\ \mathrm{V}}{600\ \mathrm{V}} = 0.5. \tag{1.3.22}$$

The total power loss of the IGBT is made up of the switching losses and the losses during operation:

$$P_{\mathrm{l,T}}(D) = u_{\mathrm{CE,on}} I_2 D + P_{\mathrm{l,sw,T}} = 2.5\ \mathrm{V} \cdot 30\ \mathrm{A} \cdot 0.5 + 120\ \mathrm{W} = 157.5\ \mathrm{W}. \tag{1.3.23}$$

The total power loss of the diode is made up of the switching losses and the losses during operation:

$$P_{\mathrm{l,D}}(D) = u_{\mathrm{F}} I_2 (1 - D) + P_{\mathrm{l,sw,D}} = 2.7\ \mathrm{V} \cdot 30\ \mathrm{A} \cdot 0.5 + 7.3\ \mathrm{W} = 47.8\ \mathrm{W}. \tag{1.3.24}$$

Exercise 02: Step-up and (synchronous) buck-boost converters

Task 2.1: Boost converter with no losses

The boost converter which is shown in Fig. 2.1.1 supplies a resistive load with a rated voltage of $U_2 = 60$ V. The converter operates in steady state and all losses of the boost converter are neglected.

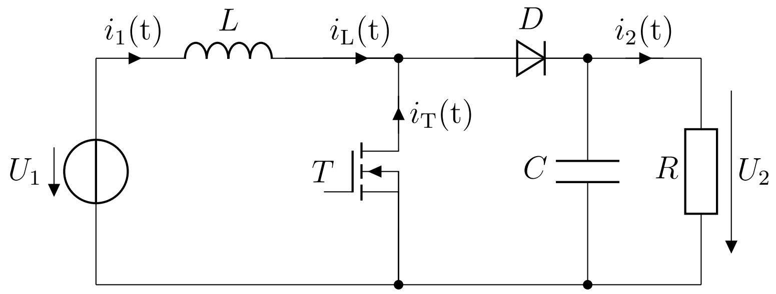


Figure 2.1.1: Boost converter with no losses.

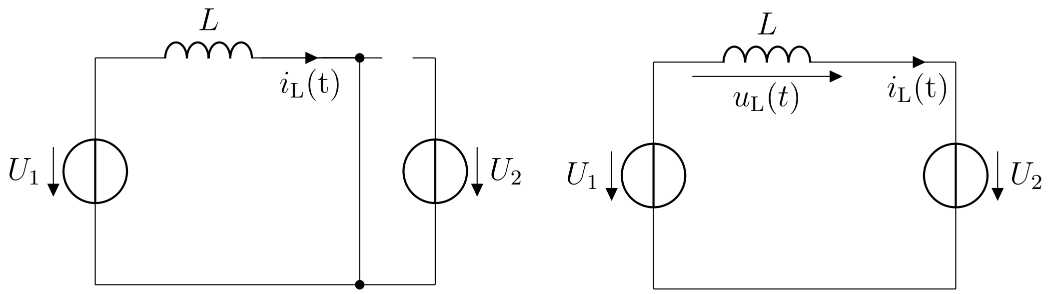
The specification of the boost converter is given in Tab. 2.1.1.

Input voltage:	$U_1 = 12$ V	Output voltage:	$U_2 = 60$ V
Output current:	$I_2 = 2$ A	Minimal output power:	$P_{2,\min} = 10$ W
Output voltage ripple:	$\triangle u_2 = 120$ mV	Switching frequency:	$f_s = 100$ kHz

Table 2.1.1: Parameters of the boost converter.

2.1.1 Derive the duty cycle D which leads to the specific output voltage of $U_2 = 60$ V.

Answer:



Solution Figure 2.1.1: ECD of the boost converter for different switching states.

The equivalent circuit diagram (ECD) is shown in Sol.-Fig. 2.1.1. As there are no losses at the transistor, the voltage U_1 is equal to u_L for the switch-on period T_{on} and is defined as

$$U_1 = \overline{u}_L = L\frac{\Delta i_L}{\Delta t}. \tag{2.1.1}$$

This is an approximation of the total differential using a difference equation with the differences Δ. This results in the component differential equations becoming average value equations. This subtask does not consider the complete period T_s, but only the switch-on period T_on. Furthermore, the initial state of the current i_L must also be considered. That is why we can completely rearrange the equation into:

$$i_\mathrm{L}(T_\mathrm{on}) = \frac{U_1}{L} T_\mathrm{on} + i_\mathrm{L}(0). \tag{2.1.2}$$

Applying Kirchhoff's second law, the voltage equation for the switch-off time T_off is given with:

$$U_1 = u_\mathrm{L} + U_2. \tag{2.1.3}$$

Now, the differential expression for u_L leads to:

$$L\frac{\Delta i_\mathrm{L}}{\Delta t} = U_1 - U_2. \tag{2.1.4}$$

The equation for the current i_L at the switch-off time T_off results from the term describing the discharge of the inductance from T_off together with the initial condition of the current at this time

$$i_\mathrm{L}(t) = \frac{U_1 - U_2}{L}(t - T_\mathrm{on}) + i_\mathrm{L}(T_\mathrm{on}). \tag{2.1.5}$$

This results in the equation for the entire current curve over the period:

$$i_\mathrm{L}(t) = \frac{U_1 - U_2}{L}(t - T_\mathrm{on}) + \frac{U_1}{L} T_\mathrm{on} + i_\mathrm{L}(0). \tag{2.1.6}$$

If one considers the steady state $i_\mathrm{L}(t = 0) = i_\mathrm{L}(t = T_\mathrm{s})$, this results in

$$i_\mathrm{L}(0) = \frac{U_1 - U_2}{L}(T_\mathrm{off}) + \frac{U_1}{L} T_\mathrm{on} + i_\mathrm{L}(0). \tag{2.1.7}$$

Next $i_\mathrm{L}(0) = 0$ cancels due to a subtraction

$$0 = \frac{U_1 - U_2}{L}(T_\mathrm{off}) + \frac{U_1}{L} T_\mathrm{on}. \tag{2.1.8}$$

Canceling out the inductance leads to

$$U_1 T_\mathrm{on} = (-U_1 + U_2) T_\mathrm{off}, \tag{2.1.9}$$

and rearranging to:

$$U_1(T_\mathrm{off} + T_\mathrm{on}) = U_2 T_\mathrm{off}. \tag{2.1.10}$$

The switch-off period T_off plus the switch-on period T_on is the total period T_s. Hence, the following equation can be found for the input-output voltage gain:

$$\frac{U_1}{U_2} = \frac{T_\mathrm{off}}{T_\mathrm{s}} = \frac{T_\mathrm{s}}{T_\mathrm{s}} - \frac{T_\mathrm{on}}{T_\mathrm{s}} = 1 - D. \tag{2.1.11}$$

And with the values given in the task, the duty cycle is calculated as follows:

$$D = 1 - \frac{U_1}{U_2} = 1 - \frac{12\ \text{V}}{60\ \text{V}} = 0.8. \tag{2.1.12}$$

2.1.2 Determine the average input current $\bar{i}_\text{L}$.

Answer:

With neglecting the losses ($P_\text{l} = 0$ W) of the boost converter and assuming a sufficient filtering of the inductor current, the average value $\bar{i}_\text{L}$ is equal I_1 and can be calculated using the power balance

$$P_2 = U_2 I_2 = 60\ \text{V} \cdot 2\ \text{A} = 120\ \text{W}. \tag{2.1.13}$$

Since no losses are considered, the power P_1 is equal to P_2. This results in the average current $\bar{i}_\text{L}$:

$$I_1 = \bar{i}_\text{L} = \frac{P_1}{U_1} = \frac{120\ \text{W}}{12\ \text{V}} = 10\ \text{A}. \tag{2.1.14}$$

An alternative solution would be to determine the average current $\bar{i}_\text{L}$ via the duty cycle

$$\frac{I_1}{I_2} = \frac{1}{1-D}. \tag{2.1.15}$$

$$I_1 = \frac{I_2}{1-D} = \frac{2\ \text{A}}{1-0.8} = 10\ \text{A}. \tag{2.1.16}$$

2.1.3 Define a suitable inductance for the coil L, so that the boost converter is operating in boundary conduction mode (BCM) when supplying the minimum output power $P_{2,\text{min}}$. Determine the maximal switch-off current i_T of the transistor T for the rated output current $I_2 = 2$ A.

Answer:

Because no losses are assumed, $P_{1,\text{min}} = P_{2,\text{min}}$ follows. The average current $I_{1,\text{min}}$ over a period to achieve the minimum input power is

$$I_{1,\text{min}} = \frac{P_{1,\text{min}}}{U_1} = \frac{10\ \text{W}}{12\ \text{V}} = 0.833\ \text{A}. \tag{2.1.17}$$

The requirement for BCM is that the current is zero at the starting point of the period $i_{1,\text{BCM}}(t = 0) = 0$ A. Due to the triangular current waveform of the inductor current i_L, the maximum current is given with:

$$i_{1,\text{BCM}}(T_\text{on}) = 2I_{1,\text{min}} = 2 \cdot 0.833\ \text{A} = 1.667\ \text{A}. \tag{2.1.18}$$

The differential equation of the inductance is defined as:

$$u_\text{L} = L\frac{\text{d}i_\text{L}(t)}{\text{d}t}. \tag{2.1.19}$$

Neglecting saturation, the equation simplifies to:

$$\overline{u}_{\mathrm{L}} = L\frac{\Delta i_{\mathrm{L}}}{\Delta t}. \tag{2.1.20}$$

Above $\frac{\Delta i_{\mathrm{L}}}{\Delta t}$ is the average current change over a time interval Δt. Only the switch-on period is considered, therefore $\Delta t = T_{\mathrm{on}}$. Furthermore, $u_{\mathrm{L}} = U_1$ and $\Delta i_{\mathrm{L}} = 2I_{1,\mathrm{min}}$. The switch-on period can be determined by the known duty cycle D and the known frequency f_{s}

$$T_{\mathrm{on}} = DT_{\mathrm{s}} = \Delta t = \frac{D}{f_{\mathrm{s}}} = \frac{0.8}{100\ \mathrm{kHz}} = 8\ \mathrm{\mu s}. \tag{2.1.21}$$

The following equation can be derived from the difference quotient equation as:

$$L = \frac{\Delta t u_{\mathrm{L}}}{\Delta i_{\mathrm{L}}} = \frac{DU_1}{f_{\mathrm{s}}2I_{1,\mathrm{min}}} = \frac{0.8 \cdot 12\ \mathrm{V}}{100\ \mathrm{kHz} \cdot 2 \cdot 1.667\ \mathrm{A}} = 57.6\ \mathrm{\mu H}. \tag{2.1.22}$$

The maximum current flows through the transistor at the end of the switch-on interval

$$I_{\mathrm{T,max}} = \frac{\Delta I_{\mathrm{L}}}{2} = \frac{1.667\ \mathrm{A}}{2} = 0.833\ \mathrm{A}. \tag{2.1.23}$$

2.1.4 Calculate a suitable capacitance to meet the output voltage ripple specification. Determine the current stress of the capacitor $I_{\mathrm{C,RMS}}$.

Answer:

The reaction of the voltage fluctuation on the current fluctuation is neglected. During on-state of the transistor, the output capacitor feed the load. Assuming an constant output current and an ideal capacitor, the output can be adopted as linearly decreasing for this interval.

The differential form of the capacitor equation is used for this task

$$i_{\mathrm{C}} = C\frac{\mathrm{d}u_{\mathrm{c}}(t)}{\mathrm{d}t}. \tag{2.1.24}$$

Simplifications can be assumed as:

$$\overline{i}_{\mathrm{C}} = C\frac{\Delta u_{\mathrm{c}}}{\Delta t}. \tag{2.1.25}$$

Here it is repeatedly considered that $\frac{\Delta u_{\mathrm{C}}}{\Delta t}$ is the average voltage change over a time interval Δt. In addition, $i_{\mathrm{C}} = I_2$ and $\Delta u_{\mathrm{c}} = \Delta u_2$ for T_{on} applies. Δt is again equal to the switch-on period and can be represented by the following equation

$$\Delta t = \frac{D}{f_{\mathrm{s}}}. \tag{2.1.26}$$

Now that everything is known, the capacitance can be determined as

$$C = \frac{I_2 \Delta t}{\Delta u_2} = \frac{I_2 D}{\Delta u_2 f_{\mathrm{s}}} = \frac{2\ \mathrm{A} \cdot 0.8}{120\ \mathrm{mV} \cdot 100\ \mathrm{kHz}} = 133.333\ \mathrm{\mu F}. \tag{2.1.27}$$

During off-state the capacitor is charged with the current $i_1 - i_2$. The average of the capacitor current is zero. An important rule during dimensioning the capacitor is the RMS value of the current. The RMS value of the current is determined using the following equation

$$I_\mathrm{C} = \sqrt{\frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} i_\mathrm{C}^2(t)\mathrm{d}t}. \tag{2.1.28}$$

Considering the switch-on and switch-offs periode, we can rewrite as:

$$I_\mathrm{C} = \sqrt{\frac{1}{T_\mathrm{s}}\int_0^{DT_\mathrm{s}} i_\mathrm{C}^2(t)\mathrm{d}t + \frac{1}{T_\mathrm{s}}\int_{DT_\mathrm{s}}^{T_\mathrm{s}} i_\mathrm{C}^2(t)\mathrm{d}t}. \tag{2.1.29}$$

While neglecting the current ripple, one receives:

$$I_\mathrm{C} = \sqrt{D(I_2)^2 + (1-D)(I_1 - I_2)^2} = \sqrt{0.8 \cdot (2\ \mathrm{A})^2 + (1-0.8)\cdot(10\ \mathrm{A} - 2\ \mathrm{A})^2} = 4\ \mathrm{A}. \tag{2.1.30}$$

The current ripple causes an additional fluctuation in the current through the capacitor, which leads to a higher RMS value. The following equation is used for the current ripple

$$I_\mathrm{C} = \sqrt{D(I_2)^2\left[1 + \frac{1}{3}\left[\frac{\Delta i_\mathrm{L}}{2I_2}\right]^2\right] + (1-D)(I_1 - I_2)^2\left[1 + \frac{1}{3}\left[\frac{\Delta i_\mathrm{L}}{2(I_1 - I_2)}\right]^2\right]} \tag{2.1.31}$$

$$I_\mathrm{C} = \sqrt{0.8 \cdot (2\ \mathrm{A})^2\left[1 + \frac{1}{3}\left[\frac{0.833\ \mathrm{A}}{2\cdot(2\ \mathrm{A})}\right]^2\right] + (1-0.8)(10\ \mathrm{A} - 2\ \mathrm{A})^2\left[1 + \frac{1}{3}\left[\frac{0.833\ \mathrm{A}}{2(10\ \mathrm{A} - 2\ \mathrm{A})}\right]^2\right]} = 4.023\ \mathrm{A}. \tag{2.1.32}$$

As the effective value (RMS) of currents increases due to fluctuations, the value of I_C in (2.1.32) is slightly higher than in (2.1.30).

Task 2.2: Boost converter with losses

Next, the impact of power losses on the above's converter behavior is investigated. For the following points an ideally smoothed input current and a ripple-free output voltage are assumed. The boost converter with losses is shown in Fig. 2.2.1.

2.2.1 From now on, the influence of the resistor R_L is considered. Derive the efficiency η and voltage ratio U_1/U_2 of the boost converter in dependence on the duty cycle D and the resistance ratio $\alpha = R_\mathrm{L}/R$. Sketch both functions, η and U_2/U_1 over the duty cycle D and analyze the findings.

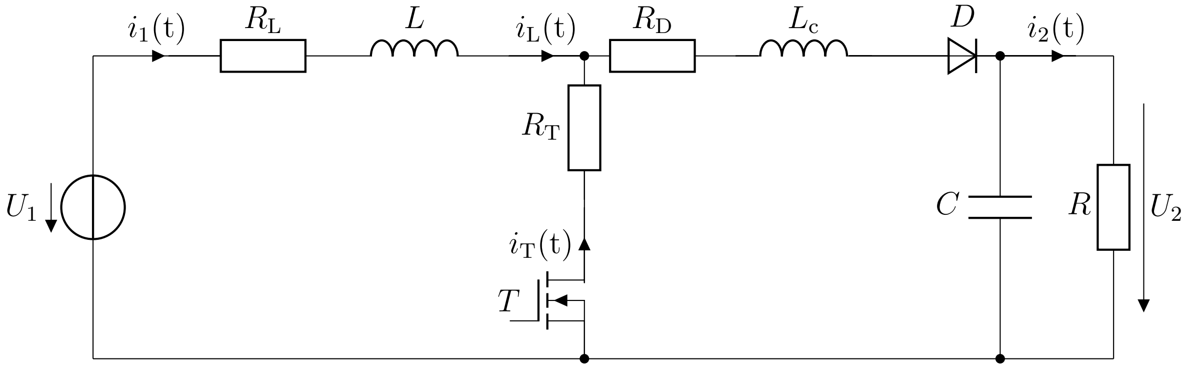


Figure 2.2.1: Boost converter with losses.

Answer:

We are looking for: $\eta = F(D, \alpha)$ and $\frac{U_2}{U_1} = F(D, \alpha)$ with $\alpha = \frac{R_\mathrm{L}}{R}$. Assuming a good smoothing of the inductor current yields to $I_\mathrm{L} = \bar{i}_\mathrm{L} = i_\mathrm{L}(t)$. The efficiency is given by the following equation

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + P_l} = \frac{U_2 I_2}{U_2 I_2 + R_\mathrm{L} I_\mathrm{L}^2}. \tag{2.2.1}$$

It must be noted that the capacitor is charged when it is switched off and discharged when it is switched on. The same applies to the inductor, which discharges when it is switched off. The following equation can be created from these boundary conditions

$$I_2 = \frac{1}{T_\mathrm{s}} \int_0^{DT_\mathrm{s}} i_\mathrm{C,discharge}(t)\mathrm{d}t - \frac{1}{T_\mathrm{s}} \int_{DT_\mathrm{s}}^{T_\mathrm{s}} i_\mathrm{C,charge}(t)\mathrm{d}t + \frac{1}{T_\mathrm{s}} \int_{DT_\mathrm{s}}^{T_\mathrm{s}} I_\mathrm{L}(t)\mathrm{d}t. \tag{2.2.2}$$

As the charging and discharging currents of the capacitor are the same, they cancel each other out

$$0 = \frac{1}{T_\mathrm{s}} \int_0^{DT_\mathrm{s}} i_\mathrm{C,discharge}(t)\mathrm{d}t - \frac{1}{T_\mathrm{s}} \int_{DT_\mathrm{s}}^{T_\mathrm{s}} i_\mathrm{C,charge}(t)\mathrm{d}t. \tag{2.2.3}$$

This simplifies the expression to

$$I_2 = \frac{1}{T_\mathrm{s}} \int_{DT_\mathrm{s}}^{T_\mathrm{s}} I_\mathrm{L}(t)\mathrm{d}t. \tag{2.2.4}$$

As described in the task description, the current I_L is independent of t, so the integral can be solved as follows:

$$I_2 = \frac{I_\mathrm{L}}{T_\mathrm{s}}(T_\mathrm{s} - (DT_\mathrm{s})) = I_\mathrm{L}(1 - D). \tag{2.2.5}$$

Next, we insert (2.2.5) into (2.2.1) and receive:

$$\eta = \frac{U_2 I_2}{U_2 I_2 + \frac{R_\mathrm{L} I_2^2}{(1-D)^2}} = \frac{U_2}{U_2 + \frac{R_\mathrm{L} I_2}{(1-D)^2}} = \frac{R I_2}{R I_2 + \frac{R_\mathrm{L} I_2}{(1-D)^2}} = \frac{R}{R + \frac{R_\mathrm{L}}{(1-D)^2}} = \frac{1}{1 + \frac{\alpha}{(1-D)^2}}. \tag{2.2.6}$$

The efficiency is defined as:

$$\eta = \frac{P_2}{P_1} = \frac{U_2 I_2}{U_1 I_\mathrm{L}}. \tag{2.2.7}$$

Rearranging leads to:

$$\frac{U_2}{U_1} = \eta\frac{I_\mathrm{L}}{I_2} = \eta\frac{1}{(1-D)} = \frac{1}{1+\frac{\alpha}{(1-D)^2}}\frac{1}{1-D} = \frac{1}{(1-D)+\frac{\alpha}{1-D}}\cdot\frac{(1-D)}{(1-D)} = \frac{1-D}{(1-D)^2+\alpha}. \tag{2.2.8}$$

This results for η as a function of D and α given as:

$$\eta(D,\alpha) = \frac{1}{1+\frac{\alpha}{(1-D)^2}}. \tag{2.2.9}$$

Moreover, the voltage transfer ration $\frac{U_2}{U_1}$ as a function of D and α follows as:

$$\frac{U_2}{U_1}(D,\alpha) = \frac{1-D}{(1-D)^2+\alpha}. \tag{2.2.10}$$

2.2.2 For $R_\mathrm{L} = 0.2\ \Omega$, determine the required duty cycle value for the given input and output voltages.

Answer:

Defining the auxillary duty cycle $D_\mathrm{P} = 1-D$ we can rewrite the voltage gain expression as: $\frac{U_2}{U_1} = \frac{D_\mathrm{P}}{(D_\mathrm{P})^2+\alpha}$. As this expression is a quadratic term, the quadratic formula can be used to solve for D_P

$$\frac{U_2}{U_1}(D_\mathrm{P})^2 - D_\mathrm{P} + \frac{U_2\alpha}{U_1} = 0, \tag{2.2.11}$$

$$D_\mathrm{P}^2 - \frac{U_1}{U_2}D_\mathrm{P} + \alpha = 0. \tag{2.2.12}$$

The first term resulting from (2.2.12) is:

$$D_\mathrm{P1} = \frac{1}{2}\frac{U_1}{U_2} + \sqrt{\left(\frac{1}{2}\frac{U_1}{U_2}\right)^2 - \alpha} = \frac{1}{2}\cdot\frac{12\ \mathrm{V}}{60\ \mathrm{V}} + \sqrt{\left(\frac{1}{2}\cdot\frac{12\ \mathrm{V}}{60\ \mathrm{V}}\right)^2 - \frac{0.2\ \Omega}{30\ \Omega}} = 0.158. \tag{2.2.13}$$

If one inserts this into the equation $D_1 = 1 - D_\mathrm{P1}$, one gets the result $D_1 = 0.842$.
The second term resulting from (2.2.12) is:

$$D_\mathrm{P2} = \frac{1}{2}\frac{U_1}{U_2} - \sqrt{\left(\frac{1}{2}\frac{U_1}{U_2}\right)^2 - \alpha} = \frac{1}{2}\cdot\frac{12\ \mathrm{V}}{60\ \mathrm{V}} - \sqrt{\left(\frac{1}{2}\cdot\frac{12\ \mathrm{V}}{60\ \mathrm{V}}\right)^2 - \frac{0.2\ \Omega}{30\ \Omega}} = 0.042. \tag{2.2.14}$$

Leading to $D_2 = 0.958$ the duty cycle is only a theoretical value as one can find from the following loss analysis: The inductor currents for the two above consdered cases are:

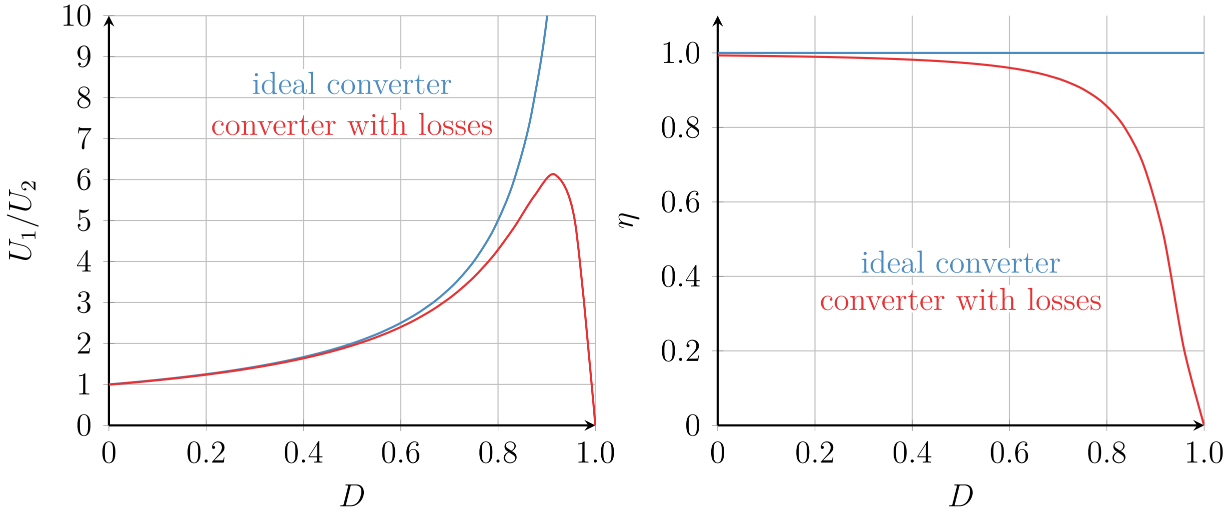
$$I_\mathrm{L1} = \frac{I_2}{1-D_1} = \frac{2\ \mathrm{A}}{1-0.842} = 12.679\ \mathrm{A}, \tag{2.2.15}$$

$$I_\mathrm{L2} = \frac{I_2}{1-D_2} = \frac{2\ \mathrm{A}}{1-0.958} = 47.321\ \mathrm{A}. \tag{2.2.16}$$

Putting these current values in relation to the losses, the following can be determined by using the relation $P = I^2R$:

$$\frac{(47.321\ \text{A})^2}{(12.679\ \text{A})^2} = 13.9296. \tag{2.2.17}$$

The losses are 14 times higher by a duty cycle of $D_2 = 0.958$ compared to $D_1 = 0.842$.
The voltage ratio dependant on the duty cycle D is shown in Sol.-Fig. 2.2.1 on the left side. Both functions for the loss-free and lossy operation are visualized. Moreover, on the right side of Sol.-Fig. 2.2.1 the resulting efficiency cures are shown.



Solution Figure 2.2.1: Voltage ratio (on the left) and the efficiency function (on the right) are visualized. Both functions are dependent on the duty cycle D.

2.2.3 Calculate the efficiencies $\eta_1(D_1)$ and $\eta_2(D_2)$ of the boost converter for an output current of 2 A and a coil resistance of $R_\text{L} = 0.2\ \Omega$.

Answer:

With the previous calculated duty cycles, the efficiency is determined with (2.2.6) for $D_1 = 0.842$ by

$$\eta_1 = \frac{1}{1 + \frac{\frac{0.2\ \Omega}{30\ \Omega}}{(1-0.842)^2}} = 0.789, \tag{2.2.18}$$

and, with the second duty cycle $D_2 = 0.958$ the efficiency results into:

$$\eta_2 = \frac{1}{1 + \frac{\frac{0.2\ \Omega}{30\ \Omega}}{(1-0.958)^2}} = 0.211. \tag{2.2.19}$$

2.2.4 In addition, consider the conduction losses of the diode D and the transistor T. Assume an equivalent resistance of $R_\text{D} = 0.5\ \text{m}\Omega$ and forward voltage of $U_\text{D} = 1$ V for the diode and an equivalent resistance of $R_\text{DS,on} = 30\ \text{m}\Omega$ for the transistor. Determine the required duty cycle value when the conduction losses are considered.

Answer:
Considering the losses, the equation is extended with the loss terms by

$$U_1 I_\mathrm{L} = U_2 I_2 + R_\mathrm{L} I_\mathrm{L}^2 + R_\mathrm{DS,on} I_\mathrm{T}^2 + U_\mathrm{th} \bar{i}_\mathrm{D} + R_\mathrm{D} I_\mathrm{D}^2, \tag{2.2.20}$$

with the average diode current $\bar{i}_\mathrm{D}$ and the RMS diode current I_D. The diode RMS current is determined as:

$$I_\mathrm{D} = \sqrt{\frac{1}{T}\int_0^T i_\mathrm{D}(t)^2 \mathrm{d}t} = \sqrt{\frac{1}{T}\left(\int_0^{T_\mathrm{on}} 0\mathrm{d}t + \int_{T_\mathrm{on}}^{T_\mathrm{s}} I_\mathrm{L}(t)^2 \mathrm{d}t\right)} = \sqrt{\frac{T_\mathrm{s} - T_\mathrm{on}}{T_\mathrm{s}} I_\mathrm{L}^2} = \sqrt{1-D} I_\mathrm{L}. \tag{2.2.21}$$

Moreover, the average diode current is calculated with

$$\bar{i}_\mathrm{D} = (1-D) I_\mathrm{L} = I_2, \tag{2.2.22}$$

and, the current through the transistor is given by:

$$I_\mathrm{T} = \sqrt{\frac{1}{T}\int_0^T i_\mathrm{T}(t)^2 \mathrm{d}t} = \sqrt{\frac{1}{T_\mathrm{s}}\left(\int_0^{T_\mathrm{on}} I_\mathrm{L}^2 \mathrm{d}t + \int_{T_\mathrm{on}}^{T_\mathrm{s}} 0\mathrm{d}t\right)} = \frac{T_\mathrm{on}}{T_\mathrm{s}} I_\mathrm{L}^2 = \sqrt{D} I_\mathrm{L}. \tag{2.2.23}$$

Next, the currents in (2.2.20) are replaced with I_2, which leads to

$$\frac{U_1 I_2}{1-D} = U_2 I_2 + R_\mathrm{L} \frac{I_2^2}{(1-D)^2} + R_\mathrm{DS,on} \frac{D I_2^2}{(1-D)^2} + U_\mathrm{th} I_2 + R_\mathrm{D} \frac{I_2^2}{1-D}, \tag{2.2.24}$$

with $I_2 = \frac{U_2}{R}$ and $D_\mathrm{p} = 1 - D$ yields:

$$U_1 D_\mathrm{p} = U_2 D_\mathrm{p}^2 + \frac{R_\mathrm{L}}{R} U_2 + (1 - D_\mathrm{p}) \frac{R_\mathrm{DS,on}}{R} U_2 + U_\mathrm{th} D_\mathrm{p}^2 + \frac{R_\mathrm{D}}{R} U_2 D_\mathrm{p}. \tag{2.2.25}$$

Rearrang the equation into the quadratic formula structure by

$$0 = (U_2 + U_\mathrm{th}) D_\mathrm{p}^2 + \left(\frac{R_\mathrm{D}}{R} U_2 - \frac{R_\mathrm{DS,on}}{R} U_2 - U_1\right) D_\mathrm{p} + \frac{R_\mathrm{DS,on} + R_\mathrm{L}}{R} U_2, \tag{2.2.26}$$

Leads to the first solution given by

$$D_\mathrm{P3} = -\frac{1}{2}\frac{\frac{R_\mathrm{D}}{R} U_2 - \frac{R_\mathrm{DS,on}}{R} U_2 - U_1}{U_2 + U_\mathrm{th}} + \sqrt{\left(\frac{1}{2}\frac{\frac{R_\mathrm{D}}{R} U_2 - \frac{R_\mathrm{DS,on}}{R} U_2 - U_1}{U_2 + U_\mathrm{th}}\right)^2 - \frac{\frac{R_\mathrm{DS,on}+R_\mathrm{L}}{R} U_2}{U_2 + U_\mathrm{th}}} = 0.1458, \tag{2.2.27}$$

which results in a duty cycle of $D_3 = 0.8542$. The second solution is calculated in the same way as follows

$$D_\mathrm{P3} = -\frac{1}{2}\frac{\frac{R_\mathrm{D}}{R} U_2 - \frac{R_\mathrm{DS,on}}{R} U_2 - U_1}{U_2 + U_\mathrm{th}} - \sqrt{\left(\frac{1}{2}\frac{\frac{R_\mathrm{D}}{R} U_2 - \frac{R_\mathrm{DS,on}}{R} U_2 - U_1}{U_2 + U_\mathrm{th}}\right)^2 - \frac{\frac{R_\mathrm{DS,on}+R_\mathrm{L}}{R} U_2}{U_2 + U_\mathrm{th}}} = 0.0517, \tag{2.2.28}$$

resulting in $D = 0.9483$.

2.2.5 Beside the conduction losses, also switching losses need to be considered in practice. In Fig. 2.2.2 the voltage and current waveforms are visualized for the turn-off event of a diode (reverse recovery effect). Therefore, calculate the turn-off losses for a fast diode with a commutation inductivity loop of $L_c = 500$ nH. The peak reverse recovery current is $\hat{i}_{rr} = 4$ A and the reverse recovery time is $t_{rr} = t_2 - t_0 = 46.6$ ns.

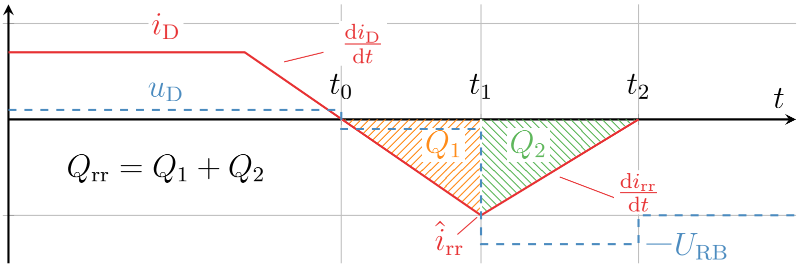


Figure 2.2.2: Turn-off behavior of a fast silicon diode.

Answer:

Switching losses occur when the diode voltage or current is not zero. In Fig. 2.2.2 the voltage and current waveforms are shown. The voltage u_D is for the interval t_0 until t_1 small and, therefore, the switching losses. At the time step t_1 the diode is blocking and the reverse breakdown voltage U_{RB} applies. Due to this much bigger value, the switching losses increases significantly, resulting that only the interval between t_1 and t_2 is responsible for the switching losses. Hence, the losses are calculated as follows

$$P_l = \frac{1}{T}\int_{t_1}^{t_2} i_{rr} U_{RB} dt = \frac{1}{T} U_{RB} \int_{t_1}^{t_2} i_{rr} dt, \qquad (2.2.29)$$

with the electric charge

$$Q_2 = \int_{t_1}^{t_2} i_{rr} dt, \qquad (2.2.30)$$

which results to the loss as:

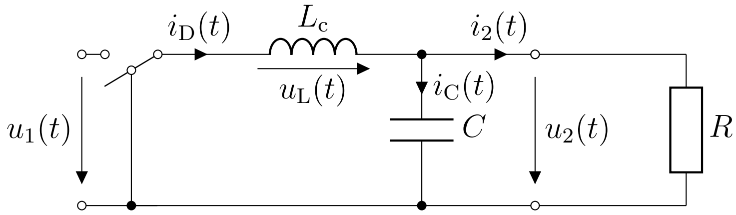
$$P_l = \frac{U_{RB} Q_2}{T}. \qquad (2.2.31)$$

With the assumption of a linear current waveform, the derivative is replaced with the difference quotient for the time interval $t_1 - t_2$ by:

$$\Delta i_D = \frac{di_D(t)}{dt}. \qquad (2.2.32)$$

The equivalent circuit diagram of the diode switch-off event is shown in Sol.-Fig. 2.2.2. The current $i_D(t)$ is dependent on the voltage U_2 by:

$$\Delta i_D \approx \frac{di_D(t)}{dt} = \frac{U_2}{L_c} = \frac{60 \text{ V}}{500 \text{ nH}} = 120 \ \frac{\text{A}}{\mu\text{s}}. \qquad (2.2.33)$$



Solution Figure 2.2.2: Equivalent circuit diagram of the diode switch-off event.

With the current slope Δi_D and the time $t_{10} = t_1 - t_0$, the maximum reverse current $\hat{i}_{rr}$ is defined as

$$\Delta i_D t_{10} = \hat{i}_{rr}, \tag{2.2.34}$$

which is rearranged to calculate the time interval with:

$$t_{10} = \frac{\hat{i}_{rr}}{i_D} = \frac{4\ \mathrm{A}}{120\ \frac{\mathrm{A}}{\mu\mathrm{s}}} = 33.3\ \mathrm{ns}. \tag{2.2.35}$$

Hence, the time interval t_{21} is given with:

$$t_{21} = t_{rr} - t_{10} = 46.6\ \mathrm{ns} - 33.3\ \mathrm{ns} = 13.3\ \mathrm{ns}. \tag{2.2.36}$$

With (2.2.30) the electrical charge Q_2 is calculated by:

$$Q_2 = \frac{1}{2} t_{21} \hat{i}_{rr} = \frac{13.3\ \mathrm{ns}}{2} \cdot 4\ \mathrm{A} = 26.6\ \mathrm{nC}. \tag{2.2.37}$$

Reverse breakdown voltage is determined form the ECD (Sol.-Fig. 2.2.2) with the second Kirchhoff's law with:

$$U_{RB} = U_2 + L_c \frac{\mathrm{d}i_{rr}(t)}{\mathrm{d}t} = 60\ \mathrm{V} + 500\ \mathrm{nH} \frac{4\ \mathrm{A}}{13.3\ \mathrm{ns}} = 210.4\ \mathrm{V}. \tag{2.2.38}$$

Therefore, the turn-off switching losses are calculated as:

$$P_{l,D1} = \frac{U_{RB} Q_2}{T_s} = \frac{210.4\ \mathrm{V} \cdot 26.6\ \mathrm{nC}}{\frac{1}{100\ \mathrm{kHz}}} = 0.56\ \mathrm{W}. \tag{2.2.39}$$

2.2.6 Determine the turn-off switching losses of a normal silicon diode with a reverse recovery work $Q_{rr} = 16\ \mu\mathrm{C}$ and a rate of current rise $\frac{\mathrm{d}i_{rr}}{\mathrm{d}t} = 40\ \frac{\mathrm{A}}{\mu\mathrm{s}}$. Compare the result with the previous subtask.
Answer:
The reverse breakdown voltage is calculated with (2.2.38), resulting in:

$$U_{RB,2} = 60\ \mathrm{V} \cdot 500\ \mathrm{nH} \cdot 40\ \frac{\mathrm{A}}{\mu\mathrm{s}} = 80\ \mathrm{V}. \tag{2.2.40}$$

Furthermore, the time intervals are necessary to calculate the diode switch-off loss, which is given by

$$T_{10} = t_1 - t_0 = \frac{\hat{i}_{\mathrm{rr}}}{\Delta i_{\mathrm{D}}} \tag{2.2.41}$$

and

$$T_{21} = t_2 - t_1 = \frac{\hat{i}_{\mathrm{rr}}}{\Delta i_{\mathrm{rr}}}. \tag{2.2.42}$$

With the determined time intervals, the corresponding electric charge is defined with

$$Q_1 = \frac{t_{10}}{2}\hat{i}_{\mathrm{rr}} \tag{2.2.43}$$

and

$$Q_2 = \frac{t_{21}}{2}\hat{i}_{\mathrm{rr}}. \tag{2.2.44}$$

With Δi_{rr} given in this subtask and with Δi_{D} from the previous subtask, the relation of the two electric charges is built to eliminate the time intervals as

$$\frac{Q_1}{Q_2} = \frac{\frac{\frac{\hat{i}_{\mathrm{rr}}}{\Delta i_{\mathrm{D}}}}{2}\hat{i}_{\mathrm{rr}}}{\frac{\frac{\hat{i}_{\mathrm{rr}}}{\Delta i_{\mathrm{rr}}}}{2}\hat{i}_{\mathrm{rr}}} = \frac{\Delta i_{\mathrm{rr}}}{\Delta i_{\mathrm{D}}} = \frac{40\ \frac{\mathrm{A}}{\mathrm{\mu s}}}{120\ \frac{\mathrm{A}}{\mathrm{\mu s}}} = \frac{1}{3}, \tag{2.2.45}$$

which means, that the charge of $Q_1 = \frac{1}{3}Q_2$. The total electric charge is defined by

$$Q_{\mathrm{rr}} = Q_1 + Q_2 = \frac{1}{3}Q_2 + Q_2 = \frac{4}{3}Q_2, \tag{2.2.46}$$

which is resorted to determine Q_2 by:

$$Q_2 = \frac{3}{4}Q_{\mathrm{rr}} = \frac{3}{4} \cdot 16 \cdot 10^{-6}\ \mathrm{As} = 12\ \mathrm{\mu C}. \tag{2.2.47}$$

As in the previous task, the main losses occur during the T_{21} time interval with the electric charge Q_2. Using (2.2.40) from the beginning, the total switch-off loss are calculated by:

$$P_{\mathrm{l,D2}} = \frac{U_{\mathrm{RB,2}}Q_2}{T_{\mathrm{s}}} = \frac{80\ \mathrm{V} \cdot 12\ \mathrm{\mu C}}{10\ \mathrm{\mu s}} = 96\ \mathrm{W}. \tag{2.2.48}$$

Comparing the switch-off losses of the two diodes with

$$\frac{P_{\mathrm{l,D2}}}{P_{\mathrm{l,D1}}} = \frac{96\ \mathrm{W}}{0.59\ \mathrm{W}} = 171.4. \tag{2.2.49}$$

which means, that the loss of the second diode are approx. 171 times higher than of the first diode. This results in a dramatic higher cooling effort for the second diode assuming the same operation area.

Task 2.3: Buck-boost converter

A wide input-to-output voltage range can be realized by the cascade of buck and boost converters with a common inductance. With this topology the output voltage can be adjusted to a value which is higher or lower than the input voltage.

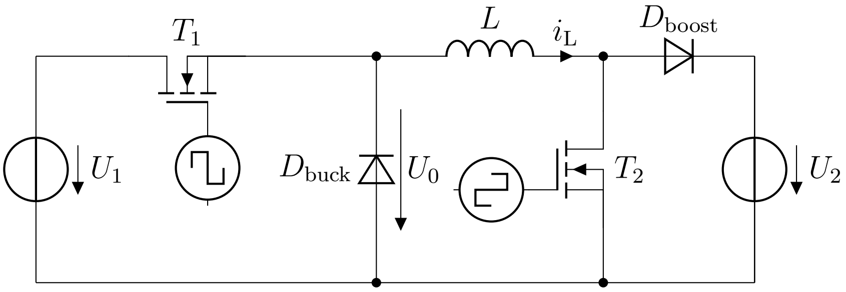


Figure 2.3.1: Buck-boost converter circuit.

Input voltage:	$U_1 = 320$ V to 720 V	Output voltage:	$U_2 = 400$ V
Output power:	$P_{out,min} = 5000$ W	Switching frequency:	$f_s = 25$ kHz

Table 2.3.1: Parameters of the circuit.

The output voltage is kept at the specified constant value by adjusting the duty cycles D_1 (of transistor T_1) and D_2 (of transistor T_2) using a control system. Both transistors operate at the same switching frequency. The ripple of the output voltage and of the current in the inductor can be ignored. The current in L is continuous. Initially, both transistors operate with the same duty cycle $D_1 = D_2 = D$ and their switching patterns are synchronized.

2.3.1 Calculate the duty cycle of the transistors T_1 and T_2 depending on the voltage transformation ratio U_2/U_1.

Answer:

For calculation of U_2 the equations for boost converter and buck converter are to consider:

$$U_2 = U_0 \frac{1}{1 - D_2} \qquad \text{and} \qquad U_0 = U_1 \cdot D_1 \tag{2.3.1}$$

and using $D_1 = D_2 = D$ leads to

$$U_2 = \frac{D}{1 - D} U_1 \qquad \text{and} \qquad D = \frac{U_2}{U_1 + U_2}. \tag{2.3.2}$$

Both equations leads to

$$D = \frac{\frac{U_2}{U_1}}{1 + \frac{U_2}{U_1}}. \tag{2.3.3}$$

2.3.2 Calculate I_L depending on D. Plot D and I_L against U_1 and enter the numerical values for $U_1 = 320$ V, $U_1 = 400$ V and $U_1 = 720$ V.

Answer:

The current I_2 though the load depends on the power:

$$I_2 = \frac{P_2}{U_2} = \frac{400\ \mathrm{W}}{400\ \mathrm{V}} = 12.5\ \mathrm{A}. \tag{2.3.4}$$

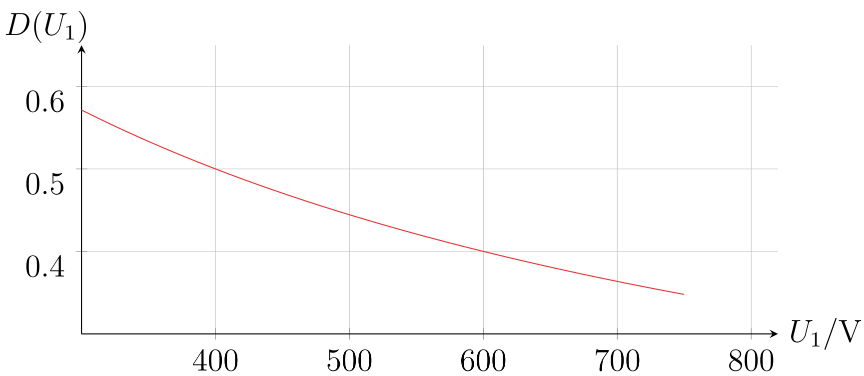
The current though the inductor depends on the duty cycle and the current I_2:

$$I_\mathrm{L} = I_2 \frac{1}{1-D} = I_2 \left(1 + \frac{U_2}{U_1}\right). \tag{2.3.5}$$

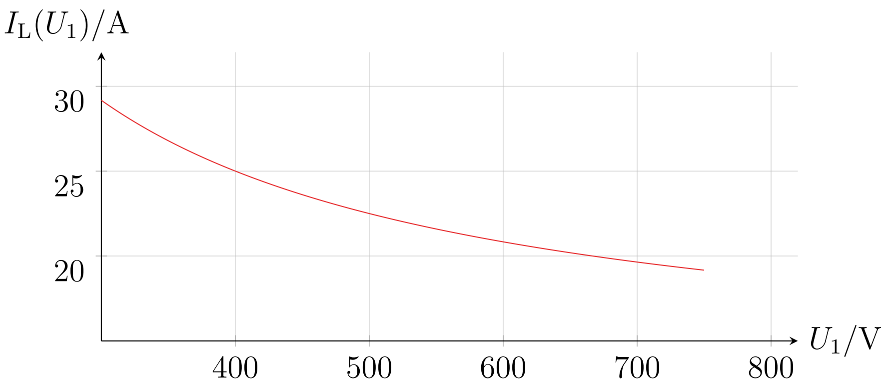
The current I_2 can be calculated based on the power and with I_2, the current through the inductor is subsequently determined. The numerical values for the three voltages are calculated with (2.3.3) and (2.3.5). The result is displayed in Sol.-Tab. 2.1. The plots are shown in Sol.-Fig. 2.3.1 and Sol.-Fig. 2.3.2

U_1	D	I_L
320 V	0.56	28 A
400 V	0.5	22.5 A
720 V	0.36	19.5 A

Solution Table 2.1: D and I_L at U_1.



Solution Figure 2.3.1: Duty cycle versus input voltage.



Solution Figure 2.3.2: Inductor current versus input voltage.

Both transistors should now be able to have different duty cycles. Assume that the transistors are switched on at the same time.

2.3.3 Graphically represent the time profiles of the voltage at L for $U_1 = 320$ V and $D_1 = 0.9$ and for $U_1 = 720$ V and $D_2 = 0.1$ for one pulse period each.

Answer:

Case 1: Using $U_1 = 320$ V and $D_1 = 0.9$ the duty cycle D_2 is obtained as

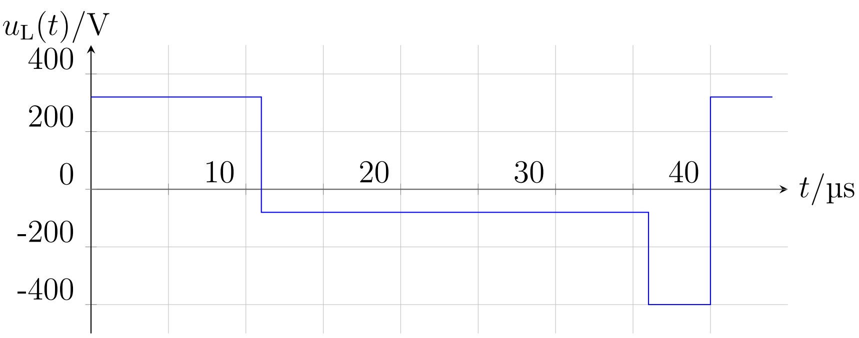
$$D_2 = 1 - D_1 \frac{U_1}{U_2} = 1 - 0.9 \cdot \frac{320\ \mathrm{V}}{400\ \mathrm{V}} = 0.28. \tag{2.3.6}$$

Sol.-Tab. 2.2 displays the inductor voltage of one period.

$U_L = U_1$	$0 \leq t \leq D_2 \cdot T_s$
$U_L = U_1 - U_2$	$D_2 \cdot T_s \leq t \leq D_1 \cdot T_s$
$U_L = -U_2$	$D_1 \cdot T_s \leq t \leq T_s$

Solution Table 2.2: U_L within the switching period.

This is displayed in Sol.-Fig. 2.3.3.

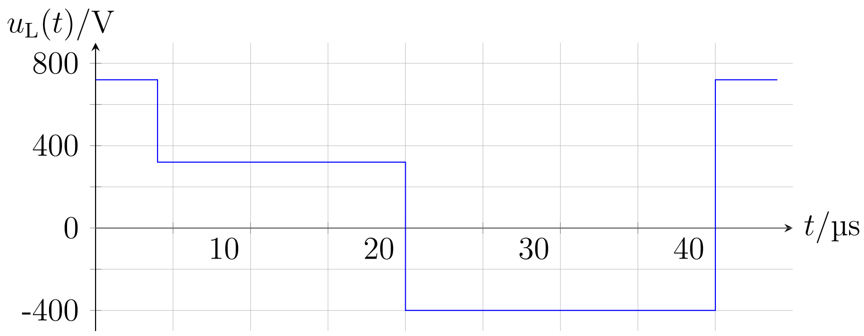


Solution Figure 2.3.3: Voltage at inductor in case 1.

Case 2: Using $U_1 = 720$ V and $D_1 = 0.1$ the duty cycle D_1 is obtained as

$$D_1 = \frac{U_2}{U_1}(1 - D_2) = \frac{400 \text{ V}}{720 \text{ V}} \cdot (1 - 0.1) = 0.5. \tag{2.3.7}$$

Again the inductor voltage is to calculate according Sol.-Tab. 2.2. The result is displayed in Sol.-Fig. 2.3.4.



Solution Figure 2.3.4: Voltage at inductor in case 2.

2.3.4 Calculate the voltage transformation ratio as a function of D_1 and D_2.

Answer:

Based on (2.3.7) we easily solve to the voltage transformation ratio:

$$\frac{U_2}{U_1}(1 - D_2) = D_1 \qquad \Rightarrow \qquad \frac{U_2}{U_1} = \frac{D_1}{1 - D_2}. \tag{2.3.8}$$

2.3.5 Express the current I_L as a function of the specified operating parameters (U_1, U_2, P_2) and as a function of D_1 and D_2.

Answer:

The dependency of I_L from I_2 and the duty cycle is leading to

$$I_L = I_2 \frac{1}{1 - D_2} = \frac{P_2}{U_2} \frac{1}{1 - D_2}. \tag{2.3.9}$$

.

2.3.6 Are the calculated relationships valid if T_1 and T_2 do not switch synchronously or operate with different clock frequencies?

Answer:

Yes, the relationsships are independent from the switching frequency and switching points. In Sol.-Fig. 2.3.3 and Sol.-Fig. 2.3.4 the area size above and below the zero-line are equal. This is kept independent of the the switching frequency and the switching points.

If the transistors T_1 and T_2 are switched on, a constant voltage drop $U_F = 2.5$ V occurs at the transistors regardless of the current. All other components are considered ideal and loss-free.

2.3.7 How should D_1 and D_2 be selected so that the losses of the overall system are minimal? The relationships calculated under subtask 3.4 and 3.5 can be used for the voltage transformation ratio and the value of I_L.

Answer:

The losses of the transistors are:

$$P_{\mathrm{loss,T1}} = D_1 U_\mathrm{F} I_\mathrm{L} = D_1 U_\mathrm{F} \frac{P_2}{U_2} \cdot \frac{1}{1-D_2} \qquad \text{and} \qquad P_{\mathrm{loss,T2}} = D_2 U_\mathrm{F} I_\mathrm{L} = D_2 U_\mathrm{F} \frac{P_2}{U_2} \cdot \frac{1}{1-D_2}. \tag{2.3.10}$$

So the losses of both transitors can be expressed by

$$P_\mathrm{loss} = (D_1 + D_2)\, U_\mathrm{F} \frac{P_2}{U_2} \cdot \frac{1}{1-D_2} \tag{2.3.11}$$

and with

$$D_1 = \frac{U_1}{U_2}(1-D_2) \qquad \Rightarrow \qquad P_\mathrm{loss} = \left(\frac{U_1}{U_2}(1-D_2) + D_2\right) U_\mathrm{F} \frac{P_2}{U_2} \cdot \frac{1}{1-D_2}. \tag{2.3.12}$$

This leads to

$$P_\mathrm{loss} = P_2 U_\mathrm{F} \left(\frac{1}{U_1} + \frac{1}{U_2} \cdot \frac{D_2}{1-D_2}\right). \tag{2.3.13}$$

The losses minimum is determined by differentiating with respect to the duty cycle D_2.

$$P_\mathrm{loss} = f(D_2) \qquad \Rightarrow \qquad \frac{\mathrm{d}}{\mathrm{d}D_2} f(D_2) = \frac{1}{(1-D_2)^2} = 0. \tag{2.3.14}$$

There is no solution in the real number space for (2.3.14). According (2.3.13) the power loss decreases, when $\frac{D_2}{1-D_2}$ decrease. The following can be conclude from the result:

- Set D_2 as small as possible.
- Activate T_2 only for boost mode.
- Set D_1 as big as as possible.

2.3.8 Plot D_1 and D_2 and the efficiency over U_1 and give numerical values for $U_1 = 320$ V, $U_1 = 400$ V and $U_1 = 720$ V.

Answer:

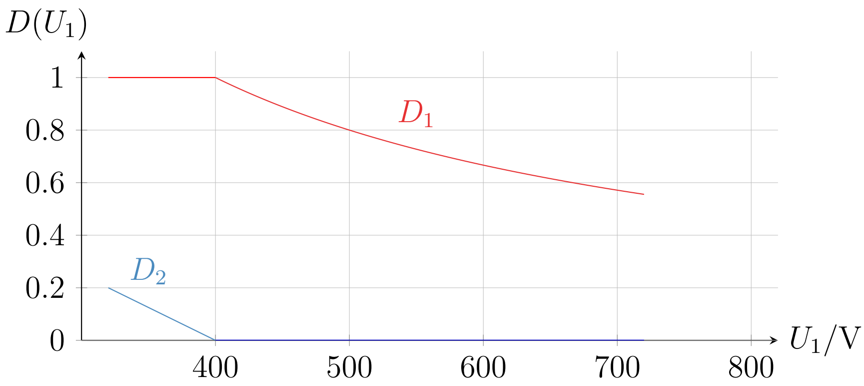
The power loss is to calculate with (2.3.13) for the 3 different voltages. The efficiency is obtained by applying

$$\eta = \frac{P_2}{P_\mathrm{loss} + P_2}. \tag{2.3.15}$$

Using both equations lead to the results displayed in Sol.-Tab. 2.3 and plotted in Sol.-Fig. 2.3.5.

U_1	D_1	D_2	P_{loss}	η_{opt}
320 V	1	0.2	46.88 W	99.071
400 V	1	0	31.25 W	99.379
720 V	0.56	0	17.36 W	99.650

Solution Table 2.3: Duty cycles, power loss and η_{opt} as fct. of U_1.



Solution Figure 2.3.5: Duty cycle versus input voltage.

2.3.9 Calculate the efficiency for the three operating points in subtask 3.2.

Answer:

Again (2.3.13) is used with the condition D_1=D_2=D. This leads to

$$P_{\text{loss}} = 2DU_{\text{F}}I_{\text{L}} = \frac{U_{\text{f}}P_2}{U_2} \cdot \frac{2D}{1-D}. \tag{2.3.16}$$

If you enter the 3 operation voltages you get the results according Sol.-Tab. 2.4.

U_1	D_1	D_2	P_{loss}	η_{sync}
320 V	0.56	0.56	78.12 W	98.462
400 V	0.5	0.5	62.50 W	98.765
720 V	0.36	0.36	35.16 W	99.300

Solution Table 2.4: Duty cycles, power loss and η_{sync} as fct. of U_1.

2.3.10 How high is the maximum efficiency gain and at which operating point does it occur? Give an explanation for the observed finding.

Answer:

The efficiency gain is calculated by $\Delta\eta$=η_{opt}-η_{sync}. You get following result:
The maximum efficiency gain is at $U_1 = 400$ V. It looks like the efficiency gain is higher within boost mode, but why is the highest efficiency gain at 400 V? Let's consider the power loss at boost mode for the two cases. The power loss needs to be expressed by a function of the voltages:

U_1	$\Delta\eta$
320 V	0.609
400 V	0.614
720 V	0.350

Solution Table 2.5: Efficiency gain at U_1.

$$P_{\mathrm{loss}} = D_1 U_{\mathrm{F}} I_{\mathrm{L}} + D_2 U_{\mathrm{F}} I_{\mathrm{L}}. \tag{2.3.17}$$

Considering the optimal efficiency the $D_1 = 1$ and $D_2 = f(U_2)$ for 320 V $\leq U_2 \leq$ 400 V (boost mode) and $D_1 = 0$ and $D_1 = f(U_2)$ for 400 V $\leq U_2 \leq$ 720 V (buck mode).
For 320 V $\leq U_2 \leq$ 400 V (boost mode)

$$I_{\mathrm{L}} = \frac{P_2}{U_1} \qquad \text{and} \qquad D_1 = 1 \qquad \text{and} \qquad D_2 = 1 - \frac{U_1}{U_2} \tag{2.3.18}$$

leads to

$$P_{\mathrm{loss,opt}} = \frac{2P_2U_{\mathrm{F}}}{U_1} - \frac{P_2U_{\mathrm{F}}}{U_2}. \tag{2.3.19}$$

For 400 V $\leq U_2 \leq$ 720 V (buck mode)

$$I_{\mathrm{L}} = \frac{P_2}{U_1} \qquad \text{and} \qquad D_1 = \frac{U_1}{U_2} \qquad \text{and} \qquad D_2 = 0 \tag{2.3.20}$$

leads to

$$P_{\mathrm{loss,opt}} = \frac{P_2U_{\mathrm{F}}}{U_1}. \tag{2.3.21}$$

For the synchronous operation mode following is valid:

$$I_{\mathrm{L}} = \frac{P_2}{U_2} \cdot \frac{1}{1-D} \qquad \text{and} \qquad D_1 = D_2 = D \qquad \text{and} \qquad D = \frac{U_2}{U_1 + U_2}. \tag{2.3.22}$$

Using (2.3.17) leads to:

$$P_{\mathrm{loss,sync}} = \frac{P_2U_{\mathrm{F}}}{U_1} + \frac{P_2U_{\mathrm{F}}}{U_1} = \frac{2P_2U_{\mathrm{F}}}{U_1}. \tag{2.3.23}$$

The efficiency can be expressed by

$$\eta = \frac{P_2}{P_{\mathrm{loss}} + P_2} = \frac{1}{1 + \frac{P_{\mathrm{loss}}}{P_2}}. \tag{2.3.24}$$

Using (2.3.19) and (2.3.21) in (2.3.24) results in:

$$\eta_{\mathrm{opt,boost}} = \frac{1}{1 + U_{\mathrm{F}}\left(\frac{2}{U_1} - \frac{1}{U_2}\right)} \qquad \text{and} \qquad \eta_{\mathrm{opt,buck}} = \frac{1}{1 + \frac{U_{\mathrm{F}}}{U_1}}. \tag{2.3.25}$$

Using (2.3.23) in (2.3.24) results in:

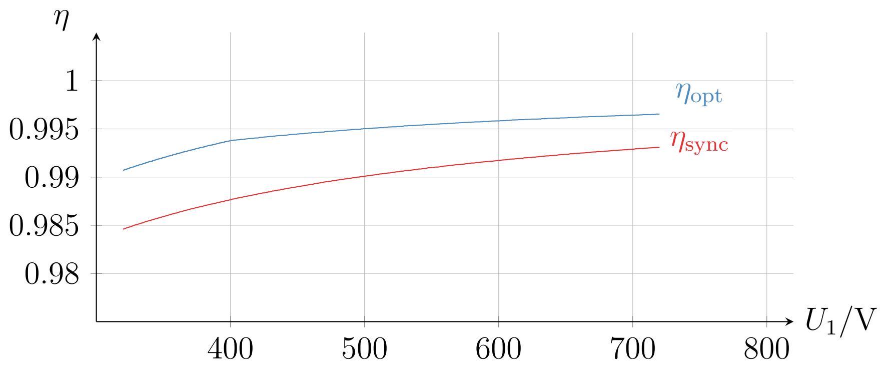
$$\eta_{\text{sync}} = \frac{1}{1 + U_\text{F}\frac{2}{U_1}}. \tag{2.3.26}$$

The efficiency gain is $\Delta\eta = \eta_\text{opt} - \eta_\text{sync}$:

$$\Delta\eta_\text{boost} = \frac{\frac{U_\text{F}}{U_2}}{\left(1 + \frac{2U_\text{F}}{U_1} - \frac{U_\text{F}}{U_2}\right)\left(1 + \frac{2U_\text{F}}{U_1}\right)}. \tag{2.3.27}$$

$$\Delta\eta_\text{buck} = \frac{\frac{U_\text{F}}{U_1}}{\left(1 + \frac{2U_\text{F}}{U_1}\right)\left(1 + \frac{U_\text{F}}{U_1}\right)}. \tag{2.3.28}$$

The two efficiencies are calculated with (2.3.25),(2.3.26) and displayed in Sol.-Fig. 2.3.6.



Solution Figure 2.3.6: Efficiency η_opt and η_sync versus input voltage.

Conclusions:

- (2.3.26) and (2.3.25) shows, that $\eta_\text{opt,boost}$ and η_sync differ only by the constant factor $\frac{U_\text{F}}{U_2}$ which has a relatively greater impact on efficiency at higher U_1.
- (2.3.26) and (2.3.25) shows, that $\eta_\text{opt,buck}$ and η_sync differ in the coefficient of $\frac{U_\text{F}}{U_1}$ in the denominator. This cause that the difference decrease with increasing U_1.
- The difference between the power losses of $P_\text{loss,opt}$ and $P_\text{loss,sync}$ in boost mode is the constant factor $\frac{P_2 U_\text{F}}{U_2}$ which has a proportionally greater effect on the efficiency with smaller U_2 power losses (here larger U_1).
- The highest efficiency gain is obtained at $U_1 = 400$ V since the input and output voltage are equal allowing to not switch at all in the optimized pattern mode (compared to $D_1 = D_2 = 0.5$ in standard synchronous mode leading to unnecessary switching).

Exercise 03: Combined step-up / step-down converters

Task 3.1: Inverting buck-boost converter

An inverting buck-boost converter (see Fig. 3.1.1) is used to generate the negative supply voltage of a control electronic unit. The input voltage is specified as $U_1 = 18$ V, the output voltage is regulated to $U_2 = 12$ V. The output power can vary in the range $P_2 = 2$ W … 15 W.

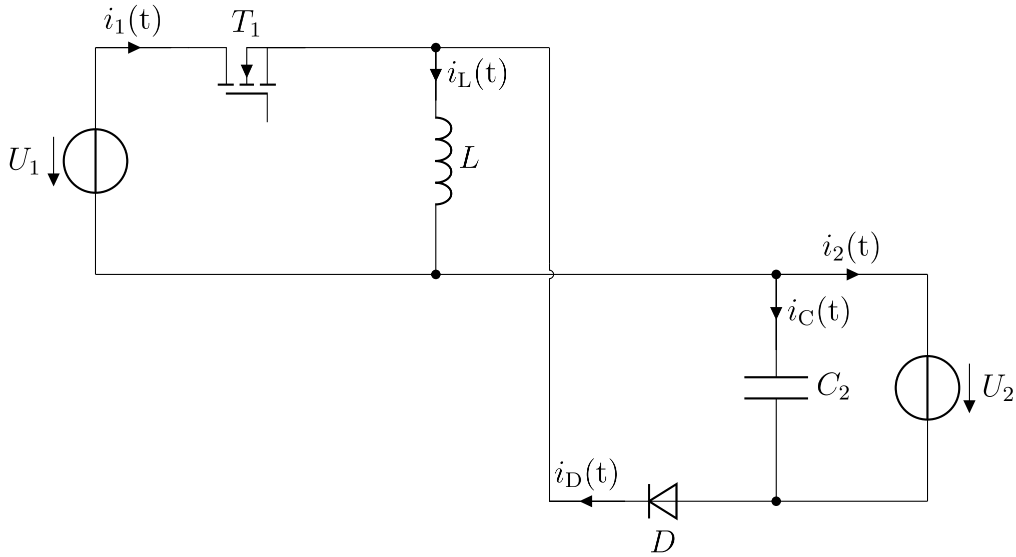


Figure 3.1.1: Inverting buck-boost converter.

3.1.1 The system should operate at boundary conduction mode (BCM) throughout the entire output power range. How should the inductance be selected so that the switching frequency is always above the hearing threshold $f_s = 20$ kHz?

Answer:

The Δ is an approximation of the total differential using a difference equation with the differences Δ. This results in the component differential equations becoming average value equations. During the transistor on time, the input voltage is equal to the inductor voltage and defined as:

$$U_1 = \overline{u}_L = L\frac{\Delta i_L}{\Delta t}. \quad (3.1.1)$$

With voltages U_1 and U_2 given in the task, the duty cycle is calculated as:

$$D = \frac{U_2}{U_1 + U_2} = \frac{12 \text{ V}}{18 \text{ V} + 12 \text{ V}} = 0.4. \quad (3.1.2)$$

During the on phase of the transistor, the total energy for one period (on and off phase) must be stored in the inductor. Therefore, the average power and the corresponding inductor current increases with the inverse value of the duty cycle. Hence, the average inductor current is given with:

$$\bar{i}_{\mathrm{L}} = \frac{P_2}{U_2}\frac{1}{D} = \frac{15\ \mathrm{W}}{12\ \mathrm{V}}\frac{5}{3} = 2.083\ \mathrm{A}. \tag{3.1.3}$$

With BCM the maximum inductor current, i.e., the peak-to-peak ripple, is defined as:

$$\Delta i_{\mathrm{L}} = 2\bar{i}_{\mathrm{L}} = 4.16\ \mathrm{A}. \tag{3.1.4}$$

Here, Δt can also be expressed as:

$$\Delta t = \frac{D}{f_{\mathrm{s}}}. \tag{3.1.5}$$

Therefore, the inductance can be determined using the following equation:

$$L = \frac{\Delta t \overline{u}_{\mathrm{L}}}{\Delta i_{\mathrm{L}}} = \frac{D\overline{u}_{\mathrm{L}}}{f_{\mathrm{s}}\Delta i_{\mathrm{L}}} = \frac{DU_1}{f_{\mathrm{s}}\Delta i_{\mathrm{L}}} = \frac{0.4 \cdot 18\ \mathrm{V}}{20\ \mathrm{kHz} \cdot 4.16\ \mathrm{A}} = 86.4\ \mathrm{\mu H}. \tag{3.1.6}$$

3.1.2 In what value range does the switching frequency f_{s} vary considering the inductance choice from the previous subtask and the given output power range?

Answer:

The inductance equation is used again for this subtask:

$$L = \frac{\Delta t \overline{u}_{\mathrm{L}}}{\Delta i_{\mathrm{L}}} = \frac{D\overline{u}_{\mathrm{L}}}{f_{\mathrm{s}}\Delta i_{\mathrm{L}}}. \tag{3.1.7}$$

This equation can be rewritten to determine the frequency as follows:

$$f_{\mathrm{s}} = \frac{DU_1}{L\Delta i_{\mathrm{L}}}. \tag{3.1.8}$$

As the output power is specified as a value range, the highest and lowest values can be used. The lowest and highest current should be determined from these two values, from which the value range of the frequency f_{s} can then be determined. The average inductor current is calculated as:

$$\bar{i}_{\mathrm{L}}(P_2 = 2\ \mathrm{W}) = \frac{P_2}{U_2}\frac{1}{D} = \frac{2\ \mathrm{W}}{12\ \mathrm{V}}\frac{5}{3} = 0.278\ \mathrm{A}, \tag{3.1.9}$$

$$\bar{i}_{\mathrm{L}}(P_2 = 15\ \mathrm{W}) = \frac{P_2}{U_2}\frac{1}{D} = \frac{15\ \mathrm{W}}{12\ \mathrm{V}}\frac{5}{3} = 2.0833\ \mathrm{A}. \tag{3.1.10}$$

With (3.1.8) the resulting switching frequencies are:

$$f_{\mathrm{s}}(P_2 = 2\ \mathrm{W}) = \frac{0.4 \cdot 18\ \mathrm{V}}{86.4\ \mathrm{\mu H} \cdot 2 \cdot 0.278\ \mathrm{A}} = 150\ \mathrm{kHz}, \tag{3.1.11}$$

$$f_{\mathrm{s}}(P_2 = 15\ \mathrm{W}) = \frac{0.4 \cdot 18\ \mathrm{V}}{86.4\ \mathrm{\mu H} \cdot 2 \cdot 2.0833\ \mathrm{A}} = 20\ \mathrm{kHz}. \tag{3.1.12}$$

The switching frequency f_s varies in the range from 20 kHz ... 150 kHz for the specified output power range in the task.

3.1.3 What is the peak value $\hat{i}_1 = \max\{i_1(t)\}$ of the transistor current?
Answer:
As the current through the transistor is the current i_1, their maximum values are the same. Since the BCM is considered in this circuit, the maximum current is the peak ripple current, which corresponds to the value of 4.16 A.

3.1.4 How does the duty cycle D change with the output power? Calculate the duty cycle values and the transistor switch-on times $T_{on} = DT_s$ for minimum and maximum output power.
Answer:
The switching period is the inverse of the switching frequency:

$$T_s = \frac{1}{f_s}. \tag{3.1.13}$$

The previously obtained maximum and minimum switching frequencies lead to:

$$T_s(P_2 = 2\ \text{W}) = \frac{1}{150\ \text{kHz}} = 6.67\ \mu\text{s}, \tag{3.1.14}$$

$$T_s(P_2 = 15\ \text{W}) = \frac{1}{20\ \text{kHz}} = 50\ \mu\text{s}. \tag{3.1.15}$$

The transistor switch-on times can be determined using

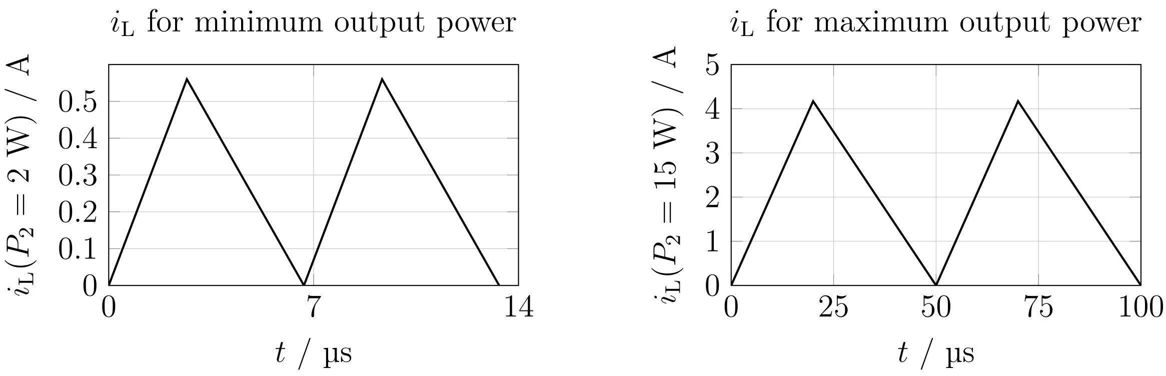
$$T_{on} = DT_s \tag{3.1.16}$$

leading to:

$$T_{on}(P_2 = 2\ \text{W}) = 0.4 \cdot 6.67\ \mu\text{s} = 2.67\ \mu\text{s}, \tag{3.1.17}$$

$$T_{on}(P_2 = 15\ \text{W}) = 0.4 \cdot 50\ \mu\text{s} = 20\ \mu\text{s}. \tag{3.1.18}$$

3.1.5 Sketch the course of the inductor current $i_L(t)$ for minimum and maximum output power.
Answer:
The solution sketch is depicted in Sol.-Fig. 3.1.1.



Solution Figure 3.1.1: Current i_L for minimum and maximum output power.

3.1.6 At which operating point does the maximum output voltage ripple Δu_2 occur (assumption: the load draws a constant current)?

Answer:

The capacitor current is given by

$$i_C(t) = i_L(t) - I_2, \tag{3.1.19}$$

with the assumption of a constant load current I_2. The voltage equation of the capacitor is defined as

$$i_C(t) = C\frac{\mathrm{d}u_C(t)}{\mathrm{d}t}, \tag{3.1.20}$$

which is rearranged into

$$u_C(t) = \frac{1}{C}\int i_C(t), \tag{3.1.21}$$

showing that the voltage ripple depends on the capacitor current, which is larger a the second operating point ($P_2 = 15$ W, Sol.-Fig. 3.1.1), leading to the highest voltage ripple.

3.1.7 How high should the output capacitance be selected to ensure $\Delta u_2 < 0.02 \cdot U_2$?

Answer:

The output capacitor C_2 is located parallel to the output voltage U_2, therefore, the maximum voltage ripple is calculated according to the task description with:

$$\Delta u_2 = \Delta u_C = 0.02U_2 = 0.02 \cdot 12\text{ V} = 0.24\text{ V}. \tag{3.1.22}$$

The relationship between the voltage ripple and the capacitance is identical to the step-up converter, which is defined as:

$$\Delta u_C = \frac{I_2}{C}DT_s. \tag{3.1.23}$$

Considering the maximum power ($P_2 = 15$ W) results into:

$$C_2 = \frac{I_2(P_2 = 15\text{ W})DT_s(P_2 = 15\text{ W})}{\Delta u_C} = \frac{1.25\text{ A} \cdot 0.4 \cdot 50\text{ µs}}{0.24\text{ V}} = 104\text{ µF}. \tag{3.1.24}$$

3.1.8 What is the maximum effective value $i_{\mathrm{C,RMS}}$ of the output capacitor current?

Answer:

The capacitor RMS current is defined as follows

$$i_{\mathrm{C,RMS}} = \sqrt{\frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} i_\mathrm{C}^2(t)\mathrm{d}t}, \tag{3.1.25}$$

with the capacitor current for the inverting buck-boost converter being:

$$i_\mathrm{C}(t) = \begin{cases} \frac{D}{1-D}I_2 + \Delta i_\mathrm{L}\frac{T_\mathrm{off}-2(t-kT_\mathrm{s})}{2T_\mathrm{off}}, & t \in [kT_\mathrm{s}, kT_\mathrm{s}+T_\mathrm{off}], \\ -I_2, & t \in [kT_\mathrm{s}+T_\mathrm{off}, (k+1)T_\mathrm{s}]. \end{cases} \tag{3.1.26}$$

For $k = 0$ the equation results into

$$i_{\mathrm{C,RMS}} = \sqrt{\frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{off}} \underbrace{\left(\frac{D}{1-D}I_2 + \Delta i_\mathrm{L}\frac{T_\mathrm{off}-2t}{2T_\mathrm{off}}\right)^2}_{a}\mathrm{d}t + \frac{1}{T_\mathrm{s}}\int_{T_\mathrm{off}}^{T_\mathrm{s}} (-I_2)^2\,\mathrm{d}t}, \tag{3.1.27}$$

where the variable a is utilized for a shorter description in the following. This first part is solved as:

$$\begin{aligned} \int_0^{T_\mathrm{off}} a\mathrm{d}t &= \int_0^{T_\mathrm{off}} \left(\frac{D}{1-D}I_2 + \Delta i_\mathrm{L}\frac{T_\mathrm{off}-2t}{2T_\mathrm{off}}\right)^2 \mathrm{d}t \\ &= \int_0^{T_\mathrm{off}} \frac{D^2}{(D-1)^2}I_2^2 + \frac{D}{(1-D)}I_2\Delta i_\mathrm{L} - 2\frac{D}{(1-D)}I_2\Delta i_\mathrm{L}\frac{t}{T_\mathrm{off}} \\ &\quad + \frac{1}{4}\Delta i_\mathrm{L}^2 - \Delta i_\mathrm{L}^2\frac{t}{T_\mathrm{off}} + \Delta i_\mathrm{L}^2\frac{t^2}{T_\mathrm{off}^2}\mathrm{d}t \\ &= \left[\frac{D^2}{(1-D)^2}I_2^2 t + \frac{D}{(1-D)}I_2\Delta i_\mathrm{L} t - \frac{D}{(1-D)}I_2 i_\mathrm{L}\frac{t^2}{T_\mathrm{off}} \right. \\ &\quad \left. + \frac{1}{4}\Delta i_\mathrm{L}^2 t - \frac{1}{2}\Delta i_\mathrm{L}^2\frac{t^2}{T_\mathrm{off}} + \frac{1}{3}\Delta i_\mathrm{L}^2\frac{t^3}{T_\mathrm{off}^2}\right]_0^{T_\mathrm{off}}. \end{aligned} \tag{3.1.28}$$

Applying the limits of the integral and rearranging the expression results into:

$$\begin{aligned} \int_0^{T_\mathrm{off}} a\mathrm{d}t &= \left[\frac{D^2}{(1-D)^2}I_2^2 + \frac{1}{12}\Delta i_\mathrm{L}^2\right]T_\mathrm{off} - 0 \\ &= \left[\frac{0.4^2}{(1-0.4)^2}\cdot(1.25\ \mathrm{A})^2 + \frac{1}{12}\cdot(4.16\ \mathrm{A})^2\right]\cdot 30\ \mathrm{\mu s} = 64.1\ \mathrm{A^2\mu s}. \end{aligned} \tag{3.1.29}$$

The calculation of the second part (transistor is open) is given by:

$$\int_{T_\mathrm{off}}^{T_\mathrm{s}} (-I_2)^2\,\mathrm{d}t = \left[I_2^2 t\right]_{T_\mathrm{off}}^{T_\mathrm{s}} = I_2^2\left(T_\mathrm{s} - T_\mathrm{off}\right) = (1.25\ \mathrm{A})^2\cdot(50\ \mathrm{\mu s} - 30\ \mathrm{\mu s}) = 31.3\ \mathrm{A^2\mu s}. \tag{3.1.30}$$

Hence, the capacitor RMS current is calculated with:

$$i_{\mathrm{C,RMS}} = \sqrt{\frac{1}{50\ \mathrm{\mu s}}\left(64.1\ \mathrm{A^2\mu s} + 31.3\ \mathrm{A^2\mu s}\right)} = 1.38\ \mathrm{A}. \tag{3.1.31}$$

3.1.9 Sketch the curves of the voltage $u_{\mathrm{T}}(t)$ at the power transistor and the current $i_{\mathrm{D}}(t)$ in the output diode for $P_2 = 2$ W. What is the maximum blocking voltage of the transistor?

Answer:

The input voltage for an activated transistor is given with

$$U_1 = u_{\mathrm{T}} + u_{\mathrm{L}}, \tag{3.1.32}$$

which is rearranged into:

$$u_{\mathrm{T}} = U_1 - u_{\mathrm{L}}. \tag{3.1.33}$$

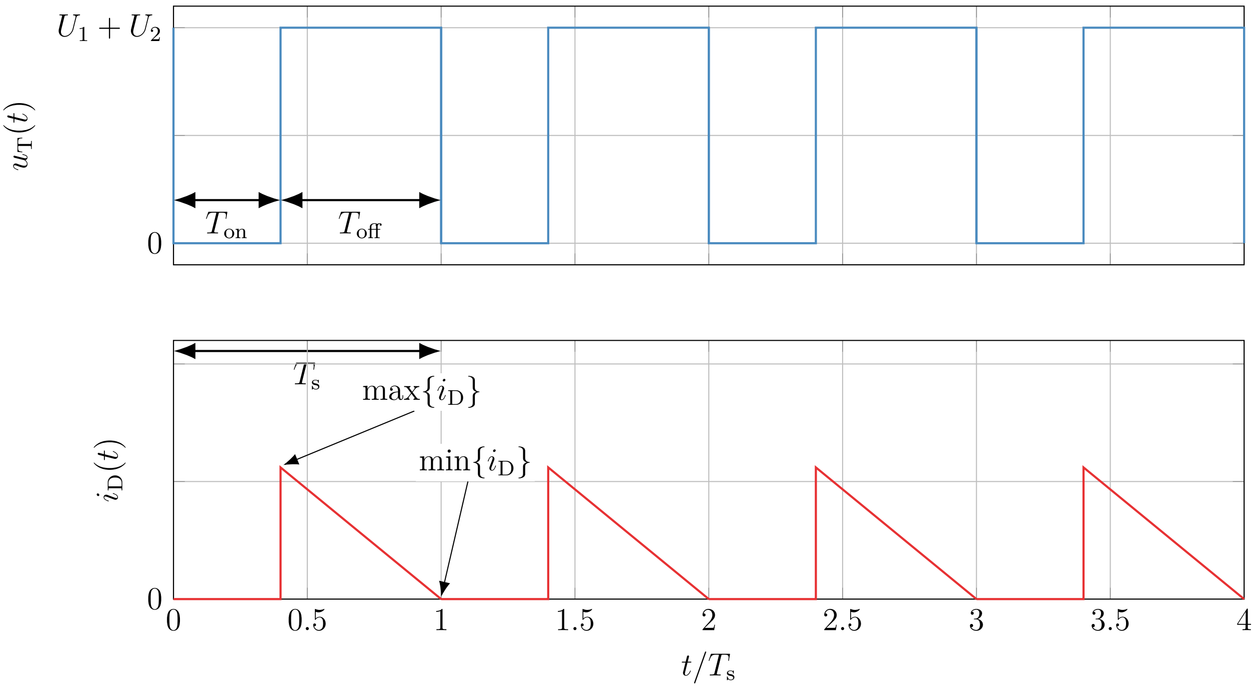
During the off time of the transistor, the inductance voltage is defined as

$$u_{\mathrm{L}} = -U_2 - u_{\mathrm{D}}, \tag{3.1.34}$$

where the diode voltage is assumed to be zero during the conduction phase. Hence, the maximum blocking voltage of the transistor is calculated with (3.1.33) and (3.1.34), which results in:

$$u_{\mathrm{T}} = U_1 - (-U_2) = U_1 + U_2 = 18\ \mathrm{V} + 12\ \mathrm{V} = 30\ \mathrm{V}. \tag{3.1.35}$$

The diode current and the blocking voltage of the transistor is shown in Sol.-Fig. 3.1.2.



Solution Figure 3.1.2: Visualization of the transistor blocking voltage (in the upper part) and diode current below.

Task 3.2: Boost-buck converter and SEPIC topology

The supply of a plasma coating system is realized by a boost converter followed by a buck converter according Fig. 3.2.1 (with common capacitance). The converter is connected to a voltage U_1 and provides a variable output voltage U_2. The parameters are displayed in Tab. 3.2.1.

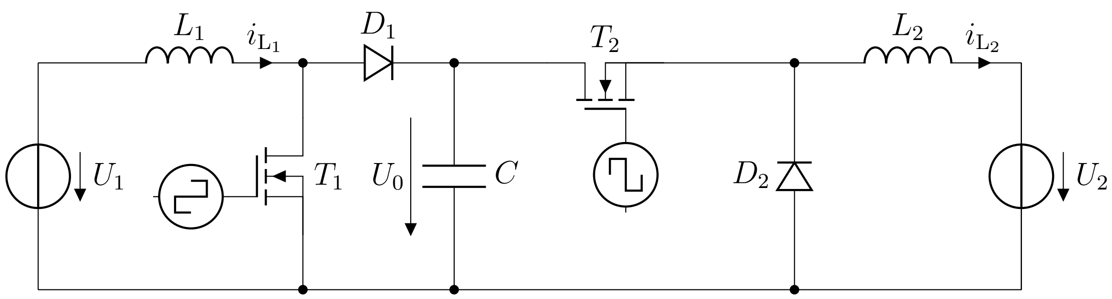


Figure 3.2.1: Boost-buck converter circuit.

The output voltage is set to the specified value by adjusting the duty cycles D_1 (of transistor T_1) and D_2 (of transistor T_2) using a control system. Both transistors operate at the same switching frequency. The switching frequency fluctuation of the intermediate circuit voltage and the currents in the inductors can be neglected unless otherwise stated. The currents in L_1 and L_2 show a continuous course. Both transistors are operated with the same duty cycle $D_1 = D_2 = D$.

3.2.1 Calculate the duty cycle D range to achieve the stated output voltage U_2 range.

Input voltage:	$U_1 = 380$ V	Output voltage:	$U_2 = 285$ V to 450 V
Output power:	$P_2 = 3000$ W	Switching frequency:	$f_s = 50$ kHz
P_2 is constant (unless otherwise stated)			

Table 3.2.1: Parameter of the boost-buck converter circuit.

Answer:

For the boost-buck converter the voltage transfer ratio delivers:

$$\frac{U_2}{U_1} = \frac{D}{1-D} \quad \Leftrightarrow \quad D = \frac{\frac{U_2}{U_1}}{1+\frac{U_2}{U_1}}. \tag{3.2.1}$$

In case of $U_2 = 285$ V, the duty cycle results in

$$D = \frac{\frac{285\text{ V}}{380\text{ V}}}{1+\frac{285\text{ V}}{380\text{ V}}} = 0.429. \tag{3.2.2}$$

3.2.2 Which intermediate circuit voltage U_0 results depending on U_2?

Answer:

The first part is the boost converter part. For the output voltage of 285 V, the intermediate circuit voltage U_0 is calculated as

$$U_0 = U_1\frac{1}{1-D} = 380\text{ V}\frac{1}{1-0.429} = 665\text{ V}. \tag{3.2.3}$$

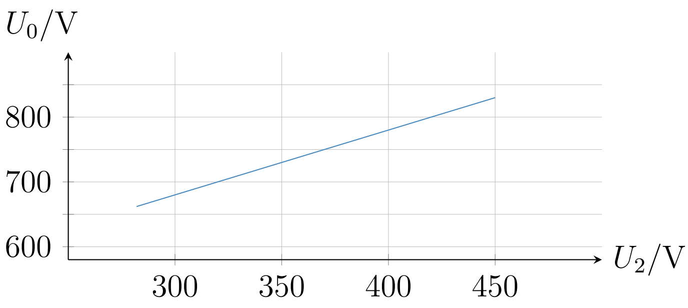
3.2.3 Plot D and U_0 against U_2 and calculate the numerical values for $U_2 = 285$ V, $U_2 = 380$ V and $U_2 = 450$ V.

Answer:

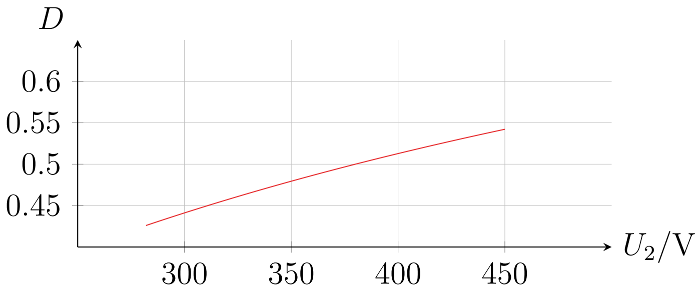
Here, (3.2.1) and (3.2.3) are used to calculate results summarized in Sol.-Tab. 3.6 and displayed in Sol.-Fig. 3.2.1 and Sol.-Fig. 3.2.2.

U_2	U_0	D
285 V	665 V	0.429
380 V	760 V	0.5
450 V	830 V	0.542

Solution Table 3.6: D and U_0 at U_2.



Solution Figure 3.2.1: Intermediate circuit voltage versus output voltage.



Solution Figure 3.2.2: Duty cycle versus output voltage.

3.2.4 What blocking voltage ratings must the transistors T_1 and T_2 and the diodes D_1 and D_2 have?

Answer:

Based on the schematic from Fig. 3.2.1, following voltage ratings of the semiconductor components can be derived:

- Reverse voltage across D_1: In case of T_1 is active, the potential at the TT_1's drain is pulled down. The diode D_1 hat to block U_0.
- Block voltage across T_1: In case of T_1 is inactive, the current of i_{L1} load the capacitor C via diode D_1. The voltage across the ideal diode in forward direction is 0 V, so that the voltage U_0 is applied to transistor T1.
- Reverse voltage across D_2: In case of T_2 is active, the voltage U_0 of capacitor C is applied to D_2.
- Block voltage across T_2: In case of T_2 is inactive, the current i_{L2} flows through D_1. The forward voltage of the ideal diode is 0 V, so that U_0 is applied to T_2.

The minimum blocking voltage of the component needs to be higher than U_0.

The converter operation continues with $D_1 = D_2 = D$. The input and output inductances have the same value $L_1 = L_2 = L = 0.5$ mH.

3.2.5 Derive the input and output current ripple Δi_{L_1} and Δi_{L_2} depending on the duty cycle.

Answer:

The current ripple depends on the duty cycle, switching period and inductor voltage. For the boost converter stage:

$$\Delta i_{\mathrm{L1}} = \frac{U_1 D}{f_\mathrm{s} L} \tag{3.2.4}$$

results while the buck converter stage's current ripple is:

$$\Delta i_{\mathrm{L2}} = \frac{(U_0 - U_2)\, D}{f_\mathrm{s} L}. \tag{3.2.5}$$

3.2.6 To what minimum value can the output power be reduced while still ensuring continuous operation across the entire output voltage range (i.e., continuous current flow in L_1 and L_2)?

Answer:

For the boost stage the power is the product of output voltage U_0 and average current calculated by using (3.2.4):

$$P_{\mathrm{lim,boost}} = U_0 \frac{\Delta i_{\mathrm{L1}}}{2} D. \tag{3.2.6}$$

For the buck stage the power is the product of output voltage U_2 and average current calculated by using (3.2.5):

$$P_{\mathrm{lim,buck}} = U_2 \frac{\Delta i_{\mathrm{L2}}}{2}. \tag{3.2.7}$$

Entering the relevant values in (3.2.6) and (3.2.7) leads to Sol.-Tab. 3.7.

U_2	$P_{\mathrm{min,boost}}$	$P_{\mathrm{min,buck}}$
285 V	930 W	929 W
380 V	1444 W	1444 W
450 V	1853 W	1854 W

Solution Table 3.7: Minimal power necessary for CCM (boost and buck stage).

The minimal power, which ensuring continuous operation across the entire output voltage range yields 1854 W.

3.2.7 At which value of the output voltage range is this limit reached first on the input side and at which value is it reached first on the output side?

Answer:

As displayed in Sol.-Tab. 3.7 the minimal power limit is caused by the boost-stage (input side) for output voltage less than 380 V. For output voltages greater than 380 V the buck-stage (output side) causes the minimal power limit for continuous conduction mode.

For comparison reasons, the single ended primary inductance converter (SEPIC) in Fig. 3.2.2 shall be considered under similar conditions as an alternative circuit.

3.2.8 What blocking voltage ratings must the transistors T_1 and the diode D have?

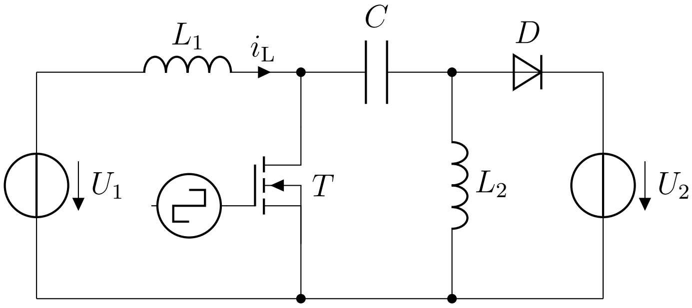


Figure 3.2.2: Single ended primary inductance converter circuit.

Answer:

For the SEPIC topology the voltage at the transistor blocking voltage is calculated by the sum of U_c and U_2.

$$U_{T,block} = U_C + U_2. \tag{3.2.8}$$

Using (3.2.18), which reflects the voltage balance during a switching period of inductor L_1 as

$$DU_1 + (1 - D)(U_1 - U_2 - U_C) = 0 \tag{3.2.9}$$

the voltage U_C is obtained by

$$U_C = \frac{U_1}{1 - D} - U_2. \tag{3.2.10}$$

The substitution of U_1 by U_2 leads to

$$U_C = U_2 \frac{(1 - D)}{D} \frac{1}{(1 - D)} - U_2 = \left(\frac{1}{D} - 1\right) U_2. \tag{3.2.11}$$

The substitution of U_C in (3.2.11) leads to

$$U_{T,block} = \left(\frac{1}{D} - 1\right) U_2 + U_2 = \frac{U_2}{D}. \tag{3.2.12}$$

The diode has to block the voltage, when the transistor is active. Applying Kirchhoff's loop rule yields:

$$U_C + U_{D,block} + U_2 = 0. \tag{3.2.13}$$

With the used loop direction (right turn) the diode blocks negative voltage, which leads to:

$$-U_{D,block} = U_C + U_2. \tag{3.2.14}$$

U_C corresponds to the input voltage U_1, because the voltage transfer ratio of the SEPIC-topology is calculated by:

$$\frac{U_2}{U_1} = \frac{D}{1 - D} \quad \Leftrightarrow \quad U_1 = \frac{1 - D}{D} U_2 = \left(\frac{1}{D} - 1\right) U_2 = U_C \tag{3.2.15}$$

The result shows, that the transistor and the diode needs to block the sum of input and output voltage, which is the same blocking voltage as for the transistors and diodes in a boost-buck converter.

3.2.9 Derive the input and output current ripple $\Delta i_{\mathrm{L_1}}$ and $\Delta i_{\mathrm{L_2}}$ depending on the duty cycle.
Answer:
The current ripple depends on the duty cycle, switching period and inductor voltage. If T is active, the current through L_1 increases. This corresponds to the ripple current

$$\Delta i_{\mathrm{L1}} = \frac{U_1 D}{f_\mathrm{s} L_1}. \tag{3.2.16}$$

Also for L_2 the current increases while T is active. In this case the voltage of the capacitor C is applied to L_2. The ripple current of L_1 is obtained by

$$\Delta i_{\mathrm{L2}} = \frac{U_\mathrm{C} D}{f_\mathrm{s} L_2} = \frac{U_1 D}{f_\mathrm{s} L_2}. \tag{3.2.17}$$

3.2.10 To what minimum value can the output power be reduced while still ensuring continuous operation across the entire output voltage range? (i.e., continuous current flow in L_1 and L_2)?
Answer:
Both inductors have the same ripple current as long as they have the same inductance. For the inductor L_1 the power is the product of output voltage U_2 and average current calculated by using (3.2.4):

$$P_\mathrm{min} = U_1 \frac{\Delta i_{\mathrm{L1}}}{2} D. \tag{3.2.18}$$

U_2	P_min
285 V	530 W
380 V	722 W
450 V	849 W

Solution Table 3.8: Minimal necessary power of SEPIC topology for CCM mode.

Entering the values in (3.2.18) leads to Sol.-Tab. 3.8. The minimal power, which ensures continuous operation across the entire output voltage range yields 849 W.

3.2.11 Describe the advantages and disadvantages of the SEPIC topology compared to the boost-buck converter. Consider the necessary components and the quality of the output voltage.
Answer:
Advantages of SEPIC-topology compared to boost-buck converter:

- Minimal power limit is much lower (nearly factor 2).
- The amount of components are reduced (only 1 transistor and 1 diode is needed).

Disadvantages of SEPIC-topology compared to boost-buck converter:

- The output current is pulsating (needs more filtering components).

Exercise 04: Isolated DC-DC converters

Task 4.1: Flyback converter

A flyback converter with an input voltage range $U_1 = 300\ \text{V} \dots 900\ \text{V}$ is used to supply a control electronics unit. The converter delivers a rated output power of $P_2 = 30\ \text{W}$ at a regulated (constant) output voltage of $U_2 = 15\ \text{V}$. The flyback converter is operated in discontinuous conduction mode with a constant switching frequency of $f_\text{s} = 50\ \text{kHz}$. The turns ratio of the transformer is $N_1/N_2 = 60/12$, the magnetizing inductance on the primary side is $L_\text{m} = 760\ \mu\text{H}$. The coupling between the primary and secondary windings is ideal and the converter operates in steady state.

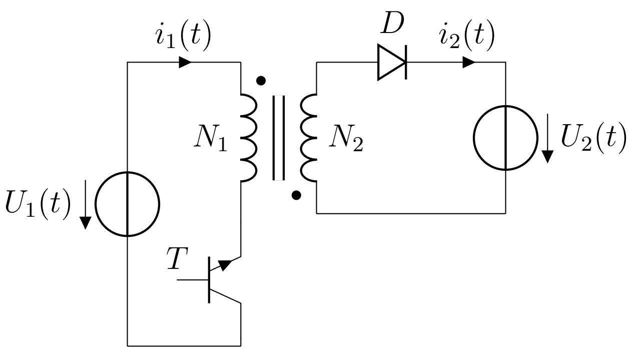


Figure 4.1.1: Flyback converter topology.

Input voltage:	$U_1 = 300\ \text{V} \dots 900\ \text{V}$	Output voltage:	$U_2 = 15\ \text{V}$
Output power:	$P_2 = 30\ \text{W}$	Transformation ratio:	$N_1/N_2 = 60/12$
Magn. inductance:	$L_\text{m} = 760\ \mu\text{H}$	Switching frequency:	$f_\text{s} = 50\ \text{kHz}$

Table 4.1.1: Parameters of the flyback converter.

4.1.1 The input voltage is $U_1 = 760\ \text{V}$ at rated output power. What is the peak value $\hat{i}_1$ of the primary current i_1? What is the peak value $\hat{i}_2$ of the secondary current i_2? Calculate the duty cycle of the transistor for this operating point.

Answer:

To determine the peak current $\hat{i}_1$, we consider the average power transfer

$$P_2 = W_\text{L} f_\text{s}, \tag{4.1.1}$$

with the stored magnetic energy

$$W_\text{L} = \frac{1}{2} L_\text{m} \hat{i}_1^2. \tag{4.1.2}$$

Inserting delivers

$$P_2 = \frac{1}{2} L_1 \hat{i}_1^2 f_\text{s}. \tag{4.1.3}$$

Solving for $\hat{i}_1$ yields the peak current

$$\hat{i}_1 = \sqrt{\frac{2P_2}{L_\mathrm{m} f_\mathrm{s}}} = \sqrt{\frac{2 \cdot 30\ \mathrm{W}}{760\ \mathrm{\mu H} \cdot 50\ \mathrm{kHz}}} = 1.257\ \mathrm{A}. \tag{4.1.4}$$

Since an ideal transformer is assumed, the secondary peak current directly follows from the primary peak current and the turns ratio:

$$\hat{i}_2 = \hat{i}_1 \frac{N_1}{N_2} = 1.257\ \mathrm{A} \cdot \frac{60}{12} = 6.28\ \mathrm{A}. \tag{4.1.5}$$

Because of DCM the voltage transfer ratio is

$$\frac{U_2}{U_1} = \frac{D^2}{2} \frac{\Delta i_\mathrm{m,max}}{\bar{i}_2}. \tag{4.1.6}$$

The yet unknown current ripple can be determined via the switch-on interval

$$\Delta i_\mathrm{m,max} = \frac{T_\mathrm{s} \cdot U_1}{L_\mathrm{m}} = \frac{\frac{1}{50\ \mathrm{kHz}} \cdot 760\ \mathrm{V}}{760\ \mathrm{\mu H}} = 20\ \mathrm{A}. \tag{4.1.7}$$

Inserting in (4.1.6) and solving for D with $\bar{i}_2 = \frac{P_2}{U_2}$ yields to:

$$D = \sqrt{\frac{2U_2\bar{i}_2}{U_1 \Delta i_\mathrm{m,max}}} = \sqrt{\frac{2 \cdot 15\ \mathrm{V} \cdot 30\ \mathrm{W}}{760\ \mathrm{V} \cdot 15\ \mathrm{V} \cdot 20\ \mathrm{A}}} = 0.063. \tag{4.1.8}$$

4.1.2 The input voltage is $U_1 = 382$ V at nominal load. Calculate and sketch the following voltage and current curves for this operating point over two cycle periods: $u_\mathrm{T}(t), u_\mathrm{s}(t), i_2(t), i_1(t)$. Here, $u_\mathrm{T}(t)$ is the transistor voltage and $u_\mathrm{s}(t)$ is the voltage on the secondary side of the transformer.

Answer:

Because of the different input voltage, the duty cycle D has changed:

$$\Delta i_\mathrm{m,max} = \frac{T_\mathrm{s} \cdot U_1}{L_\mathrm{m}} = \frac{\frac{1}{50\ \mathrm{kHz}} \cdot 382\ \mathrm{V}}{760\ \mathrm{\mu H}} = 10.05\ \mathrm{A}. \tag{4.1.9}$$

$$D = \sqrt{\frac{2U_2\bar{i}_2}{U_1 \Delta i_\mathrm{m,max}}} = \sqrt{\frac{2 \cdot 15\ \mathrm{V} \cdot 30\ \mathrm{W}}{382\ \mathrm{V} \cdot 15\ \mathrm{V} \cdot 10.05\ \mathrm{A}}} = 0.125. \tag{4.1.10}$$

Based on this result, T_on can be calculated as:

$$T_\mathrm{on} = DT_\mathrm{s} = 0.125 \cdot \frac{1}{50\ \mathrm{kHz}} = 2.5\ \mathrm{\mu s}. \tag{4.1.11}$$

The flyback converter in DCM has three different switch states $T_\mathrm{on}, T'_\mathrm{off}$ and T''_off. Because of this D' has be calculated as:

$$D' = \frac{N_2 U_1}{N_1 U_2} D = \frac{12 \cdot 382\ \mathrm{V}}{60 \cdot 15\ \mathrm{V}} D = 0.637, \tag{4.1.12}$$

$$T'_{\text{off}} = D'T_{\text{s}} = 0.637 \cdot \frac{1}{50\ \text{kHz}} = 12.7\ \text{µs}, \tag{4.1.13}$$

$$T''_{\text{off}} = T_{\text{s}} - T_{\text{on}} - T'_{\text{off}} = \frac{1}{50\ \text{kHz}} - 2.5\ \text{µs} - 12.7\ \text{µs} = 4.8\ \text{µs}. \tag{4.1.14}$$

In the following the voltage values in the three intervals are calculated: The first interval is $0 < t < T_{\text{on}}$. In this interval the transistor conducts and, therefore, the voltage $U_{\text{T}} = 0$ results. The following applies to the voltage on the primary side:

$$u_{\text{T}} = U_1 + \frac{N_1}{N_2}U_2. \tag{4.1.15}$$

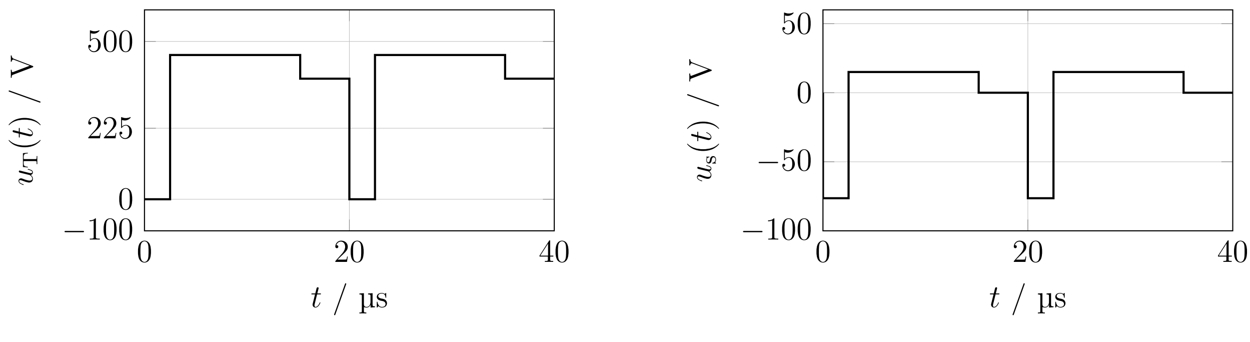
In this interval the voltage U_2 is equal to u_{s}. With the voltage $u_{\text{T}} = 0$ follows:

$$u_{\text{s}} = -U_1\frac{N_2}{N_1} = -382\ \text{V} \cdot \frac{12}{60} = -76.4\ \text{V}. \tag{4.1.16}$$

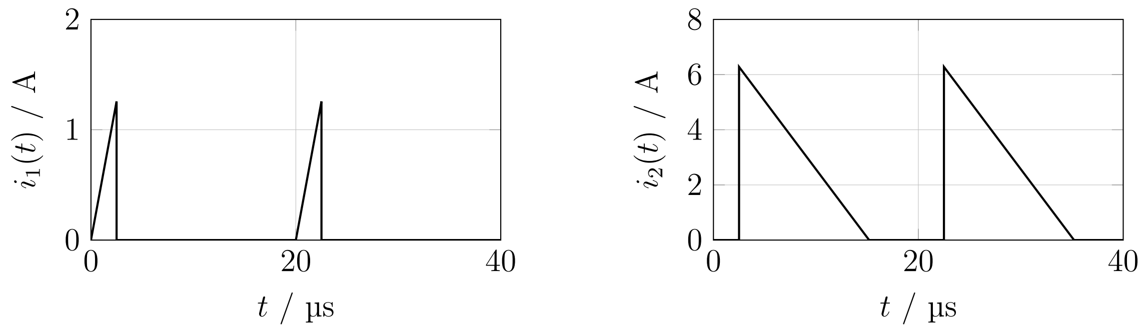
The second interval is $T_{\text{on}} < t < T'_{\text{off}}$. In this interval the transistor blocks. For this calculation (4.1.15) can be used again as:

$$u_{\text{T}} = 382\ \text{V} + \frac{60}{12} \cdot 15\ \text{V} = 457\ \text{V}. \tag{4.1.17}$$

Because of the conducting diode, for the voltage U_2 is following $U_2 = u_{\text{s}} = 15$ V. The third interval is $T_{\text{on}} + T'_{\text{off}} < t < T_{\text{s}}$. In this interval the diode is not conducting and the transistor blocks. From this follows $u_{\text{T}} = U_1$ and $u_{\text{s}} = 0$. Looking at the current $i_1(t)$ over the entire course, the current can only be present when the transistor is switched on. When the transistor is switched on, the current value rises to the peak value $\hat{i}_1 = 1.257$ A. Current $i_2(t)$ is the discharge current of the secondary winding when the diode is conducting and the transistor blocks. The discharge current starts at the peak value of the secondary side $\hat{i}_2 = 6.28$ A.



Solution Figure 4.1.1: Display of the voltages $u_{\text{T}}(t)$ and $u_{\text{s}}(t)$.



Solution Figure 4.1.2: Input and output currents.

4.1.3 Determine the mean value $\bar{i}_\mathrm{T}$ and the RMS current I_T through the transistor. Also, determine the mean value $\bar{i}_\mathrm{D}$ and the RMS current I_D through the diode. What is the maximum reverse voltage $u_\mathrm{T,max}$ of the transistor and $u_\mathrm{D,max}$ of the diode? Consider the same operation conditions as in the previous subtask.

Answer:

The current signal is in the form of a triangle. Therefore, the average value must first be determined over the T_on interval only. The value is divided by T_s to obtain the average value over the entire period. This gives:

$$\bar{i}_\mathrm{T} = \frac{1}{T_\mathrm{s}}\frac{1}{2}\hat{i}_1 T_\mathrm{on} = \frac{1}{20\ \mathrm{\mu s}} \cdot \frac{1}{2} \cdot 1.257\ \mathrm{A} \cdot 2.5\ \mathrm{\mu s} = 78.53\ \mathrm{mA}. \tag{4.1.18}$$

To determine the RMS value of the transistor current, the function of the transistor current must be set up and used, which leads to

$$I_\mathrm{T}^2 = \frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} i_1^2(t)\,\mathrm{d}t \ = \frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{on}} \frac{\hat{i}_1^2 t^2}{T_\mathrm{on}^2}\,\mathrm{d}t \ = \frac{\hat{i}_1^2 T_\mathrm{on}}{3T_\mathrm{on}}, \tag{4.1.19}$$

$$I_\mathrm{T} = \hat{i}_1\sqrt{\frac{T_\mathrm{on}}{3T_\mathrm{s}}} = 1.257\ \mathrm{A} \cdot \sqrt{\frac{2.5\ \mathrm{\mu s}}{3 \cdot 20\ \mathrm{\mu s}}} = 256.58\ \mathrm{mA}. \tag{4.1.20}$$

The mean value for the current i_D and the RMS value for i_D are determined analogously:

$$\bar{i}_\mathrm{D} = \frac{1}{T_\mathrm{s}}\frac{1}{2}\hat{i}_2 T_\mathrm{off}' = \frac{1}{20\ \mathrm{\mu s}} \cdot \frac{1}{2} \cdot 6.28\ \mathrm{A} \cdot 12.7\ \mathrm{\mu s} = 2\ \mathrm{A}, \tag{4.1.21}$$

$$I_\mathrm{D} = \hat{i}_2\sqrt{\frac{T_\mathrm{off}'}{3T_\mathrm{s}}} = 6.28\ \mathrm{A} \cdot \sqrt{\frac{12.7\ \mathrm{\mu s}}{3 \cdot 20\ \mathrm{\mu s}}} = 2.89\ \mathrm{A}. \tag{4.1.22}$$

The voltage $u_\mathrm{T,max}$ is calculated as in (4.1.15) as:

$$u_\mathrm{T,max} = U_1 + \frac{N_1}{N_2}U_2 = 382\ \mathrm{V} + \frac{60}{12} \cdot 15\ \mathrm{V} = 457\ \mathrm{V}. \tag{4.1.23}$$

(4.1.15) is used again, but now for the secondary side. This leads to:

$$u_{\mathrm{D,max}} = U_2 + \frac{N_2}{N_1}U_1 = 15\ \mathrm{V} + \frac{12}{60}\cdot 382\ \mathrm{V} = 91.4\ \mathrm{V}. \tag{4.1.24}$$

4.1.4 How much energy is transferred from the input to the output per switching period ΔE and what is the resulting average power P (consider the same operation conditions as in the previous subtask)? What happens if there is no ideal voltage source on the output side but an unloaded capacitor and the circuit is operated with $D > 0$?

Answer:

The energy on the primary side can be determined with (4.1.2):

$$W_{\mathrm{L}} = \frac{1}{2}\cdot 760\ \mathrm{\mu H}\cdot 1.257\ \mathrm{A} = 600\ \mathrm{\mu J}. \tag{4.1.25}$$

Since this is an ideal converter, no losses are assumed. To determine the average power P, the transmitted energy must be divided by the period duration T_{s}, as follows:

$$P = \frac{W_{\mathrm{L}}}{T_{\mathrm{s}}} = \frac{600\ \mathrm{\mu J}}{20\ \mathrm{\mu s}} = 30\ \mathrm{W}. \tag{4.1.26}$$

At each switching interval, energy is pushed into the capacitor, the voltage of which continues to rise until a component fails due to over voltage if no load is connected.

Task 4.2: Forward converter with asymmetric half-bridge

The schematic of a forward converter with an asymmetric half-bridge is shown in Fig. 4.2.1. For the calculations the diodes and transistors are considered as ideal components.

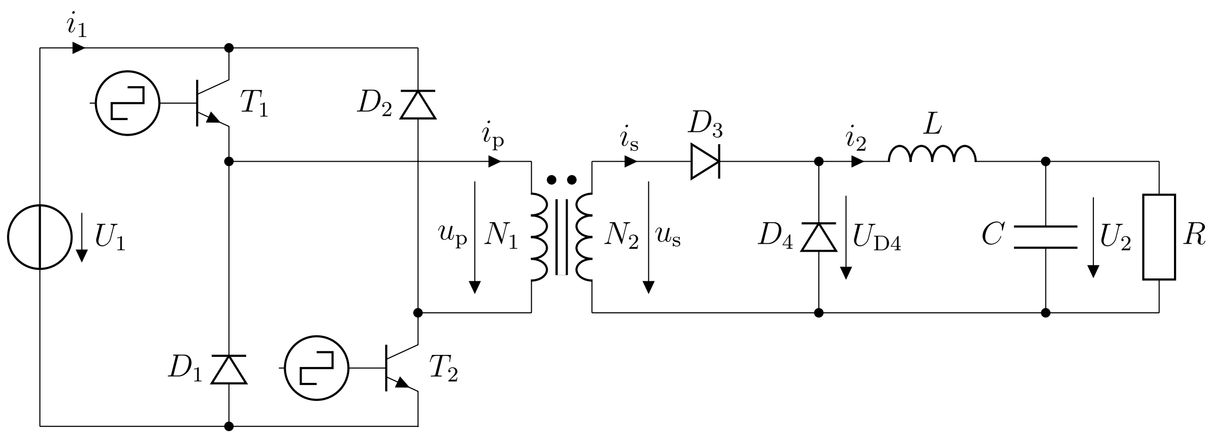


Figure 4.2.1: Forward converter with asymmetric half-bridge.

The parameters are listed in Fig. 4.2.1.

Input voltage:	$U_1 = 325$ V	Output voltage:	$U_2 = 15$ V
Output power:	$P_2 = 50$ W	Switching frequency:	$f_\mathrm{s} = 50$ kHz
Turns ratio:	$N_1/N_2 = 10$	Magnetizing inductance:	$L_\mathrm{m} = 2$ mH

Table 4.2.1: Parameter overview of the circuit.

The leakage inductance, the resistive losses, and the core losses of the transformer are negligible. The converter operates in steady-state conditions. Both transistors are controlled by the same signal.

4.2.1 At what duty cycle D does the circuit operate?

Answer:

The forward converter topology is derived from the buck-topology. Taking into account the turns ratio, the result is

$$\frac{U_2}{U_1} = D\frac{N_2}{N_1}. \tag{4.2.1}$$

This leads to

$$D = \frac{N_1}{N_2}\frac{U_2}{U_1} = 10\frac{15\ \mathrm{V}}{325\ \mathrm{V}} = 0.4615. \tag{4.2.2}$$

4.2.2 Calculate the average currents $\bar{i}_2$ and $\bar{i}_1$ over a switching cycle assuming ideal filtering of i_2.

Answer:

The current is calculated with help of the output power by

$$\bar{i}_2 = \frac{P_2}{U_2} = \frac{50\ \mathrm{W}}{15\ \mathrm{V}} = 3.33\ \mathrm{A}. \tag{4.2.3}$$

Since the losses can be neglected, it follows

$$\bar{i}_1 = \frac{P_2}{U_1} = \frac{50\ \mathrm{W}}{325\ \mathrm{V}} = 0.138\ \mathrm{A}. \tag{4.2.4}$$

4.2.3 Calculate the peak value $\hat{i}_\mathrm{m}$ of the magnetizing current i_m.

Answer:

The duty cycle is less than 0.5, i.e., the magnetizing current i_m increase while the transistors are active and decrease to 0 A before the next period starts. This leads to

$$\hat{i}_\mathrm{m} = \Delta i_\mathrm{m,max} = \frac{D \cdot U_1}{L_\mathrm{m} \cdot f_\mathrm{s}} = \frac{0.4615 \cdot 325\ \mathrm{V}}{2\ \mathrm{mH} \cdot 50\ \mathrm{kHz}} = 1.5\ \mathrm{A}. \tag{4.2.5}$$

4.2.4 Sketch the signals u_p, i_m, i_p and i_1 considering the switching-induced ripples.

Answer:

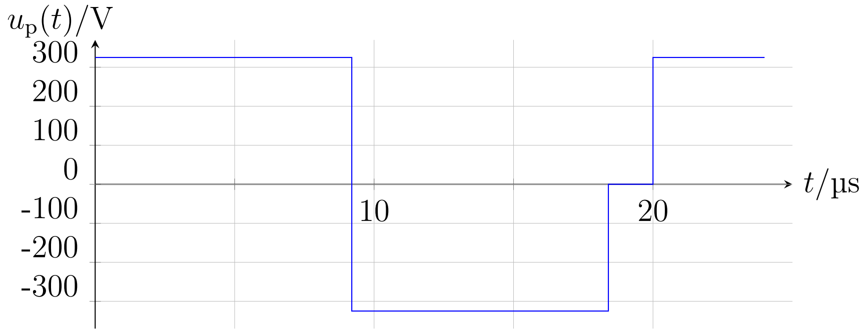
The current i_p corresponds to the sum of i_m and the current $i_\mathrm{1,trOn}$ for the transferred energy to the load. Since the losses can be neglected, the portion of the primary current cause by the secondary current is calculated by

$$\bar{i}_{1\text{sec}} = \frac{N_2}{N_1} I_2 = \frac{3.333\ \text{A}}{10} = 0.333\ \text{A}. \tag{4.2.6}$$

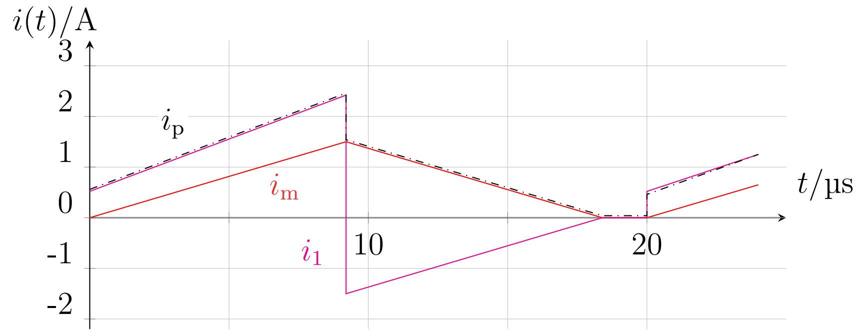
The average value of the current that belongs to the transferred energy, while switch-on phase is calculated by

$$\bar{i}_{1,\text{trOn}} = \frac{\bar{i}_{1\text{sec}}}{D} = \frac{0.333\ \text{A}}{0.4615} = 0.722\ \text{A}. \tag{4.2.7}$$

The voltage and currents are displayed in Sol.-Fig. 4.2.1 and Sol.-Fig. 4.2.2



Solution Figure 4.2.1: Voltage at primary side.



Solution Figure 4.2.2: Currents at primary side.

4.2.5 Calculate the minimal necessary input voltage U_1, if $U_2 = 20$ V shall being constant.

Answer:

Since $i_m(t)$ should not increase after a switching period, the applicable duty cycle is limited to less than or equal to 0.5. From (4.2.1) follows that at minimum input voltage the duty cycle is maximum. Using (4.2.1) leads to

$$U_1 = \frac{N_2}{N_1} \frac{U_1}{D_{\text{max}}} = \frac{20\ \text{V}}{0.5 \cdot 10} = 400\ \text{V}. \tag{4.2.8}$$

4.2.6 Determine L such that the ripple current Δi_2 is 10 % of the average output current $\bar{i}_2$.

Answer:

The ripple current Δi_2 is expressed by

$$\Delta i_2 = \frac{D \cdot U_2}{L_m \cdot f_s} = 0.1 \cdot \bar{i}_2. \tag{4.2.9}$$

Using the result of (4.2.3) this leads to

$$L_\mathrm{m} = \frac{D \cdot U_2}{0.1 \cdot \bar{i}_2 \cdot f_\mathrm{s}} = \frac{0.4615 \cdot 15\ \mathrm{V}}{0.1 \cdot 3.33\ \mathrm{A} \cdot 50\ \mathrm{kHz}} = 0.485\ \mathrm{mH}. \tag{4.2.10}$$

Task 4.3: Singled-ended forward converter (demagnetization winding)

The power supply of a data processing system shall be realized by a singled-ended forward converter.

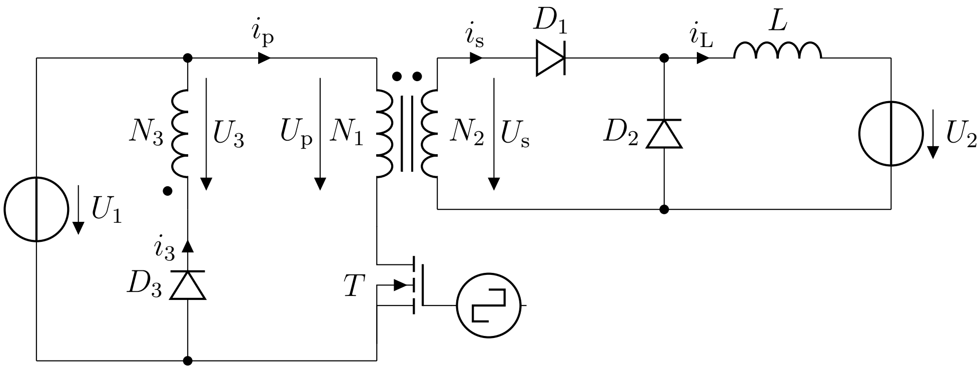


Figure 4.3.1: Single ended forward converter circuit.

The parameters are listed in Tab. 4.3.1. The output inductance L is dimensioned so that the current i_L exhibits a continuous curve shape. The transformer's leakage inductance can be neglected.

Input voltage:	$U_1 = 240\ \mathrm{V} \ldots 360\ \mathrm{V}$	Output voltage:	$U_2 = 5\ \mathrm{V}$
Output power:	$P_2 = 125\ \mathrm{W}$	Switching frequency:	$f_\mathrm{s} = 48\ \mathrm{kHz}$
Forward voltage of D_1:	$U_{\mathrm{D1,f}} = 0.4\ \mathrm{V}$	Forward voltage of D_2:	$U_{\mathrm{D2,f}} = 0\ \mathrm{V}$

Table 4.3.1: Parameters of the circuit.

4.3.1 Calculate the turns ratio N_3/N_1 limiting the maximum transistor blocking voltage to 600 V.

Answer:

The maximum blocking voltage at the transistor is calculated by

$$U_\mathrm{T,max} = U_1 + U_\mathrm{p} \quad \text{with} \quad U_\mathrm{p} = \frac{N_1}{N_3} U_3. \tag{4.3.1}$$

If the diode D_3 is active the voltage U_3 is equal U_1. Using (4.3.1) leads to

$$U_\mathrm{T,max} = U_1 + \frac{N_1}{N_3} U_1. \tag{4.3.2}$$

Solving (4.3.2) to turns ratio N_3/N_1 results in

$$\frac{N_3}{N_1} = \frac{1}{\frac{U_{\mathrm{T,max}}}{U_1} - 1} = \frac{1}{\frac{600\ \mathrm{V}}{360\ \mathrm{V}} - 1} = 1.5. \tag{4.3.3}$$

4.3.2 What is the maximum permissible duty cycle of the power transistor in this case?

Answer:

The Δi_{m} is related to the voltage at the primary coil. The voltages are

$$U_{\mathrm{p,on}} = U_1 \quad \text{and} \quad U_{\mathrm{p,off}} = U_1 \frac{N_3}{N_1} U_3. \tag{4.3.4}$$

The magnetization current is 0 A at the end of each period. For the maximum duty cycle this leads to

$$U_{\mathrm{p,on}} T_{\mathrm{p,on}} = U_{\mathrm{p,off}} T_{\mathrm{p,off}}. \tag{4.3.5}$$

The ration of $T_{\mathrm{p,on}}/T_{\mathrm{p,off}}$ is obtained by

$$\frac{T_{\mathrm{p,on}}}{T_{\mathrm{p,off}}} = \frac{D_{\mathrm{max}}}{1 - D_{\mathrm{max}}}. \tag{4.3.6}$$

Using (4.3.4) and (4.3.5) results in

$$U_{\mathrm{p,on}} \frac{D_{\mathrm{max}}}{1 - D_{\mathrm{max}}} = U_{\mathrm{p,off}}. \tag{4.3.7}$$

The cancelation of voltages $U_{\mathrm{p,on}}$ and $U_{\mathrm{p,off}}$ results in

$$\frac{N_3}{N_1} \frac{D_{\mathrm{max}}}{1 - D_{\mathrm{max}}} = 1. \tag{4.3.8}$$

The duty cycle is calculated by solving (4.3.8):

$$D_{\mathrm{max}} = \frac{\frac{N_1}{N_3}}{1 + \frac{N_1}{N_3}} = \frac{0.667}{1 + 0.667} = 0.4. \tag{4.3.9}$$

4.3.3 What turns ratio N_1/N_2 should be chosen to achieve the required secondary voltage?

Answer:

The maximal duty cycle is applied at minimum input voltage. Moreover, for the calculation of the turns ratio N_1/N_2 the forward voltage of D_1 has to taken in account. The voltage at the secondary coil is calculated by

$$U_2 = (U_{\mathrm{s}} - U_{\mathrm{D1,f}})\, D \quad \text{and} \quad U_{\mathrm{s}} = \frac{N_2}{N_1} U_1. \tag{4.3.10}$$

This leads to

$$U_2 = \left(\frac{N_2}{N_1} U_1 - U_{\mathrm{D1,f}}\right) D. \tag{4.3.11}$$

Solving (4.3.11) with respect to N_1/N_2 results in

$$\frac{N_1}{N_2} = \frac{U_1 D}{U_2 + U_{\mathrm{D1,f}} D} = \frac{240\ \mathrm{V} \cdot 0.4}{5\ \mathrm{V} + 0.4\ \mathrm{V} \cdot 0.4} = 18.6. \tag{4.3.12}$$

4.3.4 Does the duty cycle need to be adjusted when the output power changes? Over what range must the duty cycle be adjustable, considering the input voltage range?
Answer:
No, (4.3.10) shows that the output voltage depends on the turn ratio and the duty cycle, but not on the output power. The stored magnetic energy of the transformer is not used for the energy transfer. This answer is only valid the with the assumption of ideal components. The minimum duty cycle results from solving (4.3.11) with respect to D:

$$D = \frac{U_2}{\frac{N_2}{N_1} U_1 - U_{\mathrm{D1,f}}} = \frac{5\ \mathrm{V}}{\frac{N_2}{N_1} 360\ \mathrm{V} - 0.4\ \mathrm{V}} = 0.264. \tag{4.3.13}$$

The duty cycle needs to be adjustable between 0.264 and 0.4.

4.3.5 What are the resulting maximum blocking voltages of the diodes D_1 and D_2?
Answer:
The maximum blocking voltages of the diodes D_1 is calculated by

$$U_{\mathrm{D2,r}} = U_{\mathrm{s}} - U_{\mathrm{D1,f}} \quad \text{with} \quad U_{\mathrm{s}} = U_1 \frac{N_2}{N_1}. \tag{4.3.14}$$

This leads to

$$U_{\mathrm{D2,r}} = U_1 \frac{N_2}{N_1} - -U_{\mathrm{D1,f}} = 360\ \mathrm{V} \frac{1}{18.6} - 0.4\ \mathrm{V} = 18.95\ \mathrm{V}. \tag{4.3.15}$$

The maximum blocking voltages of the diodes D_1 corresponds to the secondary voltage at demagnetization. Therefore, the turn ratio of N_2/N_3 is used:

$$U_{\mathrm{D1,r}} = U_{\mathrm{s}} = U_1 \frac{N_2}{N_3} \quad \text{with} \quad \frac{N_2}{N_3} = \frac{N_2}{N_1} \frac{N_1}{N_3}. \tag{4.3.16}$$

The result is obtained by

$$U_{\mathrm{D1,r}} = 360\ \mathrm{V} \frac{1}{18.6 \cdot 1.5} = 12.9\ \mathrm{V}. \tag{4.3.17}$$

4.3.6 Determine the magnetizing inductance L_{m} to ensure that the peak value of the magnetizing current remains below 10 % of the $\bar{i}'_{\mathrm{L}}$, which corresponds to the average current $\bar{i}_{\mathrm{L}}$ through the output inductance translated to the primary side at a nominal load of $P_2 = 125$ W.
Answer:
The current through L is calculated by

$$I_2 = P_2/U_2 = \frac{125\ \mathrm{W}}{5\ \mathrm{V}} = 25\ \mathrm{A}. \tag{4.3.18}$$

This corresponds to the current at the primary side of

$$I'_{2\mathrm{p}} = I_2 \frac{N_2}{N_1}. \tag{4.3.19}$$

This leads to the current $\hat{i}_\mathrm{m}$ of

$$\hat{i}_\mathrm{m} = I'_{2\mathrm{p}} \cdot 0.1 = I_2 \frac{N_2}{N_1} \cdot 0.1 = \frac{25\ \mathrm{A}}{18.6} \cdot 0.1 = 0.1344\ \mathrm{A}. \tag{4.3.20}$$

The maximum magnetization current maximum $\hat{i}_\mathrm{m}$ corresponds to the ripple current Δi_m, because at the end of each period the magnetization current $i_\mathrm{m} = 0$ A:

$$\Delta i_\mathrm{m} = \frac{D \cdot U_1}{L_\mathrm{m} \cdot f_\mathrm{s}}. \tag{4.3.21}$$

Using (4.3.20) and solving (4.3.21) with respect to L_m yields

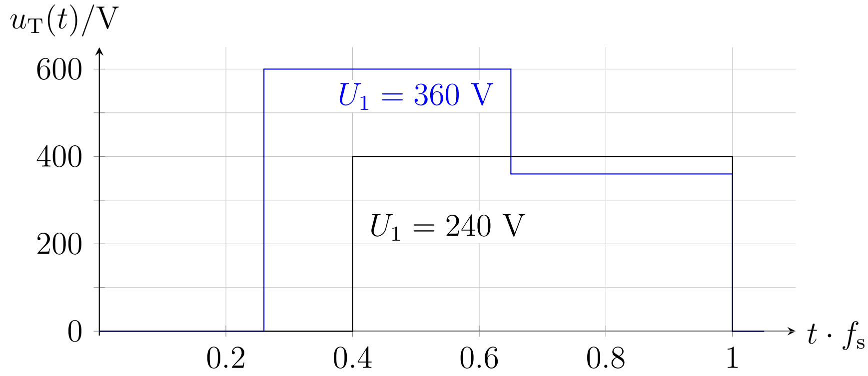
$$L_\mathrm{m} = \frac{D_\mathrm{max} \cdot U_1}{\hat{i}_\mathrm{m} \cdot f_\mathrm{s}} = \frac{0.4 \cdot 240\ \mathrm{V}}{0.1344\ \mathrm{A} \cdot 48\ \mathrm{kHz}} = 14.9\ \mathrm{mH}. \tag{4.3.22}$$

4.3.7 Sketch the signals of the voltage across the power transistor, the current through the demagnetization winding, and the current through the freewheeling diode D_2 for $U_1 = 240$ V and $U_1 = 360$ V.

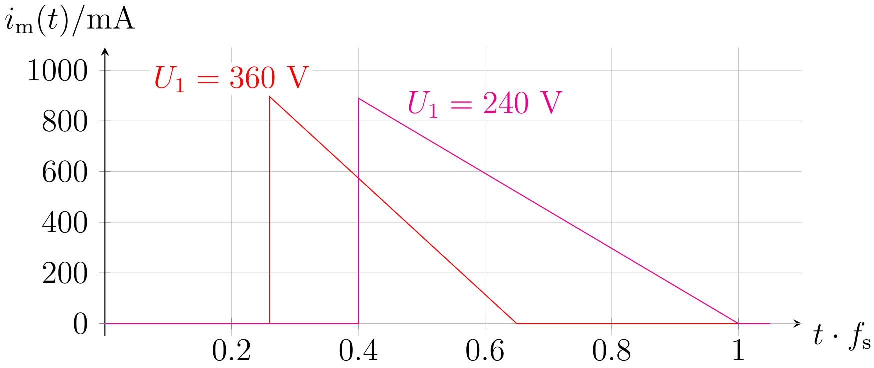
<u>Answer:</u>
The current through the demagnetization is calculated by

$$\hat{i}_3 = \frac{N_3}{N_1} \cdot \Delta i_\mathrm{m} = \frac{1}{1.5} \cdot 0.1344\ \mathrm{A} = 89.6\ \mathrm{mA}. \tag{4.3.23}$$

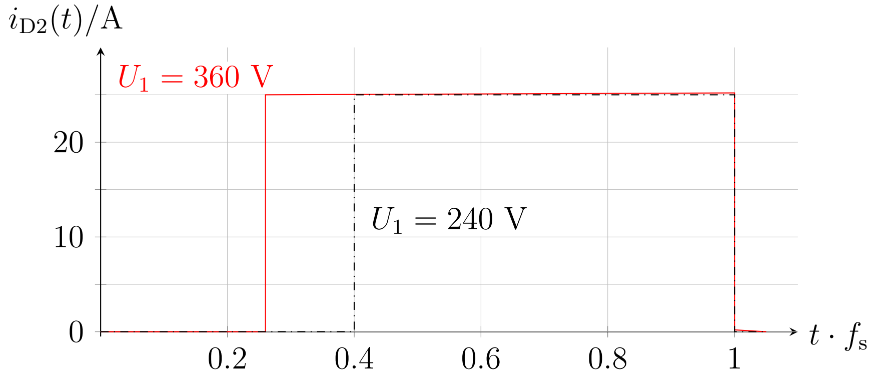
The current through freewheeling diode D_2 is taken from (4.3.18). The voltages and currents are displayed in Sol.-Fig. 4.3.1, Sol.-Fig. 4.3.2 and Sol.-Fig. 4.3.3 for the two input voltages.



Solution Figure 4.3.1: Voltage at the transistor.



Solution Figure 4.3.2: Demagnetization current of N_3.



Solution Figure 4.3.3: Current through D_2.

4.3.8 Calculate the peak magnetizing current for each case assuming a constant output current.

Answer:

The current through the primary coil N_1 is still calculated with (4.3.20) and yields 0.1344 A. Using (4.3.21) the minimal duty cycle yields

$$\hat{i}_{\mathrm{m,D_{min}}} = \Delta i_{\mathrm{m,D_{min}}} = \frac{D_{\mathrm{min}} \cdot U_1}{L_{\mathrm{m}} \cdot f_{\mathrm{s}}} = \frac{0.4 \cdot 360\ \mathrm{V}}{14.9\ \mathrm{mH} \cdot 48\ \mathrm{kHz}} = 0.1330\ \mathrm{A}. \tag{4.3.24}$$

4.3.9 Could a higher power be transferred by doubling the switching frequency of the converter?

Answer:

No, also in this case (4.3.10) shows, that the output voltage depends on the turn ratio and the duty cycle, but not on the switching frequency of the converter. The transformer does not act as energy storage. Also, this answer is valid under the assumption of ideal components.

Exercise 05: Rectifiers

Task 5.1: B2U topology with capacitive filtering

An uncontrolled single-phase, two-pulse rectifier circuit with capacitive filtering is shown in Fig. 5.1.1. All components, including the diodes, are assumed to be ideal. On the input side, the single-phase AC supply with voltage $u_1(t)$ is connected, while on the output side, a smoothing capacitor C and a constant current load I_0 are present.

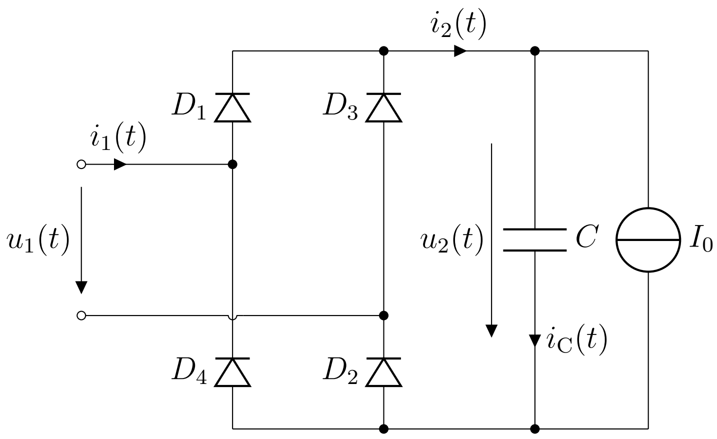


Figure 5.1.1: B2U rectifier with capacitive output filtering.

Input voltage:	$u_1(t) = 156\ \mathrm{V} \cdot \sin(\omega t)$	Load current:	$I_0 = 7.5\ \mathrm{A}$
Filter capacitance:	$C = 330\ \mathrm{\mu F}$	Frequency:	$f = 60\ \mathrm{Hz}$

Table 5.1.1: Parameters of the B2U rectifier.

The angle α represents the phase angle range between zero crossing of the supply voltage and the phase angle at which all four diodes are blocked, meaning the capacitor discharges through the load. The angle β represents the phase angle range between α and the phase angle at which two of the four diodes begin to conduct, i.e., when the capacitor is recharged from the mains supply. A steady-state operation is assumed for this task.

5.1.1 Calculate the two angles α and β. Note: For the calculation of β you can use the following simple approximation: $\sin(x) \approx x$. (This approximation is sufficiently accurate within a range of approximately $x = \pm 25°$.)

Answer:

As long as the current $i_2(t) > 0$ A, the voltage at the capacitor follows the absolute value of $u_1(t)$:

$$u_2(t) = |\hat{u}_1 \sin(\omega t)| \quad \text{for} \quad i_2(t) > 0\ \mathrm{A}. \tag{5.1.1}$$

While the conduction phase ($i_2 > 0$ A) the capacitor is charged, otherwise the capacitor provides the load current. The current of the capacitor i_C is expressed by

$$i_\mathrm{C}(t) = \begin{cases} -I_0, & i_2(t) = 0, \\ C\frac{\mathrm{d}}{\mathrm{d}t}u_2(t), & i_2(t) > 0. \end{cases} \tag{5.1.2}$$

In the conduction phase $i_2(t)$ is calculated by:

$$i_2(t) = i_\mathrm{C}(t) + I_0. \tag{5.1.3}$$

Using (5.1.1) and (5.1.2) in (5.1.3) results in

$$i_2(t) = C\frac{\mathrm{d}}{\mathrm{d}t}\left|\hat{u}_1 \sin(\omega t)\right| + I_0 = C\omega\hat{u}_1\cos(\omega t) + I_0, \quad \text{for} \quad 0 \leq \omega t < \omega t_1. \tag{5.1.4}$$

In (5.1.4) the angle ωt_1 corresponds to the angle α at which the conduction phase ends due to $i_2(t_1) = 0$ A. Inserting t_1 in (5.1.4) leads to:

$$0 = C\omega\hat{u}_1\cos(\omega t_1) + I_0 = C\omega\hat{u}_1\cos(\alpha) + I_0. \tag{5.1.5}$$

Solving (5.1.5) with respect to α results in

$$\alpha = \arccos\left(-\frac{I_0}{C\omega\hat{u}_1}\right) = \arccos(-\frac{7.5\ \mathrm{A}}{2\pi\cdot 60\ \mathrm{Hz}\cdot 330\ \mathrm{\mu F}\cdot 156\ \mathrm{V}}) = 112.8°. \tag{5.1.6}$$

Starting at angle α the capacitor is discharged by the load current I_0 up to the point ωt_2, where $|u_1(t_2)| = u_2(t_2)$. In the phase angle range $\omega t_1 \leq \omega t < \omega t_2$ the voltage $u_2(t)$ is:

$$u_2(t) = u_2(\omega t_1) + \int_{t_1}^{t} -\frac{I_0}{C}\mathrm{d}\tau = u_2(\alpha) + \int_{\alpha}^{\omega t} -\frac{I_0}{\omega C}\mathrm{d}\omega\tau = u_2(\alpha) - \frac{I_0}{\omega C}(\omega t - \alpha). \tag{5.1.7}$$

Using (5.1.7) with the condition $|u_1(t_2)| = u_2(t_2)$ leads to

$$u_2(\alpha) - \frac{I_0}{\omega C}(\omega t_2 - \alpha) = \left|\hat{u}_1\sin(\omega t_2)\right|. \tag{5.1.8}$$

This is solvable for ωt_2 in closed-form only by using the approximation $\sin(\omega t) \approx \omega t$. The voltage $|u_1(\omega t)|$ starts increasing at $\omega t > \pi$ with $\hat{u}_1\sin(\omega t - \pi)$. Using (5.1.8) we obtain

$$u_2(\alpha) - \frac{I_0}{\omega C}(\omega t_2 - \alpha) = \hat{u}_1(\omega t_2 - \pi). \tag{5.1.9}$$

Solving (5.1.9) with respect to ωt_2 leads to

$$\omega t_2 = \frac{u_2(\alpha) + \frac{I_0\alpha}{\omega C} + \hat{u}_1\pi}{\hat{u}_1 + \frac{I_0}{\omega C}} = \frac{143.4\ \mathrm{V} + \frac{7.5\ \mathrm{A}\cdot 1.969\ \mathrm{rad}}{\pi\cdot 60\ \mathrm{Hz}\cdot 330\ \mathrm{\mu F}} + 156\ \mathrm{V}\cdot\pi}{156\ \mathrm{V} + \frac{7.5\ \mathrm{A}}{\pi\cdot 60\ \mathrm{Hz}\cdot 330\ \mathrm{\mu F}}} = 3.478\ \mathrm{rad} = 199.2°. \tag{5.1.10}$$

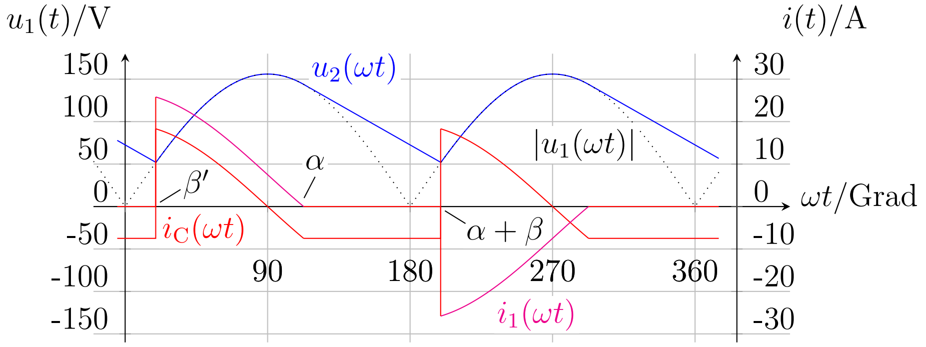
The angle β is calculated by

$$\omega t_2 = \alpha + \beta \quad \Rightarrow \quad \beta = \omega t_2 - \alpha = 199.2° - 112.8° = 86.4°. \tag{5.1.11}$$

5.1.2 Sketch the capacitor voltage $u_2(\omega t)$ considering $\omega t \in [0, ..., 2\pi]$ taking into account the previously calculated angles α and β.

Answer:

Sol.-Fig. 5.1.1 displays the voltage at the capacitor.



Solution Figure 5.1.1: Voltage at primary side.

5.1.3 Calculate the currents $i_1(\omega t)$ and $i_C(\omega t)$ and add them to the previous plot.

Answer:

Two cases are to consider for the currents of $i_1(\omega t)$ and $i_C(\omega t)$: If $\alpha < \omega t < \alpha + \beta$ and $\pi + \alpha < \omega t < \pi + \alpha + \beta$, the capacitor supplies the load current I_0. During the remaining phase angle range, the input supplies the load current I_0 and charge the capacitor. For $\alpha < \omega t < (\alpha + \beta)$ and $(\pi + \alpha) < \omega t < (\pi + \alpha + \beta)$ the current $i_C(\omega t)$ corresponds to the current $-I_0$ while the diode bridge blocks:

$$i_C(\omega t) = -I_0 \quad \text{and} \quad i_1(\omega t) = i_2(\omega t) = 0 \text{ A}. \tag{5.1.12}$$

For $(\alpha + \beta - \pi) \leq \omega t \leq \alpha$ and $(\alpha + \beta) \leq \omega t \leq (\pi + \alpha)$ the current $i_C(\omega t)$ results in

$$\begin{aligned} i_C(\omega t) &= C\frac{\mathrm{d}(u_2(t))}{\mathrm{d}t} = C\frac{\mathrm{d}(|\hat{u}_1 \sin(\omega t)|)}{\mathrm{d}t} = \omega C\, |\hat{u}_1 \cos(\omega t)| \\ &= 377 \text{ Hz} \cdot 330 \text{ µF} \cdot |156 \text{ V} \cos(\omega t)| = 19.4 \text{ A}\, |\cdot \cos(\omega t)|\,. \end{aligned} \tag{5.1.13}$$

The current $i_2(\omega t)$ is the sum of I_0 and $i_C(\omega t)$:

$$i_2(\omega t) = i_C(\omega t) + I_0 = 19.4 \text{ A}\, |\cos(\omega t)| + 7.5 \text{ A}. \tag{5.1.14}$$

The current $i_1(\omega t)$ depends on the diodes, which conduct:

$$\begin{aligned} i_1(\omega t) &= i_2(\omega t) \quad \text{for} \quad (\alpha+\beta-\pi) \leq \omega t \leq \alpha, \\ i_1(\omega t) &= -i_2(\omega t) \quad \text{for} \quad (\alpha+\beta) \leq \omega t \leq \pi+\alpha. \end{aligned} \tag{5.1.15}$$

The currents $i_\mathrm{C}(\omega t)$ and $i_1(\omega t)$ are added to Sol.-Fig. 5.1.1.

5.1.4 Assume the smoothing capacitor is very large, i.e., $C \to \infty$. What is the average active power P_0 absorbed by the current source? What will P_0 be if $C = 330\ \mu\mathrm{F}$?

Answer:

Considering (5.1.6) the term $I_0/\omega C$ becomes zero, if $C \to \infty$. This results in $\alpha = 90°$, which leads to

$$u_2(90°) = \hat{u}_1(90°) = \hat{u}_1. \tag{5.1.16}$$

For a capacitor with $C \to \infty$ the voltage decrease is zero, so that the voltage $u_2(t)$ is constant. In this case the power absorbed by the load current is obtained by

$$P_0 = \hat{u}_1 \cdot I_0 = 156\ \mathrm{V} \cdot 7.5\ \mathrm{A} = 1167\ \mathrm{W}. \tag{5.1.17}$$

For the capacitor with 330 µF again the two phase angle ranges are to distinguish. First, the phase angle range $(\alpha+\beta-\pi) \leq \omega t \leq \alpha$ is considered. The start angle of the conduction phase is calculated by

$$\beta' = \alpha + \beta - \pi = 1.969\ \mathrm{rad} + 1.51\ \mathrm{rad} - \pi = 0.337\ \mathrm{rad} = 19.3°. \tag{5.1.18}$$

Using β' the absorbed average power of the load current is calculated by:

$$P_{0,1} = \frac{\hat{u}_1 \cdot I_0 \cdot \omega}{\alpha - \beta'} \int_{\beta'}^{\alpha} \sin(\omega t)\mathrm{d}t. \tag{5.1.19}$$

This leads to

$$\begin{aligned} P_{0,1} &= \frac{\hat{u}_1 \cdot I_0}{\alpha - \beta'} \left(\cos(\beta') - \cos(\alpha)\right) \\ &= \frac{156\ \mathrm{V} \cdot 7.5\ \mathrm{A}}{1.969\ \mathrm{rad} - 0.337\ \mathrm{rad}} \left(\cos(0.337\ \mathrm{rad}) - \cos(1.969\ \mathrm{rad})\right) = 952\ \mathrm{W}. \end{aligned} \tag{5.1.20}$$

For the phase angle range $\alpha < \omega t < (\alpha + \beta)$ the voltage $u_2(\omega t)$ decreases linear and the current is independent of the voltage. This means that the power is a linear function of the voltage. Using $|\sin(\alpha+\beta)| = \sin(\beta')$ the average power is obtained by the load current multiplied with the average voltage:

$$P_{0,2} = I_0 \cdot \hat{u}_1 \frac{\sin(\alpha) + \sin(\beta')}{2} = 7.5\ \mathrm{A} \cdot 156\ \mathrm{V} \cdot \frac{\sin(112.8°) + \sin(19.3°)}{2} = 733\ \mathrm{W}. \tag{5.1.21}$$

The total absorbed average power of the load current results from the weighted sum of $P_{0,1}$ and $P_{0,2}$:

$$P_0 = \frac{(180° - \beta) \cdot P_{0,1} + \beta \cdot P_{0,2}}{180°} = \frac{(180° - 86.5°) \cdot 952\text{ W} + 86.5° \cdot 733\text{ W}}{180°} = 847\text{ W}. \qquad (5.1.22)$$

5.1.5 What is the minimum blocking voltage ratings of the diodes to ensure that the rectifiers is not damaged?

Answer:

If voltage $|u_1(t)| < u_2(t)$ the diode bridge blocks and the connection points of $u_1(t)$ floats within the voltage range of $u_2(t)$. The minimum blocking voltage ratings is associated with a conducting diode pair. The maximum of $u_1(t)$ corresponds to $\hat{u}_1 = 156$ V. In this case D_1 and D_2 conduct and the voltage drops at D_3 and D_4 in blocking direction are $\hat{u}_1 = 156$ V. The minimum of $u_1(t)$ corresponds to $-\hat{u}_1 = -156$ V. In this case D_3 and D_4 conduct and the voltage drops at D_1 and D_2 in blocking direction are $\hat{u}_1 = 156$ V. This leads to the result, that the minimum blocking voltage ratings of all diodes yields 156 V.

Task 5.2: PFC rectifier

Due to the constantly increasing load on the grid with harmonics as a result of the use of power converters, the regulations regarding the permissible harmonic content of the current consumption of electrical consumers are being tightened. It is therefore necessary, e.g. for the rectification of single-phase AC mains voltage, to design power converters with a high power factor. A variant of a PFC rectifier circuit is shown in Fig. 5.2.1. The prerequisite for the use of the boost converter is: $u_2 = U_2 > u'(t)$. The boost converter is operated with a pulse width modulated (PWM)-based controller for which the switching frequency f_T has a constant value $f_T = 20$ kHz. CCM is assumed as the operating mode.

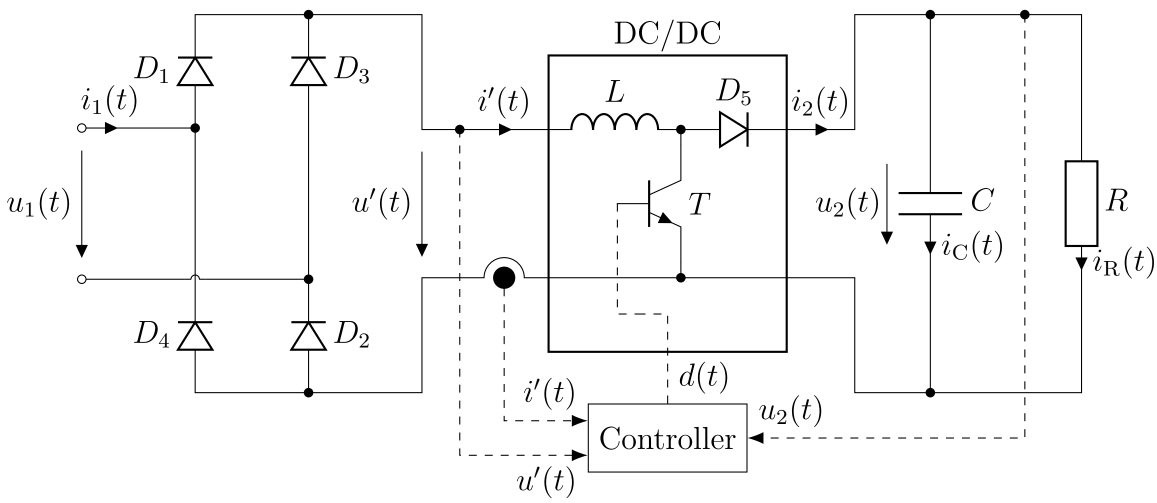


Figure 5.2.1: PFC rectifier with single-phase diode bridge and a cascaded DC/DC boost converter.

5.2.1 Specify the voltage transformation ratio $m(t) = \frac{u_2(t)}{u'(t)}$ as a function of the duty cycle $d(t)$.

Input voltage:	$u_1(t) = \hat{u}_1 \sin(\omega t) = \sqrt{2} \cdot 230\ \mathrm{V} \cdot \sin(\omega t)$	Output voltage:	$u_2(t) = 400\ \mathrm{V}$
Output power:	$P_2 = 4\ \mathrm{kW}$	Grid frequency:	$f = 50\ \mathrm{Hz}$
Inductance:	$L = 570\ \mathrm{\mu H}$	Switching frequency:	$f_\mathrm{s} = 20\ \mathrm{kHz}$

Table 5.2.1: Parameters of the PFC rectifier.

Answer:

The equation for the average voltage U_L during a pulse period is given by:

$$U_\mathrm{L} = (d(t)u'(t) + (1 - d(t))(u'(t) - U_2)). \tag{5.2.1}$$

The average voltage over the inductance for a (quasi) steady-state operation is $U_\mathrm{L} = 0$ V:

$$0 = (d(t)u'(t) + (1 - d(t))(u'(t) - U_2))f_\mathrm{s}. \tag{5.2.2}$$

Rewriting (5.2.2) delivers the voltage transformation ratio:

$$m(t) = \frac{U_2}{u'(t)} = \frac{1}{1 - d(t)}. \tag{5.2.3}$$

5.2.2 Specify the conduction time of the transistor and the diode as a function of the transformation ratio M and the time t, with the assumption $u_2(t) \approx U_2$.

Answer:

Using $M = \frac{U_2}{\hat{u}_1}$, the transformation ratio is given by:

$$m(t) = \frac{U_2}{u_1(t)} = \frac{U_2}{\hat{u}_1 \sin(\omega t)} = M\frac{1}{\sin(\omega t)}. \tag{5.2.4}$$

If the transistor T is conducting, the duty cycle $d(t)$ is derived from (5.2.3) as:

$$d(t) = 1 - \frac{1}{m(t)} = 1 - \frac{u_1(t)}{U_2} = 1 - \frac{\hat{u}_1 \sin(\omega t)}{U_2} = 1 - \frac{1}{M}\sin(\omega t). \tag{5.2.5}$$

If the diode D_5 is conducting, the turn-off duty cycle $1 - d(t)$ is derived from (5.2.3) as:

$$1 - d(t) = \frac{1}{m(t)} = \frac{1}{M}\sin(\omega t). \tag{5.2.6}$$

5.2.3 Calculate the maximum amplitude of the switching-induced ripple of the mains current i_1 for the specified operating point. Note: Consider the conductive state of T and set the voltage across the inductance as a function of the phase angle (ωt) and the conduction time of the transistor according to the previous subtask.

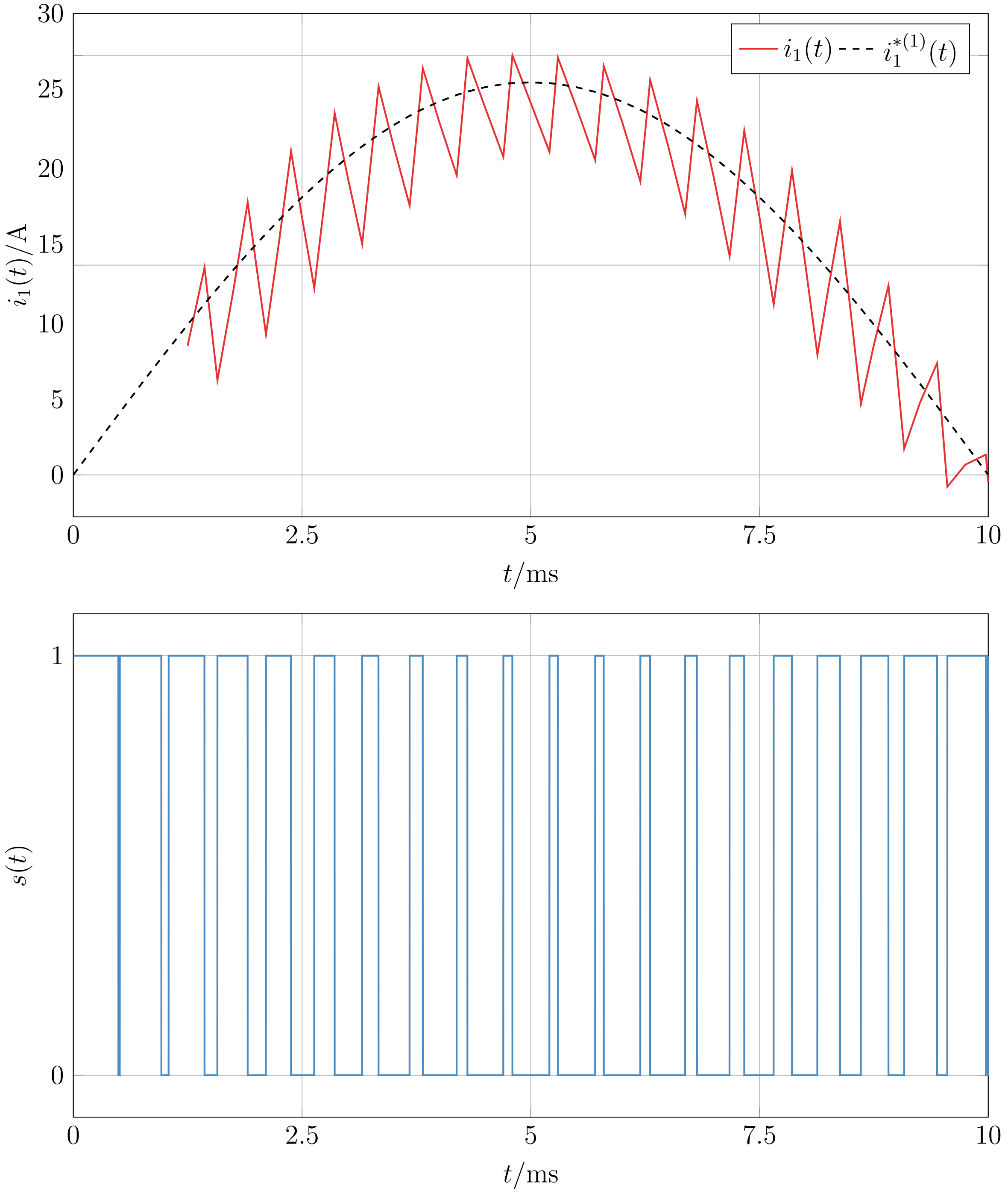


Figure 5.2.2: Incomplete representation of the mains current $i_1(t)$ and control signal $s(t)$ for power transistor T.

Answer:

To solve this task, the component equation of the inductor is used:

$$u_{\mathrm{L}}(t) = L\frac{\mathrm{d}i'(t)}{\mathrm{d}t}. \tag{5.2.7}$$

As a simplification, it is assumed that the Δ is an approximation of the total differential by a difference equation with the differences Δ. This leads to the component differential equations becoming mean value equations such as:

$$\frac{\Delta i'(t)}{\Delta t} = \frac{U_{\mathrm{L}}}{L}. \tag{5.2.8}$$

When the transistor T is conducting, $U_{\mathrm{L}} = u_1(t) = \hat{u}_1 \sin(\omega t)$ follows. This means for $\Delta i'(t)$:

$$\Delta i'(t) = \frac{U_{\mathrm{L}}\Delta t}{L} = \frac{\hat{u}_1 \sin(\omega t) d(t) T_{\mathrm{s}}}{L} = \Delta i_1'(t). \tag{5.2.9}$$

Using $d(t) = 1 - \frac{1}{M}\sin(\omega t)$:

$$\Delta i_1'(t) = \frac{U_2 \hat{u}_1 \sin(\omega t)}{U_2 L}(1 - \frac{1}{M}\sin(\omega t))T_{\mathrm{s}}. \tag{5.2.10}$$

Using $\frac{\hat{u}_1}{U_2} = \frac{1}{M}$, (5.2.10) becomes:

$$\Delta i_1'(t) = \frac{U_2 T_{\mathrm{s}}}{LM}\sin(\omega t)(1 - \frac{\sin(\omega t)}{M}). \tag{5.2.11}$$

To be able to calculate the maximum value of the mains current i_1, the following equation has to be set up from (5.2.11) as:

$$\frac{U_2 T_{\mathrm{s}}}{L}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{M}\sin(\omega t)\left(1 - \frac{1}{M}\right)\sin(\omega t)\right] = 0. \tag{5.2.12}$$

The first derivation is as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{M}\sin(\omega t)\left(1 - \frac{1}{M}\right)\sin(\omega t)\right] = \frac{-\omega\cos(\omega t)(2\sin(\omega t) - M)}{M^2} = 0. \tag{5.2.13}$$

If the term $\frac{M}{2} = \sin(\omega t)$ is applied to (5.2.13) this becomes zero:

$$\frac{-\omega\cos(\omega t)(2\frac{M}{2} - M)}{M^2} = 0. \tag{5.2.14}$$

The ratio $\frac{M}{2} = \sin(\omega t)$ can be used in (5.2.11) to calculate the maximum $\Delta i_1'(t)$ as follows:

$$\Delta i_1'(t) = \frac{U_2 T_{\mathrm{s}} M}{2LM}\left(1 - \frac{M}{2M}\right) = \frac{U_2\frac{1}{f_{\mathrm{s}}}}{4L} = \frac{400\ \mathrm{V}\cdot 50\ \mathrm{\mu s}}{4\cdot 570\ \mathrm{\mu H}} = 8.77\ \mathrm{A}. \tag{5.2.15}$$

5.2.4 Complete the current curve for a switching frequency $f_{\mathrm{s2}} = 2$ kHz and an inductance $L = 5$ mH in Fig. 5.2.2. Note: At the time $t = 0$ is $i' = 0$. The switch-on and switch-off times are determined by the control signal of the transistor T and are summarized for the first 4 switching times in Tab. 5.2.2.

	$i=1$	2	3	4
$T_{\mathrm{i,OFF}}$	490 µs	960 µs	1432 µs	1904 µs
$T_{\mathrm{i,ON}}$	506 µs	1040 µs	1573 µs	2104 µs

Table 5.2.2: Switching times $T_{\mathrm{i,OFF}}$ and $T_{\mathrm{i,ON}}$ for different i-values.

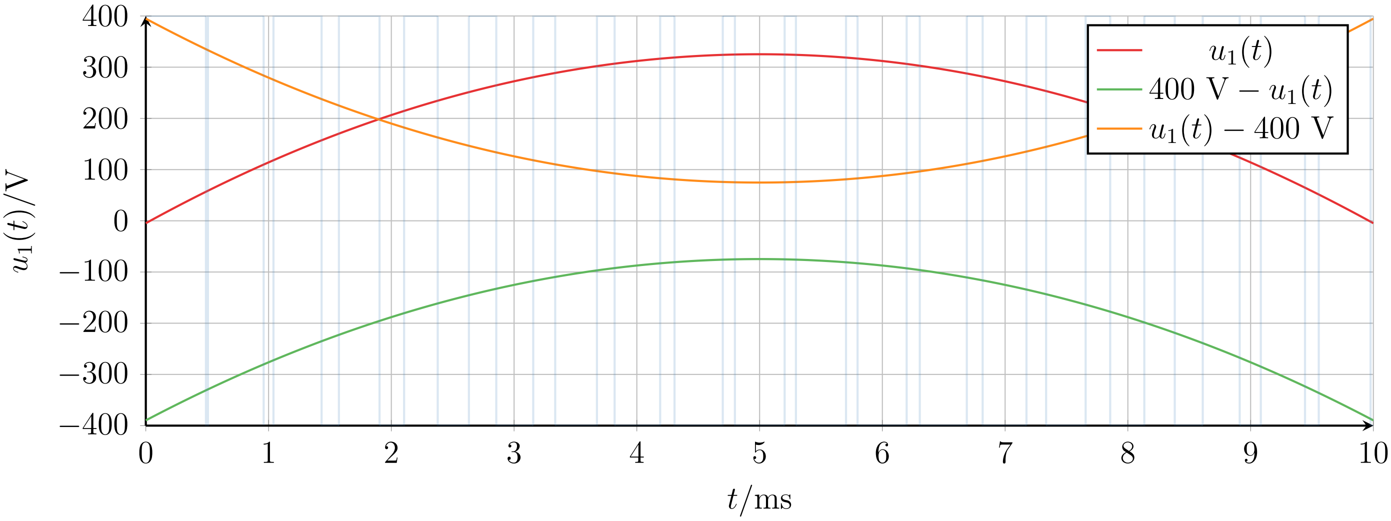


Figure 5.2.3: Voltage curve $u_1(t)$ with signal for transistor T.

Answer:
The component equation of the inductor is used to determine the current values $i'(t)$ as:

$$\frac{\mathrm{d}}{\mathrm{d}t} i'(t) = \frac{1}{L}\hat{u}_1 \sin(\omega t). \tag{5.2.16}$$

(5.2.16) must be solved for i' by integration the ODE. To do this, the right-hand side of the equation must be integrated, with the initial state of the current $i' = 0$, to:

$$i'(t) = \frac{\hat{u}_1 \cos(\omega t)}{\omega L}. \tag{5.2.17}$$

The first interval is from $0 < t < T_{1,\mathrm{off}} = 490$ µs for conducting transistor T. Inserting into (5.2.17) used with the upper and lower limit leads to:

$$i'(T_{1,\mathrm{off}}) = \frac{\hat{u}_1}{L\omega}(1-\cos(\omega T_{1,\mathrm{off}})) = \frac{\sqrt{2}\cdot 230\ \mathrm{V}}{5\ \mathrm{mH}\cdot 2\pi\cdot 50\ \mathrm{Hz}}\cdot(1-\cos(2\pi\cdot 50\ \mathrm{Hz}\cdot 490\ \mathrm{\mu s})) = 2.45\ \mathrm{A}. \tag{5.2.18}$$

For the interval $T_{1,\mathrm{off}} < t < T_{1,\mathrm{on}} = 506$ µs in which the diode D_5 conducts, the voltage $U_{\mathrm{L},1}$ is first determined as:

$$U_{\mathrm{L},1} = u_1(t) - U_2 = \hat{u}_1 \sin(\omega t) - U_2 = \sqrt{2}\cdot 230\ \mathrm{V}\cdot\sin(2\pi\cdot 50\ \mathrm{Hz}\cdot 490\ \mathrm{\mu s}) - 400\ \mathrm{V} = -350\ \mathrm{V}. \tag{5.2.19}$$

The voltage $U_{\mathrm{L},1}$ is used to calculate the current $i'(T_{1,\mathrm{on}})$ as:

$$i'(T_{1,\mathrm{on}}) = i'(T_{1,\mathrm{off}}) - \frac{U_{\mathrm{L},1}}{L}(T_{1,\mathrm{on}} - T_{1,\mathrm{off}}) = 2.45\ \mathrm{A} - \frac{350\ \mathrm{V}}{5\ \mathrm{mH}} \cdot (506\ \mu\mathrm{s} - 490\ \mu\mathrm{s}) = 1.33\ \mathrm{A}. \quad (5.2.20)$$

The second interval $T_{1,\mathrm{on}} < t < T_{2,\mathrm{off}} = 960\ \mu\mathrm{s}$, in which the transistor T conducts, have to be inserted into (5.2.17) with the upper and lower limit. This leads to:

$$i'(T_{2,\mathrm{off}}) = \frac{\hat{u}_1}{L\omega}(\cos(\omega T_{1,\mathrm{on}}) - \cos(\omega T_{2,\mathrm{off}})) + i'(T_{1,\mathrm{on}}). \quad (5.2.21)$$

With inserted values, the following follows for (5.2.21):

$$i'(T_{2,\mathrm{off}}) = \frac{\sqrt{2} \cdot 230\ \mathrm{V}}{5\ \mathrm{mH} \cdot 2\pi \cdot 50\ \mathrm{Hz}} \cdot (\cos(2\pi \cdot 50\ \mathrm{Hz} \cdot 506\ \mu\mathrm{s}) - \cos(2\pi \cdot 50\ \mathrm{Hz} \cdot 960\ \mu\mathrm{s})) + 1.33\ \mathrm{A} = 8.07\ \mathrm{A}. \quad (5.2.22)$$

For the second interval $T_{2,\mathrm{off}} < t < T_{2,\mathrm{on}} = 1040\ \mu\mathrm{s}$, in which the diode D_5 conducts, the voltage $U_{\mathrm{L}.2}$ is first determined as:

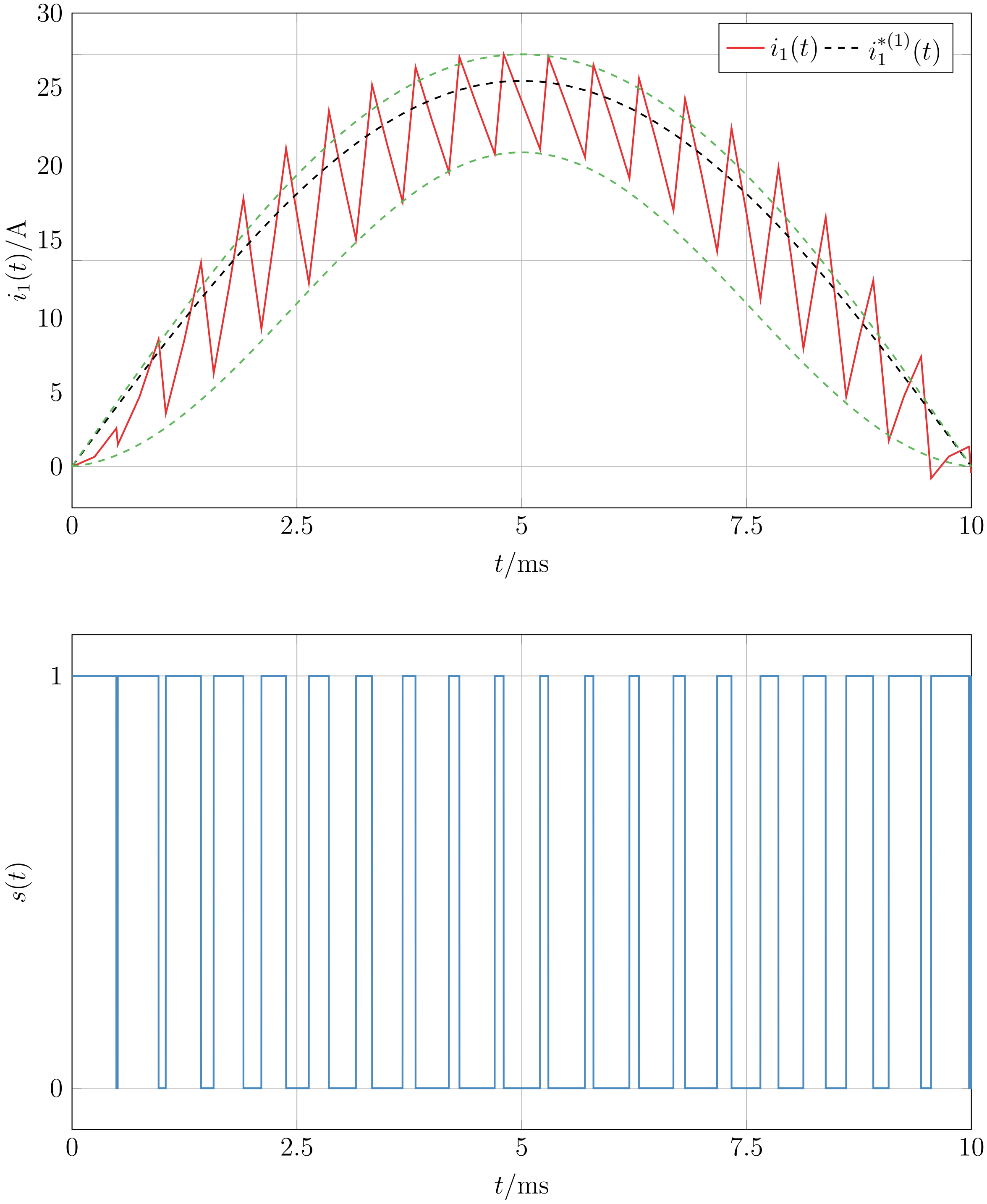
$$U_{\mathrm{L},2} = u_1(t) - U_2 = \hat{u}_1 \sin(\omega t) - U_2 = \sqrt{2} \cdot 230\ \mathrm{V} \cdot \sin(2\pi \cdot 50\ \mathrm{Hz} \cdot 960\ \mu\mathrm{s}) - 400\ \mathrm{V} = -303\ \mathrm{V}. \quad (5.2.23)$$

The voltage $U_{\mathrm{L},2}$ is used to calculate the current $i'(T_{2,\mathrm{on}})$ as:

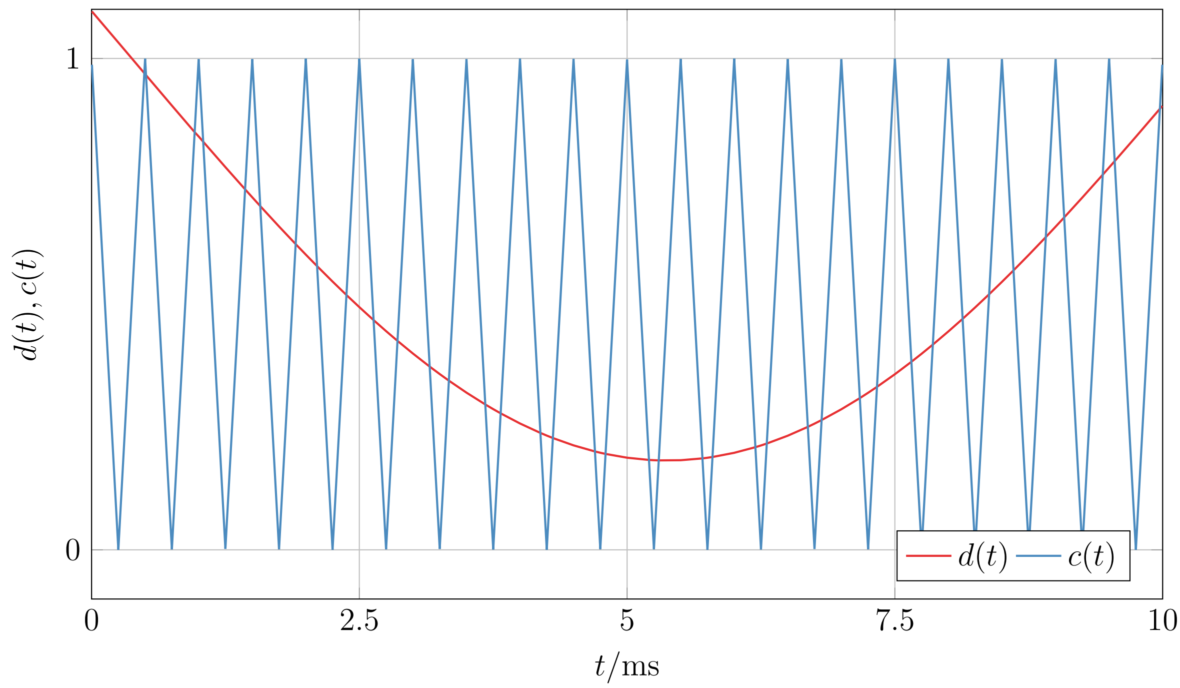
$$i'(T_{2,\mathrm{on}}) = i'(T_{2,\mathrm{off}}) - \frac{U_{\mathrm{L},2}}{L}(T_{2,\mathrm{on}} - T_{2,\mathrm{off}}) = 8.07\ \mathrm{A} - \frac{306\ \mathrm{V}}{5\ \mathrm{mH}} \cdot (1040\ \mu\mathrm{s} - 960\ \mu\mathrm{s}) = 3.32\ \mathrm{A}. \quad (5.2.24)$$

The values determined in this task for mains current $i'(t)$ must be entered in Fig. 5.2.2. This results in Sol.-Fig. 5.2.1. Figure Sol.-Fig. 5.2.2 shows that with this type of PWM current open-loop control, an nonphysical duty cycle signal of $d(t) > 1$ is necessary at the beginning of the period. This results in a open-loop control deviation, that is, a mismatch between the reference and the actual current cruves. An alternative to prevent this deviation would be to implement a feedback control system. This closed loop allows the deviation caused by $d(t) > 1$ to be compensated for downstream.

5.2.5 Approximately sketch the envelope of the current ripple in Fig. 5.2.2. Sketch the voltage across the inductor in Fig. 5.2.3 and enter the average value of the voltage as an approximation. How would the switch-on/switch-off ratio of the transistor have to be changed before and after the current peak in order to bring the average actual current value closer to the current setpoint?



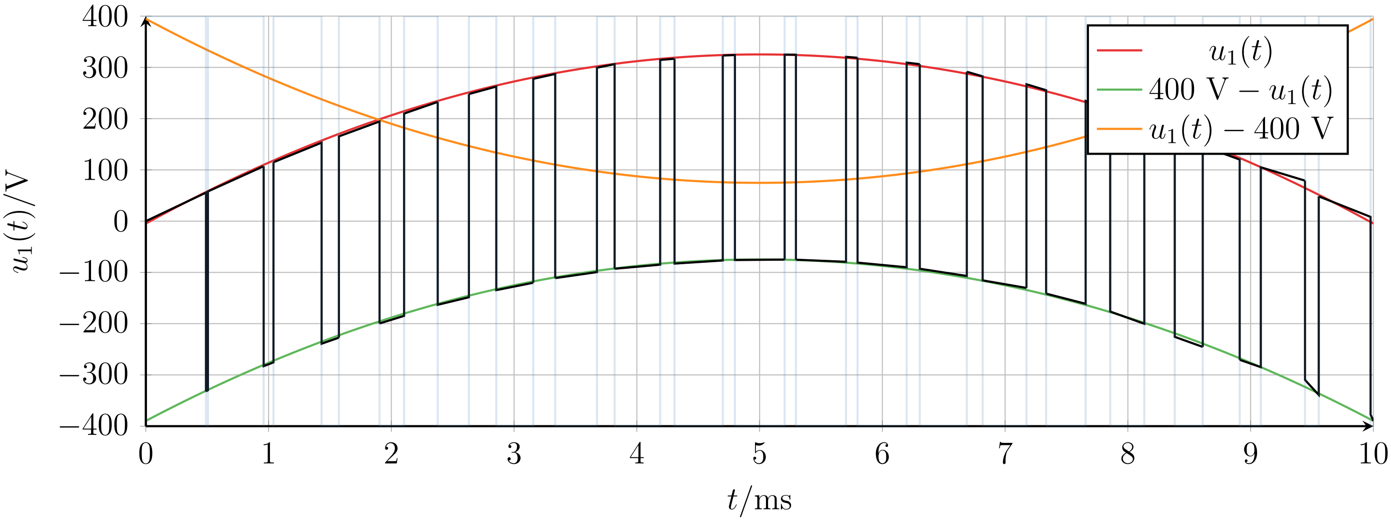
Solution Figure 5.2.1: Overall representation of the mains current i' and transistor T control signal.



Solution Figure 5.2.2: Reference signal $c(t)$ and duty cycle signal d(t).

Answer:

The envelope of the current ripple is sketched in Sol.-Fig. 5.2.1. In addition, the voltage at the inductor is sketched in Sol.-Fig. 5.2.3. The mean value of the voltage across the inductance is zero. To bring the average actual setpoint value closer to the setpoint current value, the transistor must be switched on for longer before the current maximum in Sol.-Fig. 5.2.1 and the switch-on phase must be shortened after the current maximum. By adjusting the switching ratio in this way, the current can be better regulated.



Solution Figure 5.2.3: Voltage curve $u_1(t)$ with signal for transistor T and voltage across the inductor.

5.2.6 Dimension the output capacitance C in a way that the amplitude of the output voltage ripple is $\Delta u_2 < 0.05\hat{u}_2$. Note: assume an idealized lossless converter, with the assumption $u_2(t) \approx U_2$.

Answer:

For the input power $p_1(t)$ follows $p_1(t) = P_2 - \Delta p_2$. The eqaution means the input power is equal to the power at the load minus the power at the capacitor. The part Δp_2 is zero on average, because it is the pulsating power across capacitor C. From this consideration it can be concluded for $p_1(t)$:

$$p_1(t) = \hat{u}_1 \sin(\omega t)\hat{i}_1 \sin(\omega t) = \hat{u}_1\hat{i}_1 \sin^2(\omega t). \tag{5.2.25}$$

By using the trigonometric addition formula $\sin^2(\omega t) = \frac{1}{2}(1 - \cos(2\omega t))$ follows:

$$p_1(t) = \frac{\hat{u}_1\hat{i}_1}{2}(1 - \cos(2\omega t)). \tag{5.2.26}$$

The power delivered across the load R is equal to the output power P_2 which is given as follows:

$$P_2 = \frac{\hat{u}_1\hat{i}_1}{2} = U_2 I_2. \tag{5.2.27}$$

The pulsating power across capacitor C must be calculated as follows:

$$\Delta p_2 = \frac{\hat{u}_1\hat{i}_1}{2}\cos(2\omega t). \tag{5.2.28}$$

The current through the capacitor is given by the following equation, using $P = UI$:

$$i_\mathrm{C}(t) = C\frac{\mathrm{d}u_\mathrm{c}}{\mathrm{d}t} = \frac{\Delta p_2}{U_2} = \frac{P_2\cos(2\omega t)}{U_2}. \tag{5.2.29}$$

This follows from (5.2.29) for U_C:

$$\frac{\mathrm{d}u_\mathrm{C}}{\mathrm{d}t} = \frac{P_2\cos(2\omega t)}{CU_2}, \tag{5.2.30}$$

$$u_\mathrm{C} = \frac{1}{C}\int\frac{P_2\cos(2\omega t)}{U_2}\,\mathrm{d}t = \frac{P_2\sin(2\omega t)}{2\omega CU_2}. \tag{5.2.31}$$

The capicator voltage u_C is equal to $\hat{u}_\mathrm{C}\sin(2\omega t)$, from this follows for (5.2.31):

$$\Delta u_2 = \hat{u}_\mathrm{C} = \frac{P_2}{2\omega CU_2}. \tag{5.2.32}$$

Finally the capacitance can be calculate with $\Delta u_2 < 0.05\hat{u}_2$ as:

$$C = \frac{P_2}{2\cdot 2\pi f U_2\Delta u_2} = \frac{U_2 I_2}{2\cdot 2\pi f U_2\ 0.05U_2} = \frac{400\ \mathrm{V}\cdot 10\ \mathrm{A}}{4\pi\cdot 50\ \mathrm{Hz}\cdot 400\ \mathrm{V}\cdot 0.05\cdot 400\ \mathrm{V}} = 796\ \mathrm{\mu F}. \tag{5.2.33}$$

5.2.7 Calculate the RMS value of the current through the capacitor I_C. Note: the mean value of the current through the capacitor is $\bar{i}_\mathrm{C} = 0$!

Answer:

Kirchhoff's junction rule can be used for the current $i_2(t)$:

$$i_2(t) = i_\mathrm{C}(t) + i_\mathrm{R}(t). \tag{5.2.34}$$

Since an RMS value must be calculated, the equation is squared to avoid the square root. When solving the equation, the first binomial formula must be used:

$$i_2^2(t) = (i_\mathrm{C}(t) + i_\mathrm{R}(t))^2 = i_\mathrm{C}^2(t) + 2i_\mathrm{R}(t)i_\mathrm{C}(t) + i_\mathrm{R}^2(t). \tag{5.2.35}$$

Calculate RMS value:

$$I_2^2(t) = \frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} i_2^2(t)\,\mathrm{d}t = \frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} i_\mathrm{C}^2(t)\,\mathrm{d}t + \frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} 2i_\mathrm{C}(t)i_\mathrm{R}(t)\,\mathrm{d}t + \frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} i_\mathrm{R}^2(t)\,\mathrm{d}t \tag{5.2.36}$$

Solving the equation, the term $\frac{1}{T_\mathrm{s}}\int_0^{T_\mathrm{s}} 2i_\mathrm{C}(t)i_\mathrm{R}(t)\,\mathrm{d}t$ is zero due to the periodicity of $i_\mathrm{C}(t)$:

$$I_2^2(t) = \frac{T_\mathrm{s}I_\mathrm{C}^2(t)}{T_\mathrm{s}} + \frac{T_\mathrm{s}I_\mathrm{R}^2(t)}{T_\mathrm{s}} = I_\mathrm{C}^2(t) + I_\mathrm{R}^2(t). \tag{5.2.37}$$

Finally, (5.2.37) must be converted to $I_\mathrm{C}^2(t)$:

$$I_\mathrm{C}^2(t) = I_2^2(t) - I_\mathrm{R}^2(t). \tag{5.2.38}$$

5.2.8 The current carrying capacity of the capacitor is $\frac{10\ \mathrm{A}}{1\ \mathrm{mF}}$. How large must its capacitance be selected to stay below this threshold with $I_\mathrm{C} = 10.44$ A? Is the permissible output voltage fluctuation or is it the current carrying capacity that determines the capacitance?
Answer:
Calculation of the capacitance C:

$$C = \frac{I_\mathrm{C}}{10\ \frac{\mathrm{A}}{\mathrm{mF}}} = \frac{10.44\ \mathrm{A}}{10\ \frac{\mathrm{A}}{\mathrm{mF}}} = 1.04\ \mathrm{mF}. \tag{5.2.39}$$

The dimensioned capacitance for an output voltage ripple $\Delta u_2 < 0.05\hat{u}_2$ was determined as 796 µF. If these two capacitances are compared, it becomes clear that the current carrying capacity of the capacitor is the determining factor.

5.2.9 What is the current load (effective and average value) of the mains diodes?
Answer:
Due to the rectifier behavior, the current $\bar{i}_{\mathrm{D},1-4}$ can be calculated as an average value over a half-period. So the average value is given by:

$$\bar{i}_{\mathrm{D},1-4} = \frac{1}{2\pi}\int_0^{\pi} \hat{i}_1 \sin(\omega t)\,\mathrm{d}t \ = \frac{\hat{i}_1}{2\pi}[-\cos(\pi) - [-\cos(0)]] = \frac{\hat{i}_1}{\pi}. \tag{5.2.40}$$

The effective value is given by:

$$I_{\mathrm{D},1-4}^2 = \frac{1}{2\pi}\int_0^{\pi} \hat{i}_1^2 \sin^2(\omega t)\,\mathrm{d}t \ = \frac{\hat{i}_1^2}{2\pi}\left[\frac{\pi - \frac{\sin(2\pi)}{2}}{2} - \frac{-\frac{\sin(0)}{2}}{2}\right], \tag{5.2.41}$$

$$I_{\mathrm{D},1-4} = \sqrt{\frac{\hat{i}_1^2}{2\pi}\frac{\pi}{2}} = \frac{\hat{i}_1^2}{2}. \tag{5.2.42}$$

Exercise 06: Controlled rectifiers

Task 6.1: M3C converter at an RL-load

A controlled three-pulse midpoint circuit feeds an ohmic-inductive load. The load inductance L is infinitely large such that a pure direct current I_2 is taken from the converter. The load resistance is $R = 5\ \Omega$. The converter's ideal transformer is connected to the symmetrical three-phase grid with $U_\mathrm{N} = 230$ V (effective value of phase voltage) and $U_\mathrm{N,LL} = 400$ V (line-to-line voltage). The secondary side phase voltages point an effective value of $U_{1,i} = 230\ \mathrm{V}, \forall i = \mathrm{a,b,c}$. The thyristors and commutation can be assumed to be ideal.

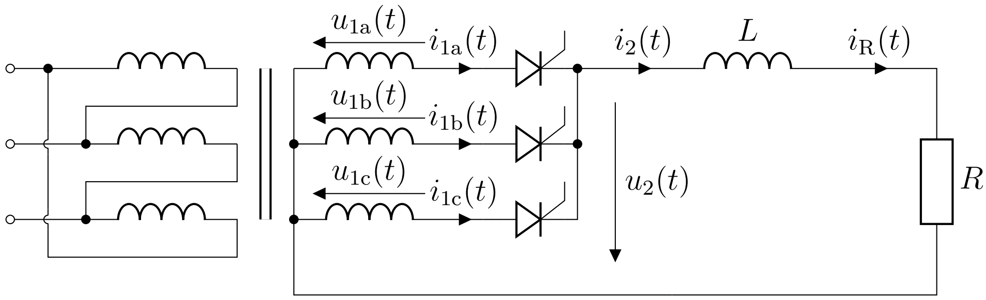


Figure 6.1.1: M3C topology with an input three-phase transformer and an RL-load.

6.1.1 Calculate the firing angle α such that an active power of $P = 6$ kW is delivered to the load. How big is the resulting load current $i_2(t) = I_2$?

Answer:

We can calculate $\bar{u}_2(\alpha)$ and I_2 from the given active power delivered, P, using

$$P = \frac{\bar{u}_2^2(\alpha)}{R} = \bar{u}_2(\alpha) I_2, \tag{6.1.1}$$

which would result in

$$\bar{u}_2(\alpha) \approx 173.2\ \mathrm{V}, I_2 \approx 34.6\ \mathrm{A}. \tag{6.1.2}$$

The average output voltage for an arbitrary α is calculated by

$$\bar{u}_2(\alpha) = \frac{3\sqrt{3}}{2\pi}\hat{u}_1 \cos(\alpha), \tag{6.1.3}$$

where

$$\hat{u}_1 = \sqrt{2}U_1 = \sqrt{2} \cdot 230\ \mathrm{V} \approx 325.27\ \mathrm{V}. \tag{6.1.4}$$

Hence, solving (6.1.3) with respect to α results in

$$\alpha = \arccos\left(\frac{2\pi\bar{u}_2(\alpha)}{3\sqrt{3}\hat{u}_1}\right) \approx 0.871\ \mathrm{rad}. \tag{6.1.5}$$

6.1.2 Draw the normalized control characteristic curve ${U_2(\alpha)}/{U_2(\alpha=0)}$ and mark the operating point $P = 6$ kW at $R = 5\ \Omega$.

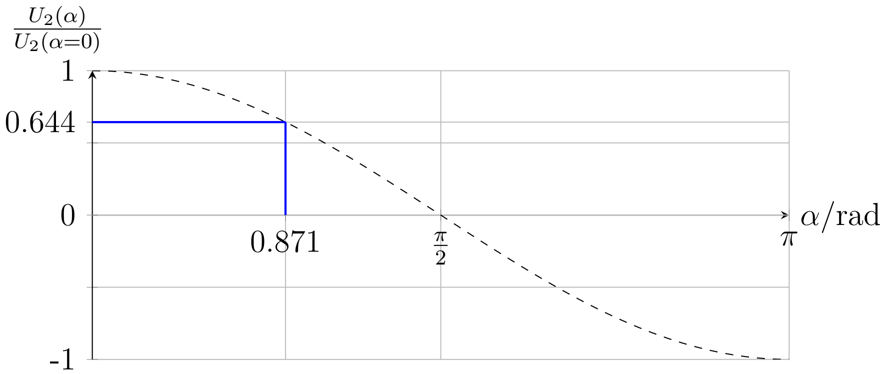
Answer:

Substituting $\alpha = 0$ in (6.1.3) results in

$$\bar{u}_2(\alpha = 0) = \frac{3\sqrt{3}}{2\pi}\hat{u}_1. \tag{6.1.6}$$

Dividing (6.1.3) by (6.1.6) leads to

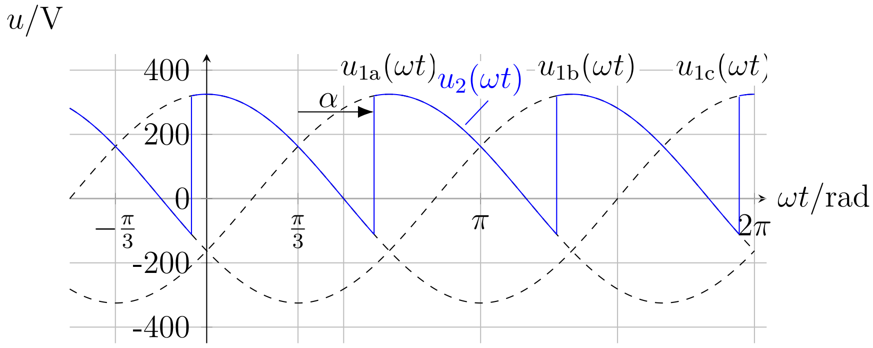
$$\frac{\bar{u}_2(\alpha)}{\bar{u}_2(\alpha = 0)} = \cos(\alpha). \tag{6.1.7}$$



Solution Figure 6.1.1: Normalized curve for $\frac{U_2(\alpha)}{U_2(\alpha=0)}$ with $\alpha = 0.871$.

6.1.3 Draw the curve of the converter's output voltage $u_2(t)$ for the calculated control angle α, from subtask 6.1.1, within a plot of the three-phase transformer's secondary voltages.

Answer:

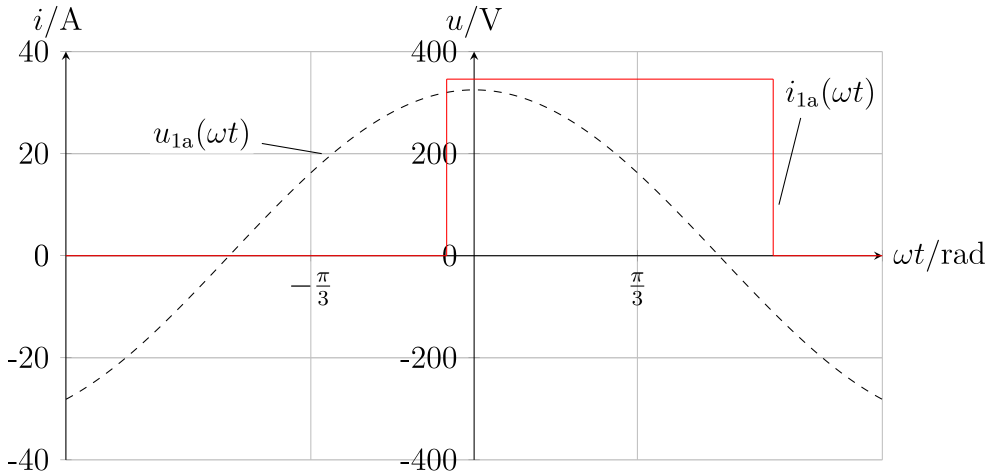


Solution Figure 6.1.2: Output voltage $u_2(t)$ for $\alpha = 0.871$.

6.1.4 Calculate the effective value $I_{1a}^{(1)}$ of the fundamental current component $i_{1a}^{(1)}(t)$ and add the latter to the previous plot. How big is the phase shift φ_{1a} between $u_{1a}(t)$ and $i_{1a}^{(1)}(t)$?

Answer:

Observing the $i_{1a}(\omega t)$ signal in Fig. 6.1.3, one can see that it is an even signal. Hence, we expect that the fundamental component of the Fourier Series would contain only cosine component ($a_1 \neq 0$) and no sine component ($b_1 = 0$).



Solution Figure 6.1.3: Input current $i_{1a}(\omega t)$.

For simplification, we can position the y-xis in the middle of the conducting period and make use of the period's symmetry. This would simplify the calculations by integrating over one half of the period and then multiply by 2. In this case the integration would be from 0 to $\frac{\pi}{3}$, as one conducting period is equal to $\frac{2\pi}{3}$. Thus, a_1 ($\hat{i}_{1a}^{(1)}$) can be calculated as:

$$a_1 = \hat{i}_{1a}^{(1)} = \frac{2}{\pi}\int_0^{\frac{\pi}{3}} I_2 \cos(\omega t)\mathrm{d}\omega t = \left[\frac{2}{\pi}\sin(\omega t)\right]_0^{\frac{\pi}{3}} = \frac{1}{\pi}I_2\sqrt{3}. \quad (6.1.8)$$

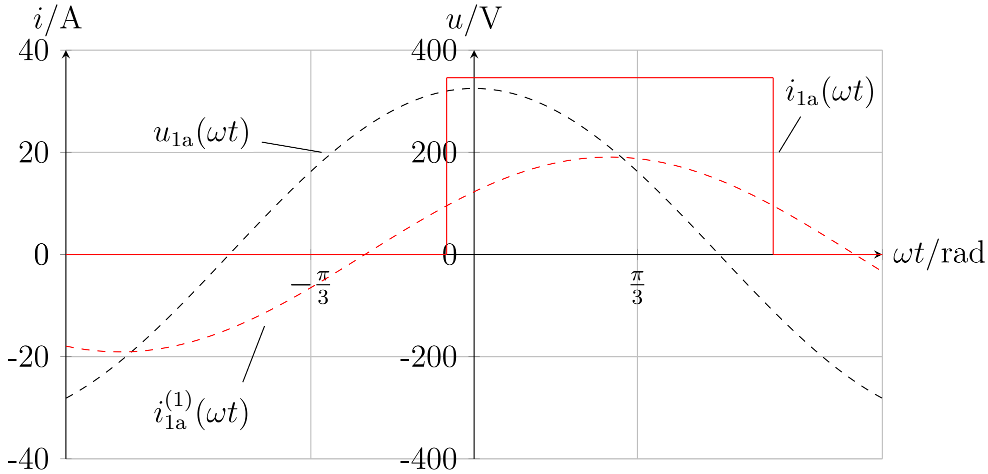
Then the effective value $I_{1a}^{(1)}$ is

$$I_{1a}^{(1)} = \frac{\hat{i}_{1a}^{(1)}}{\sqrt{2}} \approx 13.5 \text{ A}. \quad (6.1.9)$$

Fig. 6.1.4 shows the fundamental component $i_{1a}^{(1)}(\omega t)$ compared to the input voltage $u_{1a}(\omega t)$.
The phase shift of $i_{1a}^{(1)}(\omega t)$ can be calculated by calculating the phase of the peak of the signal, which can be calculated as:

$$\varphi_{1a} = (-\frac{\pi}{3} + \alpha) + (\frac{2\pi}{3} \cdot \frac{1}{2}) = -\frac{\pi}{3} + \alpha + \frac{\pi}{3} = \alpha, \quad (6.1.10)$$

where -$\frac{\pi}{3} + \alpha$ is the start of the conducting period, while $\frac{2\pi}{3} \cdot \frac{1}{2}$ is half of the period of conduction.



Solution Figure 6.1.4: Fundamental current component $i_{1a}^{(1)}(\omega t)$ for $\alpha = 0.871$.

6.1.5 Calculate the fundamental reactive power $Q^{(1)}$ drawn by the converter from the grid.

Answer:

The fundamental reactive power drawn for one full period (2π) can be calculated using

$$Q^{(1)} = 3U_1 I_1^{(1)} \sin(\alpha) = 3 \cdot 230\ \text{V} \cdot 13.5\ \text{A} \cdot \sin(0.871) \approx 7.12\ \text{kVA}. \tag{6.1.11}$$

Task 6.2: B6C converter at a motor load

In a lifting drive, a permanent magnet DC motor is supplied by a B6C converter circuit. The B6C-topology is connected to the three-phase grid. With the assumption of $L \to \infty$ the motor operates with constant nominal current and constant nominal voltage when lifting as well as lowering the load. This corresponds to a terminal voltage of $u_{\text{mot,up}}(t) = U_{\text{mot}}$ when lifting the load and $u_{\text{mot,down}}(t) = -U_{\text{mot}}$ when lowering it. In order to generate the necessary torque, the motor absorbs the current $i_{\text{mot}}(t) = I_{\text{mot}}$.

Input voltages ($i = \text{a, b, c}$):	$U_{1,i} = 230$ V (phase voltage)
	$U_{1,\text{LL},i} = 400$ V (line-to-line voltage)
Nom. motor current:	$I_{\text{mot}} = 20$ A
Nom. motor voltage:	$U_{\text{mot}} = 466$ V
Grid frequency:	$f = 50$ Hz

Table 6.2.1: Parameters of the lifting drive with B6C converter.

6.2.1 Calculate the firing angle α_{up} required for lifting and the firing angle α_{down} for lowering the load to operate the motor at rated voltage.

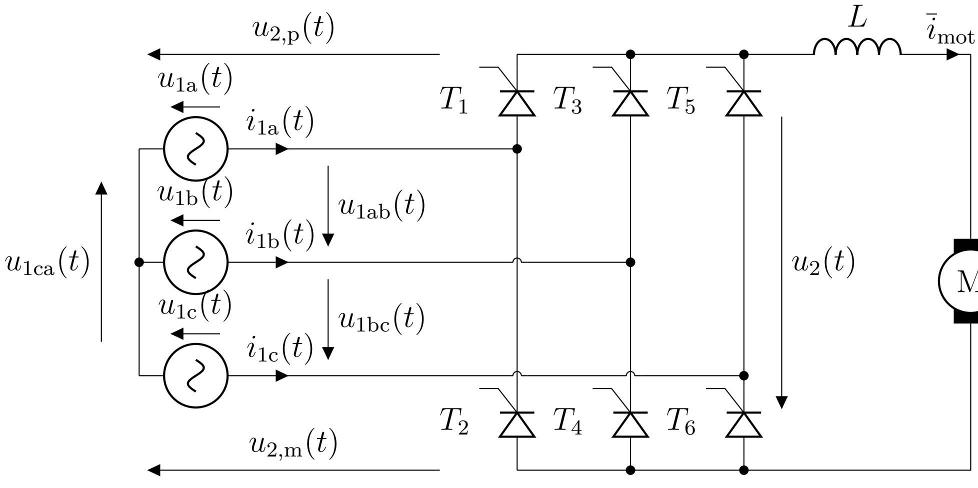


Figure 6.2.1: B6C converter at a motor load.

Answer:

The average voltage $\overline{u}_2$ is calculated by

$$\overline{u}_2 = \hat{u}_{1,\mathrm{LL}}\frac{\pi}{p}\sin(\frac{p}{\pi})\cos(\alpha). \tag{6.2.1}$$

In case of B6C-topology p is equal to 6. The maximum value of $u_{1,\mathrm{LL}}(t)$ is calculated by

$$\hat{u}_{1,\mathrm{LL}} = \sqrt{2}\sqrt{3}\cdot U_1 = \sqrt{2}\sqrt{3}\cdot 230\ \mathrm{V} = 563\ \mathrm{V}. \tag{6.2.2}$$

The voltage $\overline{u}_2$ corresponds to U_{mot}. Solving (6.2.1) with respect to α results in

$$\alpha = \arccos(\frac{\overline{u}_2\cdot p}{\hat{u}_{1,\mathrm{LL}}\pi\sin(\frac{p}{\pi})}). \tag{6.2.3}$$

The following applies to the relevant cases:

$$\begin{aligned}\alpha_{\mathrm{up}} &= \arccos(\frac{466\ \mathrm{V}\cdot 6}{563\ \mathrm{V}\pi\sin(\frac{6}{\pi})}) = 30^\circ\\ \alpha_{\mathrm{down}} &= \arccos(\frac{-466\ \mathrm{V}\cdot 6}{563\ \mathrm{V}\pi\sin(\frac{6}{\pi})}) = 150^\circ\end{aligned}. \tag{6.2.4}$$

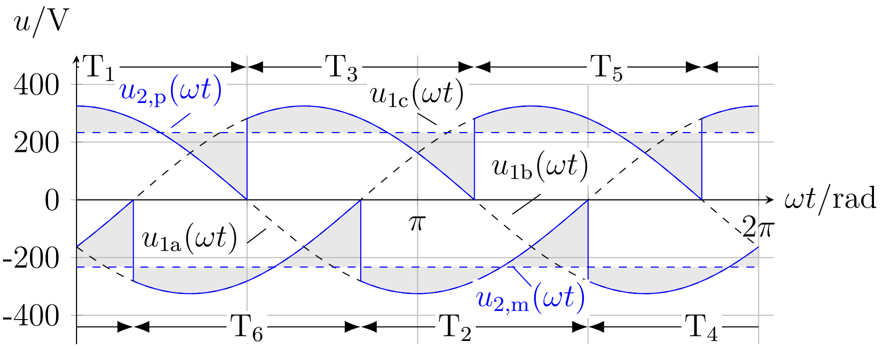
6.2.2 Sketch following signals for the two calculated firing angles α_{up} and α_{down}:

- The output voltage $u_{2,\mathrm{p}}(t)$ and $u_{2,\mathrm{m}}(t)$ of the two partial converters (reference point is neutral) and shade the effective voltage-time area for the two cases,
- The output voltage $u_2(t)$ and the mean voltage $\overline{u}_2$,
- The current $i_{1\mathrm{a}}(t)$ and it's fundamental $i_{1\mathrm{a}}^{(1)}(t)$,
- The voltage of thyristor $u_{\mathrm{T1}}(t)$,

- Indicate which thyristors are conducting during a pulse interval.

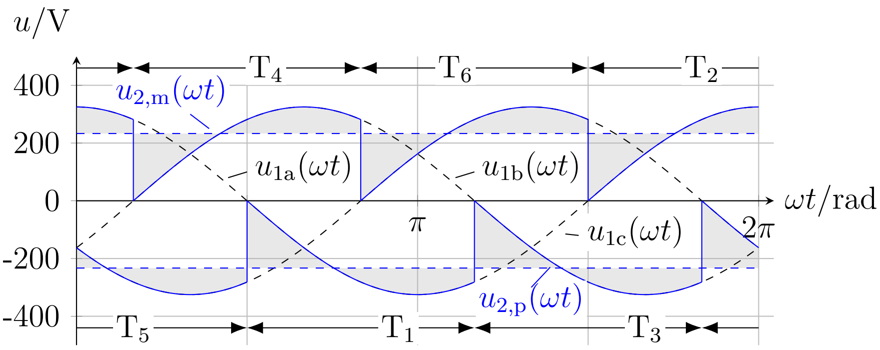
Answer:

In Sol.-Fig. 6.2.1 the output voltages $\overline{u}_{2,\mathrm{p}} = 233$ V and $\overline{u}_{2,\mathrm{m}} = -233$ V are entered as dashed blue lines. The ranges in which the respective thyristor conducts are marked with $\mathrm{T_i}$.



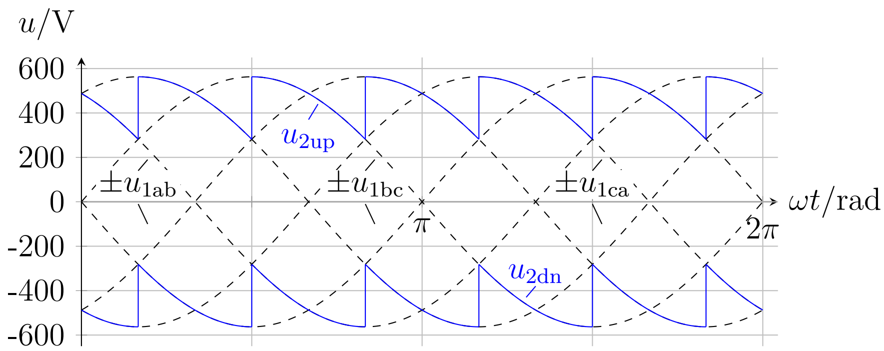
Solution Figure 6.2.1: Output voltage $u_{2,\mathrm{p}}(t)$ and $u_{2,\mathrm{m}}(t)$ for raising the load.

In Sol.-Fig. 6.2.2 the output voltages are $\overline{u}_{2,\mathrm{p}} = -233$ V and $\overline{u}_{2,\mathrm{m}} = 233$ V.



Solution Figure 6.2.2: Output voltage $u_{2,\mathrm{p}}(t)$ and $u_{2,\mathrm{m}}(t)$ for lowering the load.

In Sol.-Fig. 6.2.3 the output voltage $\overline{u}_2$ is depicted for both cases.

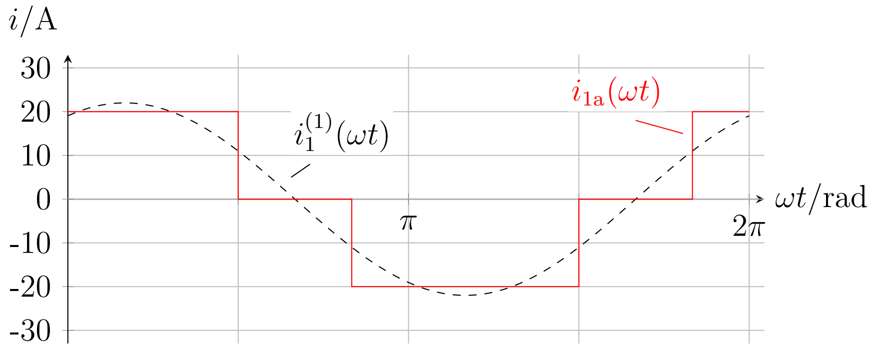


Solution Figure 6.2.3: Output voltage $u_{2\mathrm{up}}(\omega t)$ for raising and $u_{2\mathrm{dn}}(\omega t)$ lowering the load.

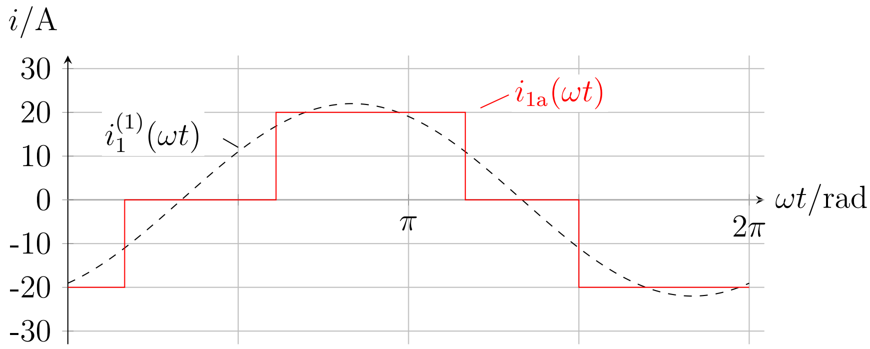
The amplitude of the fundamental $i_{1\mathrm{a}}^{(1)}(t)$ is determined by calculating the Fourier coefficient. To simplify the calculation the current signal $i_{1\mathrm{a}}(t)$ is shift by $\frac{2\pi}{3}$rad to the left, so that it is axially symmetric. Due to the symmetry the integration is performed only from 0 rad to π rad:

$$\begin{aligned}\hat{i}_{1a}^{(1)} &= \frac{2}{\pi}\int_0^{\pi} i_{1a}(\omega t)\cos(\omega t)\mathrm{d}\omega t = \frac{2}{\pi}\left(\int_0^{\frac{\pi}{3}} I_{\mathrm{mot}}\cos(\omega t)\mathrm{d}\omega t - \int_{\frac{2\pi}{3}}^{\pi} I_{\mathrm{mot}}\cos(\omega t)\mathrm{d}\omega t\right)\\ &= \frac{2I_{\mathrm{mot}}}{\pi}\left(\sin(\frac{\pi}{3}) + \sin(\frac{2\pi}{3})\right) = \frac{2\cdot 20\ \mathrm{A}}{\pi}\sqrt{3} = 22\ \mathrm{A}.\end{aligned} \tag{6.2.5}$$

The result is valid for raising and lowering the load, because in both cases the current signal forms are identical (except for some phase shift which does not change the fundamental amplitude).

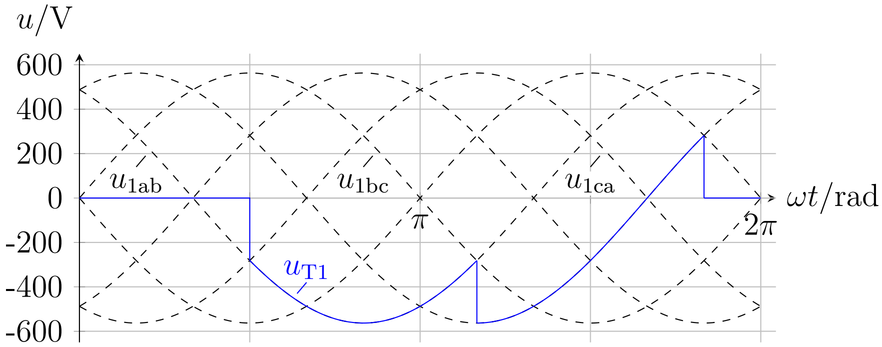


Solution Figure 6.2.4: Output current $i_{1a}(t)$ and it's fundamental amplitude for raising the load.

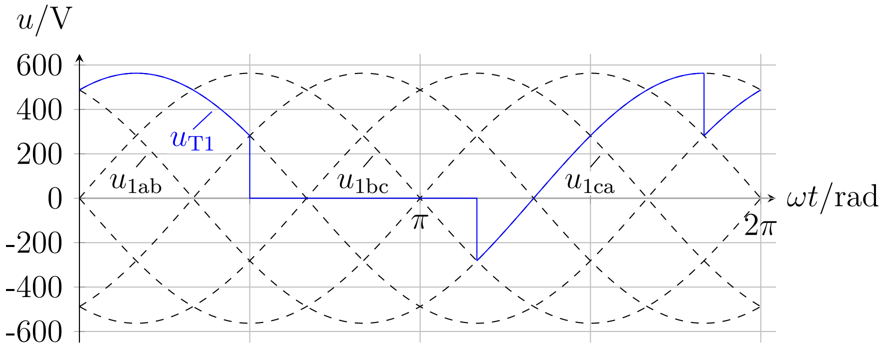


Solution Figure 6.2.5: Output current $i_{1a}(t)$ and it's fundamental amplitude for lowering the load.

The voltage $u_{T1}(t)$ is displayed for raising the load in Sol.-Fig. 6.2.6 and for lowering the load in Sol.-Fig. 6.2.7.



Solution Figure 6.2.6: Voltage $u_{T1}(\omega t)$ for raising the load.



Solution Figure 6.2.7: Voltage $u_{T1}(\omega t)$ for lowering the load.

6.2.3 Calculate the active power P, the fundamental reactive power $Q^{(1)}$ and the fundamental apparent power $S^{(1)}$ for the two considered operating points. Represent P, $Q^{(1)}$ and $S^{(1)}$ in the complex plane.

Answer:

$$\begin{aligned} P_{\mathrm{up}} &= U_{\mathrm{mot}} I_{\mathrm{mot}} = 466\ \mathrm{V} \cdot 20\ \mathrm{A} = 9320\ \mathrm{W}. \\ P_{\mathrm{down}} &= -U_{\mathrm{mot}} I_{\mathrm{mot}} = -466\ \mathrm{V} \cdot 20\ \mathrm{A} = -9320\ \mathrm{W}. \end{aligned} \tag{6.2.6}$$

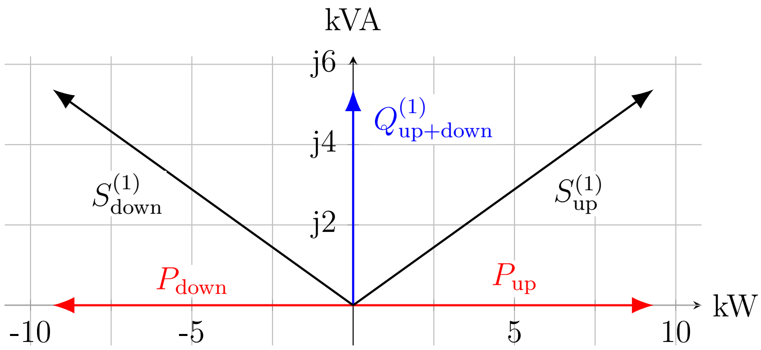
The input voltage $U_1^{(1)}$ corresponds to U_1, so that the apparent power results to

$$\begin{aligned} S_{\mathrm{up}}^{(1)} &= 3U_1 \frac{\hat{i}_{1\mathrm{a}}^{(1)}}{\sqrt{2}} = 230\ \mathrm{V} \cdot \frac{22\ \mathrm{A}}{\sqrt{2}} = 10760\ \mathrm{VA}. \\ S_{\mathrm{down}}^{(1)} &= 3U_1 \frac{\hat{i}_{1\mathrm{a}}^{(1)}}{\sqrt{2}} = 230\ \mathrm{V} \cdot \frac{22\ \mathrm{A}}{\sqrt{2}} = 10760\ \mathrm{VA}. \end{aligned} \tag{6.2.7}$$

The reactive power is calculated by

$$\begin{aligned} Q^{(1)}_{\mathrm{up}} &= 3U_1 \frac{\hat{i}^{(1)}_{1\mathrm{a}}}{\sqrt{2}} \sin\left(\alpha_{\mathrm{up}}\right) = 230\ \mathrm{V} \cdot \frac{22\ \mathrm{A}}{\sqrt{2}} \sin\left(\frac{\pi}{6}\right) = 5380\ \mathrm{VA}. \\ Q^{(1)}_{\mathrm{down}} &= 3U_1 \frac{\hat{i}^{(1)}_{1\mathrm{a}}}{\sqrt{2}} \sin\left(\alpha_{\mathrm{down}}\right) = 230\ \mathrm{V} \cdot \frac{22\ \mathrm{A}}{\sqrt{2}} \sin\left(\frac{5\pi}{6}\right) = 5380\ \mathrm{VA}. \end{aligned} \tag{6.2.8}$$

Sol.-Fig. 6.2.8 shows the power in the complex plane.



Solution Figure 6.2.8: Power within the complex plane.

6.2.4 Calculate the fundamental current ratio $g = \frac{I^{(1)}}{I}$ and the power factor λ for the two considered operating points.

Answer:

The fundamental current ratio results to

$$g = \frac{\frac{\hat{i}^{(1)}_{1\mathrm{a}}}{\sqrt{2}}}{I_{\mathrm{mot}}} = \frac{\frac{22\ \mathrm{A}}{\sqrt{2}}}{20\ \mathrm{A}} = 0.78. \tag{6.2.9}$$

The power factor λ is calculated by

$$\begin{aligned} \lambda_{\mathrm{up}} &= g \cdot \cos\left(\alpha_{\mathrm{up}}\right) = 0.78 \cdot \cos\left(\frac{\pi}{6}\right) = 0.68. \\ \lambda_{\mathrm{down}} &= g \cdot \cos\left(\alpha_{\mathrm{down}}\right) = 0.78 \cdot \cos\left(\frac{5\pi}{6}\right) = -0.68. \end{aligned} \tag{6.2.10}$$

Exercise 07: Transistor-based AC-DC converters

Task 7.1: Single-phase AC-DC converter

The following ideal single-phase converter

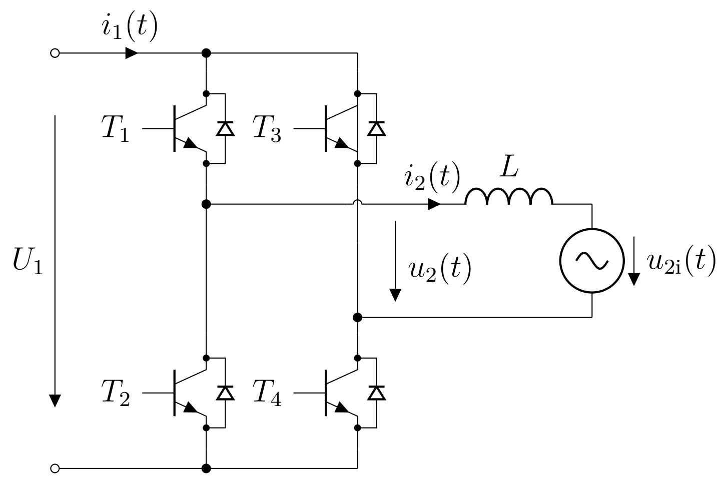


Figure 7.1.1: Single-phase AC-DC converter.

configured in a bridge topology and supplies a load consisting of an inductor and an internal load voltage. The converter consists of four transistors arranged in full bridge configuration.

Input DC voltage:	$U_1 = 200$ V
Inductance:	$L = 4.8$ mH
Internal load voltage:	$u_{2i}(t) = 150\sin(\omega t - \frac{\pi}{6})$
Reference angular frequency:	$\omega_2 = 2\pi \cdot 50$ Hz

Table 7.1.1: Parameters of the single-phase AC-DC converter.

The converter is modulated using PWM with a modulation index of $m = 0.75$. Assuming ideal operation of the switching components, perform the following tasks:

7.1.1 Draw the converter's output voltage $u_2(t)$ belonging to Fig. 7.1.2. and its fundamental component $u_2^{(1)}(t)$. How large is the phase difference φ_{2i} of the voltage fundamental component $u_2^{(1)}(t)$ compared to the internal load voltage $u_{2i}(t)$?

Answer:

Considering the modulation in Fig. 7.1.2, the output voltage $u_2(t)$ can either be $+U_1$ or $-U_1$ based on the maximum between the carrier signal $c(t)$ and reference $s^*(t)$, as shown in the figure. Moreover, the amplitude of the fundamental component $u_2^{(1)}(t)$ of the output voltage can be calculated using the modulation index m as:

$$m = \frac{\hat{u}_2^{(1)}}{U_1}, \hat{u}_2^{(1)} = 0.75U_1 = 150 \text{ V}, \tag{7.1.1}$$

$$\varphi_{2i} = \frac{\pi}{6} \text{ rad}. \tag{7.1.2}$$

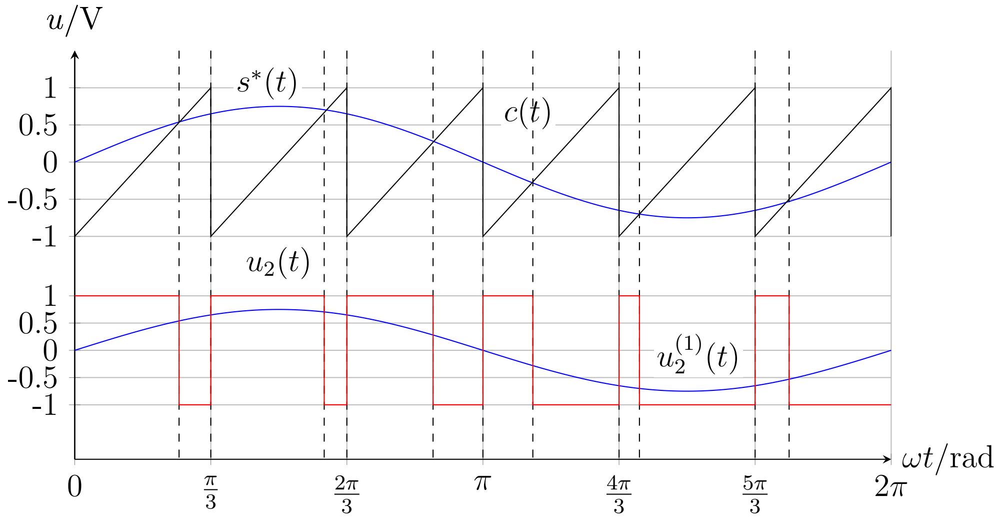


Figure 7.1.2: Output voltage $u_2(t)$ and fundamental voltage component $u_2^{(1)}(t)$.

7.1.2 Calculate the amplitude $\hat{i}_2^{(1)}$ and the phase angle $\varphi^{(1)}$ of the current fundamental component $i_2^{(1)}(t)$ compared to $u_{2\mathrm{i}}(t)$, and draw $u_2^{(1)}(t)$ as well as $i_2^{(1)}(t)$ in Fig. 7.1.3.

Answer:

Starting with the fundamental component of the output voltage $u_2^{(1)}(t)$, which can be expressed as:

$$u_2^{(1)}(t) = u_{2\mathrm{i}}(t) + u_\mathrm{L}^{(1)}(t) = u_{2\mathrm{i}}(t) + \mathrm{j}\omega_2 L i_2^{(1)}(t). \tag{7.1.3}$$

Solving for $i_2^{(1)}(t)$ leads to:

$$i_2^{(1)}(t) = \frac{u_2^{(1)}(t) - u_{2\mathrm{i}}(t)}{\mathrm{j}\omega_2 L}, \tag{7.1.4}$$

which can be represented in phasor as

$$\underline{i}_2^{(1)} = \frac{\hat{u}_2^{(1)} e^{\mathrm{j}\cdot 0} - \hat{u}_{2\mathrm{i}} e^{-\mathrm{j}\cdot\frac{\pi}{6}}}{\mathrm{j}\omega_2 L} = \frac{\hat{u}_2^{(1)} - \hat{u}_{2\mathrm{i}}\cos(\frac{\pi}{6}) + \mathrm{j}\hat{u}_{2\mathrm{i}}\sin(\frac{\pi}{6})}{\mathrm{j}\omega_2 L}. \tag{7.1.5}$$

After simplification, the final expression for the fundamental current $\underline{i}_2^{(1)}$ is

$$\underline{i}_2^{(1)} = \frac{\hat{u}_{2\mathrm{i}}\sin(\frac{\pi}{6})}{\omega_2 L} - \mathrm{j}\frac{\hat{u}_2^{(1)} - \hat{u}_{2\mathrm{i}}\cos(\frac{\pi}{6})}{\omega_2 L} = \hat{i}_2^{(1)} e^{\mathrm{j}\cdot\varphi_\mathrm{i}}. \tag{7.1.6}$$

The magnitude $\hat{i}_2^{(1)}$ can be calculated using

$$\hat{i}_2^{(1)} = \frac{1}{\omega_2 L}\sqrt{(\hat{u}_{2\mathrm{i}}\sin(\frac{\pi}{6}))^2 + (\hat{u}_2^{(1)} - \hat{u}_{2\mathrm{i}}\cos(\frac{\pi}{6}))^2} \approx 51.5\ \mathrm{A}, \tag{7.1.7}$$

while the phase $\varphi^{(1)}$ can be obtained from

$$\varphi^{(1)} = \arctan(\frac{-\hat{u}_2^{(1)} + \hat{u}_{2\mathrm{i}}\cos(\frac{\pi}{6})}{\hat{u}_{2\mathrm{i}}\sin(\frac{\pi}{6})}) \approx -0.26 \text{ rad}, \tag{7.1.8}$$

where

$$\hat{u}_2^{(1)} = \hat{u}_{2\mathrm{i}} = mU_1 = 150 \text{ V}. \tag{7.1.9}$$

Consequently, the expressions for $u_2^{(1)}(t)$ and $i_2^{(1)}(t)$ are

$$\begin{aligned} u_2^{(1)}(t) &= mU_1\sin(\omega_2 t), \\ i_2^{(1)}(t) &= \hat{i}_2^{(1)}\sin(\omega_2 t - \varphi^{(1)}). \end{aligned} \tag{7.1.10}$$

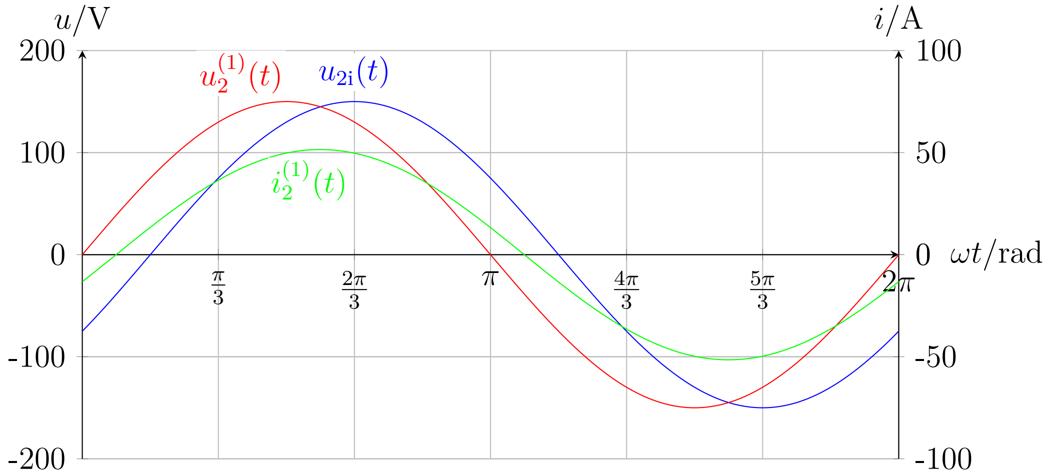


Figure 7.1.3: Internal load voltage $u_{2\mathrm{i}}(t)$, fundamental voltage and current components $u_2^{(1)}(t)$ and $i_2^{(1)}(t)$, respectively.

7.1.3 In Fig. 7.1.4, draw the voltage harmonics $u_2^{(\mathrm{h})}(t)$. Try to sketch, approximately, the current harmonics $i_2^{(\mathrm{h})}(t)$ by counting the voltage time area squares. One square corresponds to a voltage time area of 25.9 mVs. The starting point is marked with **x**.

Hint: The current harmonics $i_2^{(\mathrm{h})}(t)$ are free of any bias.

Answer:

The harmonics $u_2^{(\mathrm{h})}(t)$ can be calculated from the output voltage $u_2(t)$ and the fundamental component $u_2^{(1)}(t)$ with

$$u_2^{(\mathrm{h})}(t) = u_2(t) - u_2^{(1)}(t), \tag{7.1.11}$$

where $u_2^{(1)}(t)$ has already been calculated using (7.1.1) as

$$u_2^{(1)}(t) = mU_1\sin(\omega_2 t). \tag{7.1.12}$$

Hence,

$$u_2^{(\mathrm{h})}(t) = \begin{cases} -mU_1 \sin(\omega_2 t) + U_1 & \text{when } T_1 \text{ and } T_4 \text{ are conducting,} \\ -mU_1 \sin(\omega_2 t) - U_1 & \text{when } T_2 \text{ and } T_3 \text{ are conducting.} \end{cases} \tag{7.1.13}$$

Consequently, the change in the harmonics of the output current $\Delta i_2^{(\mathrm{h})}$ between t_0 and t_1 can be expressed as

$$\Delta i_2^{(\mathrm{h})} = \frac{1}{L}\int_{t_0}^{t_1} u_2^{(\mathrm{h})}(t)\mathrm{d}t = \frac{1}{L}\int_{t_0}^{t_1} u_2(t) - u_2^{(1)}(t)\mathrm{d}t. \tag{7.1.14}$$

For hand sketching, a simple graphical solution can be obtained by counting the number of squares below $u_2^{(\mathrm{h})}(t)$, hence, each step $\Delta i_2^{(\mathrm{h})}$ can be approximated with:

$$\Delta i_2^{(\mathrm{h})} = \frac{1}{L} \cdot (\text{number of squares}) \cdot 25.9 \text{ mVs}. \tag{7.1.15}$$

For example, the change in the current staring from **x** until the time of the first switching t_1 from U_1 to $-U_1$ can be calculated as:

$$\Delta i_2^{(\mathrm{h})} = \frac{1}{4.8 \text{ mH}} \cdot 14 \cdot 25.9 \text{ mVs} \approx 75.5 \text{ A}, \tag{7.1.16}$$

where 14 is, approximately, the number of squares below $u_2^{(\mathrm{h})}(t)$ in this time interval. So, the current at the first switching time t_1 is:

$$i_2^{(\mathrm{h})}(t_1) = i_2^{(\mathrm{h})}(t_0) + \Delta i_2^{(\mathrm{h})} = -44.4 + 75.5 \approx 31 \text{ A}. \tag{7.1.17}$$

The next steps $\Delta i_2^{(\mathrm{h})}$ in the trajectory of $i_2^{(\mathrm{h})}(t)$ can be calculated in the same way using the corresponding 'number of squares'.

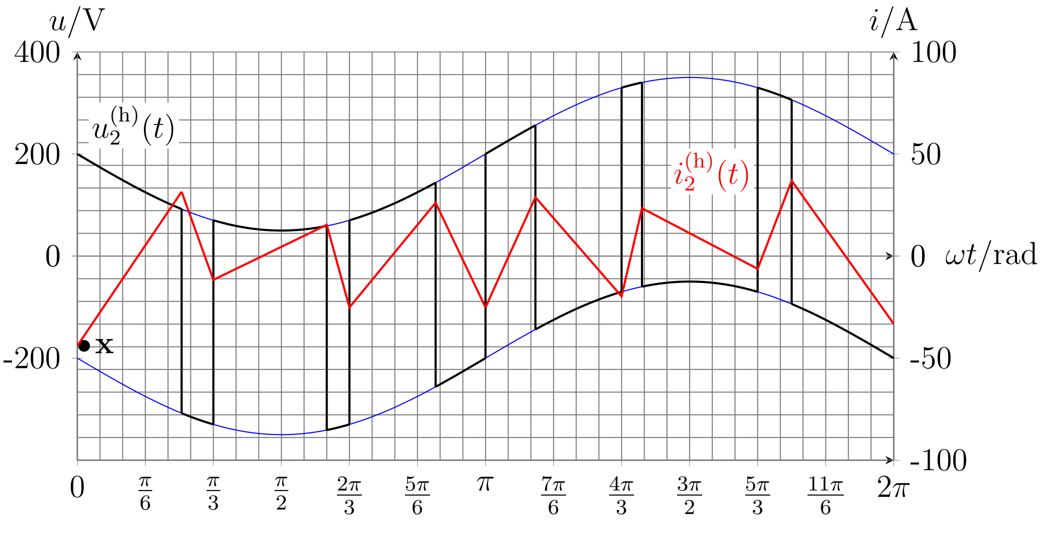


Figure 7.1.4: Harmonics of the output voltage and current.

7.1.4 Draw the converter's output current $i_2(t)$, its fundamental component $i_2^{(1)}(t)$ and its harmonics $i_2^{(h)}(t)$ in Fig. 7.1.5.

Answer:

The fundamental component $i_2^{(1)}(t)$ and the harmonics $i_2^{(h)}(t)$ have already been calculated in subtasks 7.1.2 and 7.1.3. Hence, the output current $i_2(t)$ of the converter can be calculated using

$$i_2(t) = i_2^{(1)}(t) + i_2^{(h)}(t). \tag{7.1.18}$$

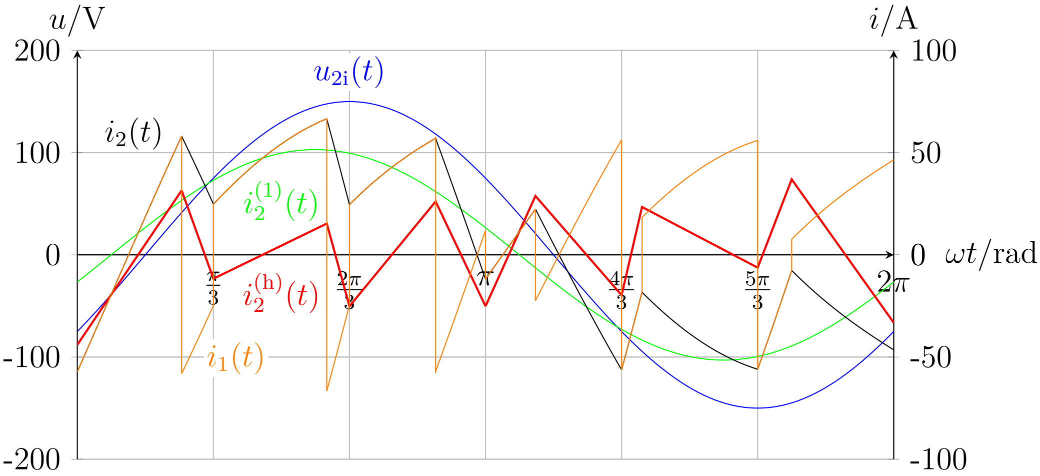


Figure 7.1.5: Output current $i_2(t)$, its fundamental component $i_2^{(1)}(t)$, its harmonics $i_2^{(h)}(t)$, and input current $i_1(t)$.

7.1.5 Mark the input current $i_1(t)$ of the AC-DC converter in Fig. 7.1.5.

Answer:

The input current $i_1(t)$ can be calculated from $i_2(t)$ using

$$i_1(t) = s(t)i_2(t), \tag{7.1.19}$$

where $s(t)$ is the switching function. Fig. 7.1.5 shows the input current's trajectory compared to the output current $i_2(t)$.

Task 7.2: Symmetrical 3-phase rectifier

A rectifier in three-phase bridge topology shall supply a symmetrical three-phase load in star connection. The load is represented by an inductance and a sinusoidal internal (or inner) voltage per phase. The inverter is operated with the fundamental frequency modulation (also known as six-step mode) and the switching elements are considered as ideal. The schematic is depicted in Fig. 7.2.1.

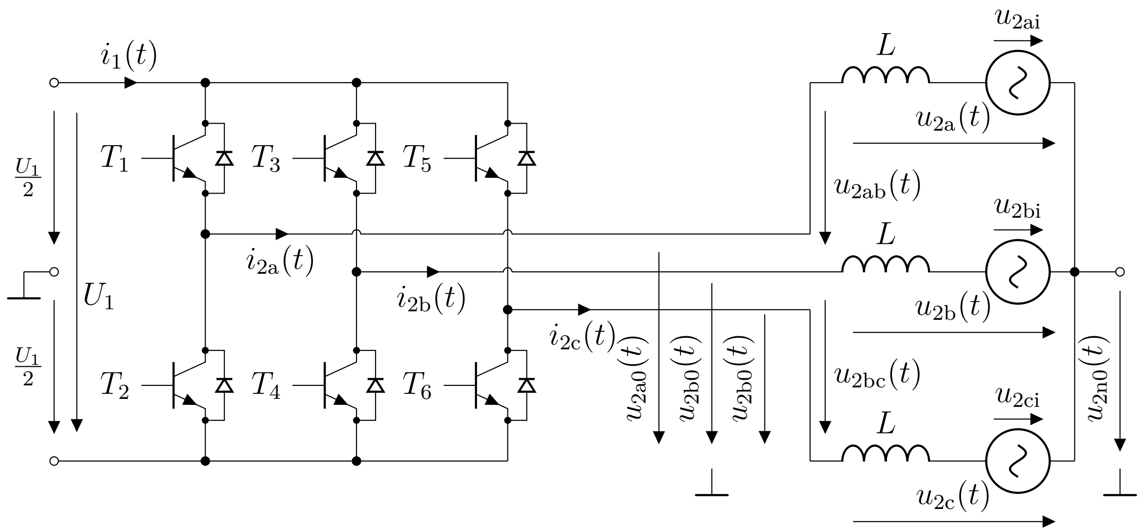


Figure 7.2.1: Three-phase inverter in six-step mode.

Input voltages:	$U_1 = 510$ V
Internal voltages:	$u_{2ai}(t) = \sqrt{2} \cdot 220\ \mathrm{V} \cdot \sin(\omega_1 t)$
Angular load frequency:	$\omega_1 = 2\pi \cdot 30\ \frac{1}{\mathrm{s}}$
11 Inductance per phase:	$L = 10$ mH
Phase angle between $u_{2ai}(t)$ and $i_{2ai}^{(1)}(t)$	$\varphi_{2a}^{(1)} = 30°$

Table 7.2.1: Parameters of three-phase inverter in six-step mode.

7.2.1 Create a table with all possible switching states for fundamental frequency modulation. Use the following notation:

$$\{s_\mathrm{a}(t), s_\mathrm{b}(t), s_\mathrm{c}(t)\} = \begin{cases} s_i(t) = +1 & \text{upper position,} \\ s_i(t) = -1 & \text{lower position.} \end{cases}$$

Sketch the switching states in the correct chronological order for one period. Calculate and sketch the voltages $u_{2a0}(t)$, $u_{2b0}(t)$ and $u_{2c0}(t)$ depending on these switching states.

Answer:

Each half bridge has got the 2 states '+1' and '-1', which results in $2^3 = 8$ combinations according table Tab. 7.2.2. To simplify the entries following is defined: $U_{1h} = \frac{U_1}{2}$. The correct chronological order is displayed in table Tab. 7.2.3.

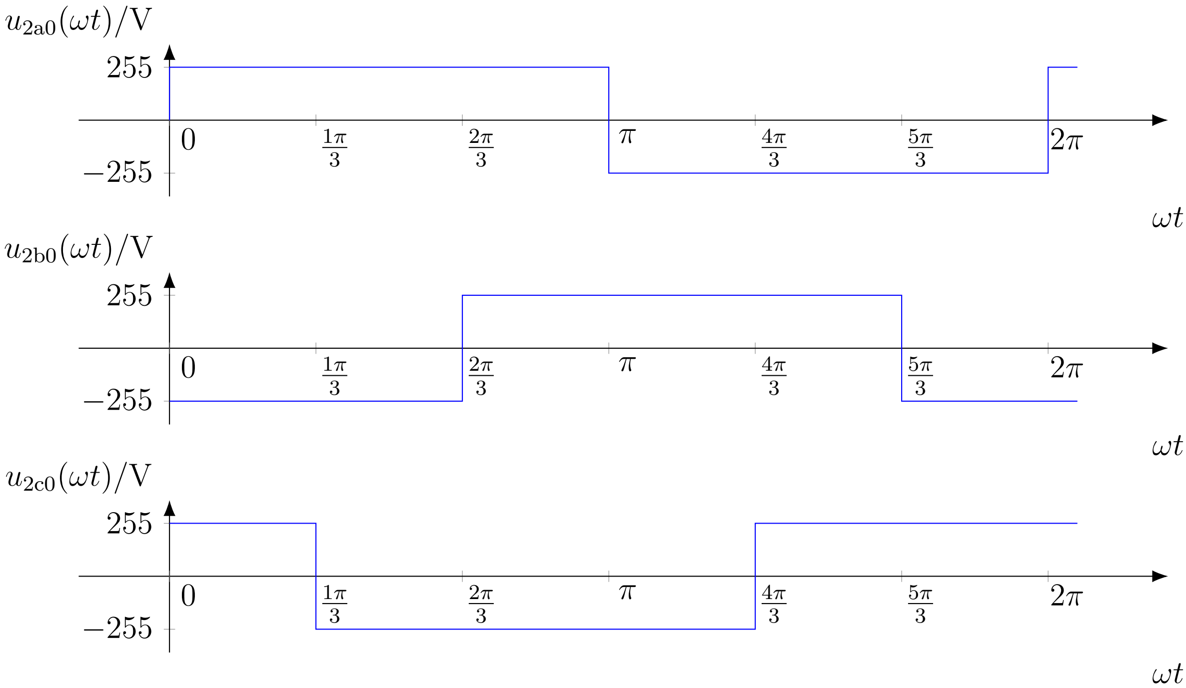
$s_a(t)$	$s_b(t)$	$s_c(t)$
-1	-1	-1
-1	-1	+1
-1	+1	-1
-1	+1	+1
+1	-1	-1
+1	-1	+1
+1	+1	-1
+1	+1	+1

Table 7.2.2: Possible switching states.

$s_a(t)$	$s_b(t)$	$s_c(t)$	u_{2a0}	u_{2b0}	u_{2c0}
+1	-1	+1	U_{1h}	$-U_{1h}$	U_{1h}
+1	-1	-1	U_{1h}	$-U_{1h}$	$-U_{1h}$
+1	+1	-1	U_{1h}	U_{1h}	$-U_{1h}$
-1	+1	-1	$-U_{1h}$	U_{1h}	$-U_{1h}$
-1	+1	+1	$-U_{1h}$	U_{1h}	U_{1h}
-1	-1	+1	$-U_{1h}$	$-U_{1h}$	U_{1h}

Table 7.2.3: Used switching states and voltages.

The switching state (-1,-1,-1) and (+1,+1,+1) are not used. In this case the chained voltages are zero. This additional degree of freedom is applied at a higher switching frequency in order to reduce the amplitude of the output voltage on average. In case of block switching, these switching states are not used, since the switching only occurs twice per period. This results in the maximum possible voltage (square wave) at the output. The voltages $u_{2a0}(\omega t)$, $u_{2b0}(\omega t)$, $u_{2c0}(\omega t)$,$u_{2ab}(\omega t)$ and $u_{2bc}(\omega t)$ are displayed in Sol.-Fig. 7.2.1.



Solution Figure 7.2.1: Voltage signals of $u_{2a0}(\omega t)$, $u_{2b0}(\omega t)$ and $u_{2c0}(\omega t)$.

7.2.2 The internal voltages $u_{2ai}(t)$, $u_{2bi}(t)$ and $u_{2ci}(t)$ are from a symmetrical voltage system, i.e., the following is always applicable: $u_{2ai}(t) + u_{2bi}(t) + u_{2ci}(t) = 0$ V. Show that this equation is also applicable for the voltages $u_{2a}(t)$, $u_{2b}(t)$ and $u_{2c}(t)$ under the same conditions.

Answer:

In the case of a symmetrical three-phase load where the current sum at the load star point is zero, the following results:

$$u_{2a}(t) + u_{2b}(t) + u_{2c}(t) = 0\ \mathrm{V} \quad i_{2a}(t) + i_{2b}(t) + i_{2c}(t) = 0\ \mathrm{A}. \tag{7.2.1}$$

This leads to

$$u_{2a}(t) = L\frac{\mathrm{d}i_{2b}(t)}{\mathrm{d}t} + u_{2ai}(t) \quad u_{2b}(t) = L\frac{\mathrm{d}i_{2b}(t)}{\mathrm{d}t} + u_{2bi}(t) \quad u_{2c}(t) = L\frac{\mathrm{d}i_{2c}(t)}{\mathrm{d}t} + u_{2ci}(t). \tag{7.2.2}$$

Using (7.2.1) leads to

$$u_{2a}(t) + u_{2b}(t) + u_{2c}(t) = L\frac{\mathrm{d}}{\mathrm{d}t}\left(i_{2a}(t) + i_{2b}(t) + i_{2c}(t)\right) + \left(u_{2ai}(t) + u_{2bi}(t) + u_{2ci}(t)\right) = 0\ \mathrm{V}. \tag{7.2.3}$$

This derivation is valid under following conditions:

- The induction L is constant.
- The internal voltages $u_{2ai}(t)$, $u_{2bi}(t)$ and $u_{2ci}(t)$ are purely sinusoidal (no harmonics), symmetrical (sum equals zero) and independent of the currents $i_{2a}(t)$, $i_{2b}(t)$ and $i_{2c}(t)$.

7.2.3 Calculate and sketch the voltages $u_{2ab}(t)$, $u_{2bc}(t)$, $u_{2a}(t)$ and the star-to-ground voltage $u_{2n0}(t)$ depending on the switching states.
Answer:
The voltage $u_{2ab}(t)$ is calculated by

$$u_{2ab}(t) = u_{2a0}(t) - u_{2b0}(t). \tag{7.2.4}$$

In similar way the voltage $u_{2bc}(t)$ is calculated by

$$u_{2bc}(t) = u_{2b0}(t) - u_{2c0}(t). \tag{7.2.5}$$

The voltage $u_{2a}(t)$ is obtained by

$$u_{2a}(t) = u_{2ab}(t) + u_{2b}(t). \tag{7.2.6}$$

Additional voltage $u_{2a}(t)$ is obtained by

$$u_{2a}(t) = u_{2ab}(t) + u_{2bc}(t) + u_{2c}(t). \tag{7.2.7}$$

The addition of (7.2.6) and (7.2.7) results in

$$2u_{2a}(t) = 2u_{2ab}(t) + u_{2bc}(t) + \left(u_{2a}(t) + u_{2b}(t) + u_{2c}(t)\right) - u_{2a}(t). \tag{7.2.8}$$

Solving (7.2.8) with respect to $u_{2a}(t)$ leads to

$$u_{2a}(t) = \frac{2}{3}u_{2ab}(t) + \frac{1}{3}u_{2bc}(t). \tag{7.2.9}$$

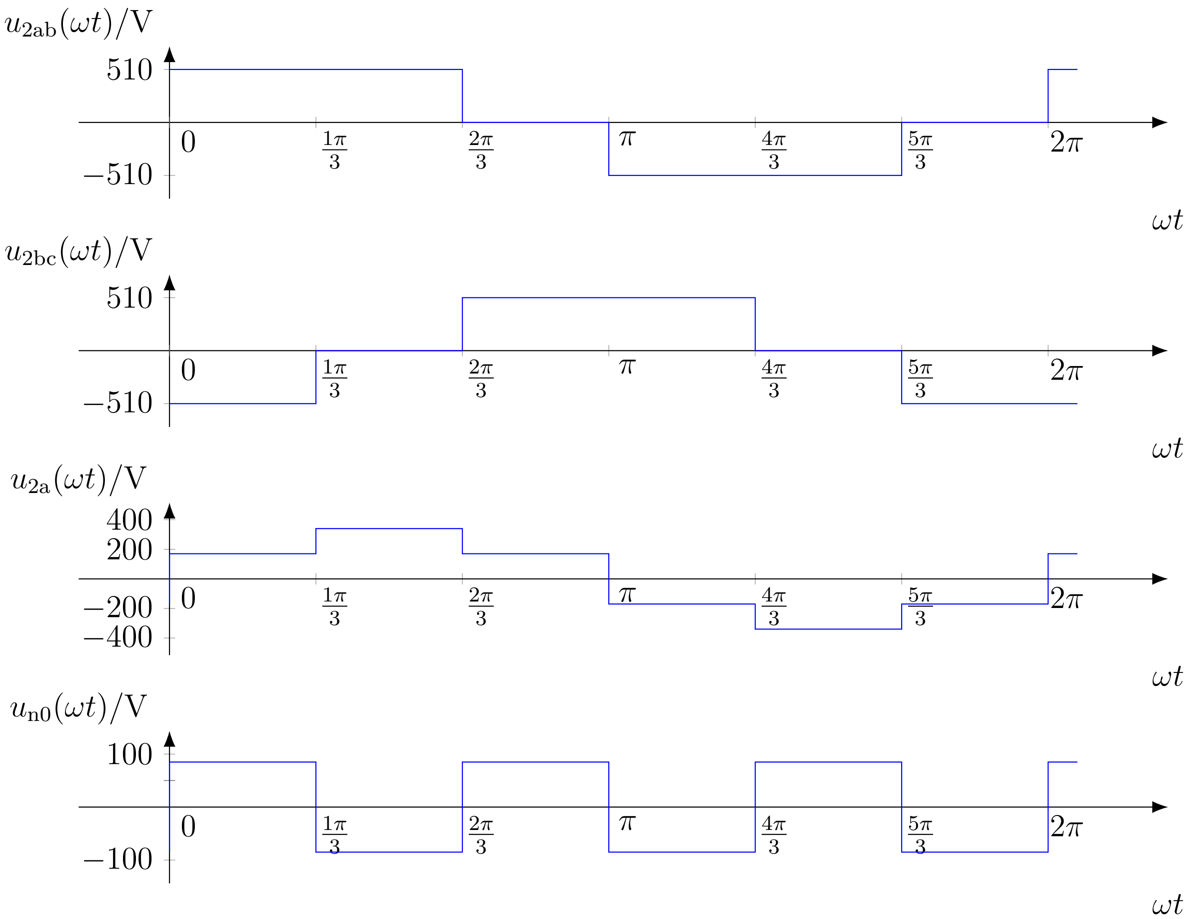
The voltage $u_{0,n}(t)$ is obtained by

$$u_{0,n}(t) = u_{2a0}(t) - u_{2a}(t) = u_{2a0}(t) - \frac{2}{3}u_{2ab}(t) - \frac{1}{3}u_{2bc}(t). \tag{7.2.10}$$

Using (7.2.4) and (7.2.5) leads to

$$\begin{aligned} u_{0,n}(t) &= u_{2a0}(t) - \frac{2}{3}\left(u_{2a0}(t) - u_{2b0}(t)\right) - \frac{1}{3}\left(u_{2b0}(t) - u_{2c0}(t)\right) \\ u_{0,n}(t) &= \frac{1}{3}\left(u_{2a0}(t) + u_{2b0}(t) + u_{2c0}(t)\right). \end{aligned} \tag{7.2.11}$$

The voltages $u_{2ab}(\omega t)$, $u_{2bc}(\omega t)$, $u_{2a}(\omega t)$ and $u_{2n0}(\omega t)$ are depicted in Sol.-Fig. 7.2.2.



Solution Figure 7.2.2: Voltage signals of $u_{2a}(\omega t)$, $u_{2n0}(\omega t)$, $u_{2ab}(\omega t)$ and $u_{2bc}(\omega t)$.

7.2.4 Decompose the voltage $u_{2a}(t)$ into a Fourier series and sketch the spectral lines related to the amplitude of the fundamental signal up to order $k = 13$.

Hint:

$$b_k = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} f(x)\sin(kx)\mathrm{d}x \quad k = \text{odd}.$$

The formula above applies to the Fourier coefficients of an odd and alternating function.

Answer:

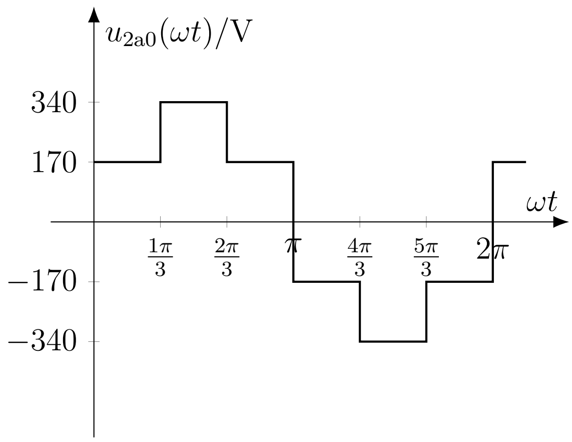
In the case of odd and alternating functions corresponding to $f(x) = -f(x+\pi)$ the Fourier coefficients are:

$$\begin{aligned} a_\mathrm{k} &= 0 \\ b_\mathrm{k} &= \frac{4}{\pi}\int_0^{\pi/2} f(x)\sin(x)\mathrm{d}x \quad k = \text{odd} \\ f(x) &= \sum_k \left(b_k \sin(kx)\right). \end{aligned} \tag{7.2.12}$$

The coefficients b_k are the amplitudes of the respective harmonic. The voltage $u_{2\mathrm{a}}(t)$ needs only to be integrated up to $\pi/2$. Only the terms with odd order numbers are taken into account. Apply this to the current signal is expressed by

$$b_\mathrm{k} = \frac{4}{\pi}\int_0^{\pi/3} \frac{U_1}{3}\sin(kt)\mathrm{d}t + \frac{4}{\pi}\int_{\pi/3}^{\pi/2} \frac{2U_1}{3}\sin(kt)\mathrm{d}t. \tag{7.2.13}$$

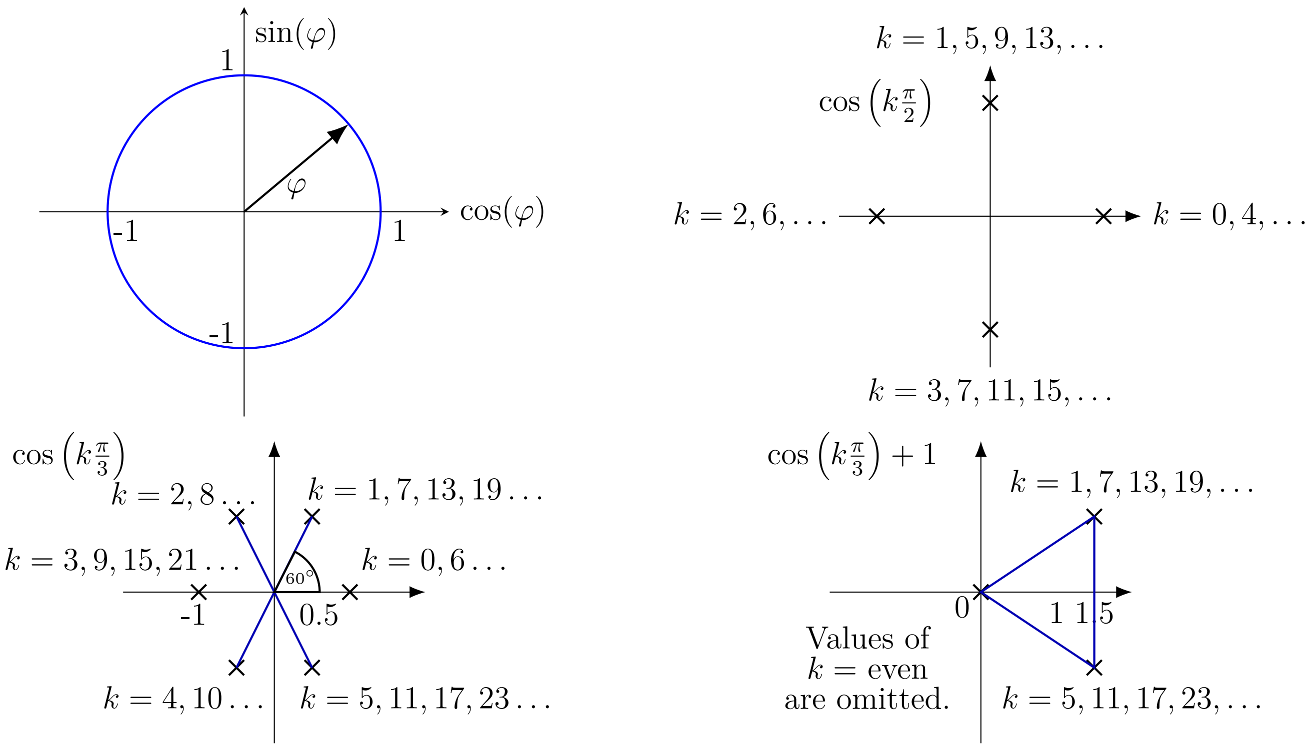
Signal ratio $u_{2\mathrm{a}0}(\omega t)/U_1$ is depicted in Sol.-Fig. 7.2.3 for one period.



Solution Figure 7.2.3: Section of the voltage curve $u_{2\mathrm{a}0}(\omega\mathrm{t})$.

$$\begin{aligned} b_\mathrm{k} &= \frac{4U_1}{3k\pi}\Big[-\cos(kt)\mathrm{d}t\Big]_0^{\pi/3} + \frac{4U_1}{3k\pi}\Big[-2\cos(kt)\mathrm{d}t\Big]_{\pi/3}^{\pi/2} \\ &= \frac{4U_1}{3k\pi}\left(-\cos(k\frac{\pi}{3}) + \cos(0) - 2\cos(k\frac{\pi}{2}) + 2\cos(0)\right) \\ &= \frac{4U_1}{3k\pi}\left(\cos(k\frac{\pi}{3}) + 1 - 2\cos(k\frac{\pi}{2})\right) \\ \text{with } &\cos(k\frac{\pi}{3}) + 1 = 1.5 \quad \text{for } k = n\cdot 6 \pm 1 \text{ is odd} \\ \text{and } &\cos(k\frac{\pi}{2}) = 0 \quad \text{for } k = \text{ odd.} \end{aligned} \tag{7.2.14}$$

The result is displayed in the complex plane in Sol.-Fig. 7.2.4.

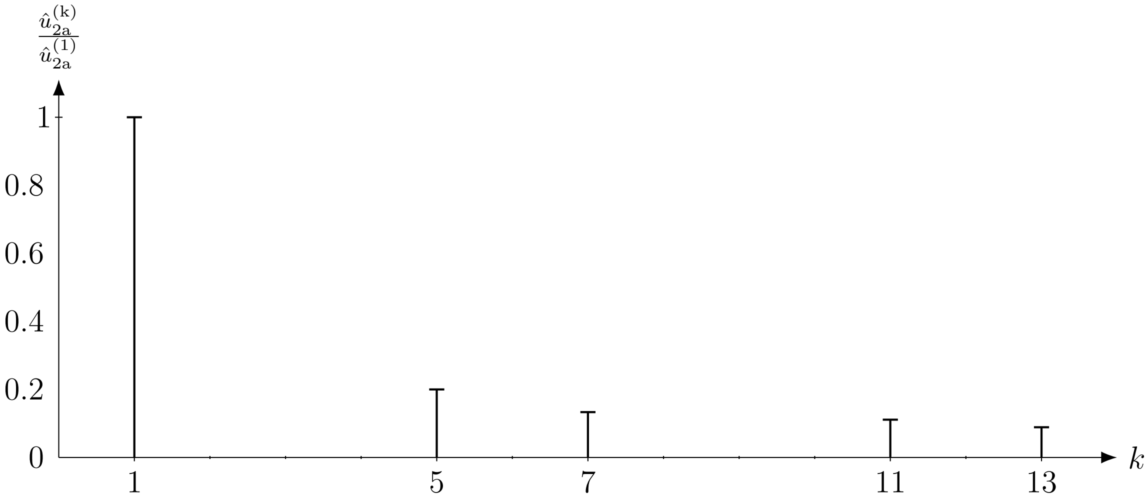


Solution Figure 7.2.4: Graphical solution of the cos terms within complex plane.

Sol.-Fig. 7.2.3 leads to

$$\hat{u}_{2a0,\mathrm{k}} = b_\mathrm{k} = \frac{4U_1}{3k\pi}\cdot\frac{3}{2} = \frac{2U_1}{k\pi}. \tag{7.2.15}$$

The amplitudes are depicted in Sol.-Fig. 7.2.5.



Solution Figure 7.2.5: Normalization to the amplitude of the fundamental oscillation.

The relation between fundamental and harmonic amplitude is calculated by

$$\frac{\hat{u}_{2a0,1}}{\hat{u}_{2a0,k}} = \frac{1}{k}$$
$$\text{with } k = n \cdot 6 \pm 1 \text{ and } n = 1, 2, 3.... \tag{7.2.16}$$

7.2.5 Based on subtask 7.2.4, calculate the fundamental amplitude $\hat{i}_{\mathrm{a}}^{(1)}$ using a vector diagram and complex alternating current calculations. From this, determine the total active power fed to the load.

Answer:

According (7.2.15) the fundamental voltage is calculated as:

$$\hat{u}_{2a0}^{(1)}(t) = \frac{2U_1}{\pi} = \frac{2 \cdot 510 \text{ V}}{\pi} = 324.68 \text{ V}. \tag{7.2.17}$$

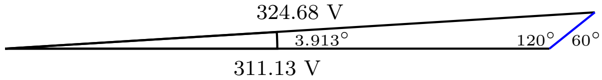
The amplitude of $u_{2ai}(t)$ is given with

$$\hat{u}_{2ai} = \sqrt{2} \cdot 220 \text{ V} = 311.13 \text{ V}. \tag{7.2.18}$$

The angle between the fundamental voltage drop across the inductance L and $i_{2a}^{(1)}(t)$ is 90°. If this is taken in account the angle between $u_{2ai}(t)$ and the fundamental voltage drop across the inductance L is calculated by

$$\alpha = 90^\circ + \varphi_{2ai}^{(1)} = 90^\circ + 30^\circ = 120^\circ. \tag{7.2.19}$$

A triangle is formed by the inverter voltage $u_{2a}^{(1)}(t)$, the voltage $u_{2ai}(t)$ and the voltage drop across



Solution Figure 7.2.6: Illustration for determining the angle using the sine theorem.

the inductance L. Two sides and one angle are known. By applying the sine theorem results in

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} \quad \text{with } a = 324.68\text{ V} \quad \alpha = 120° \quad b = 311.13\text{ V}. \tag{7.2.20}$$

Solving (7.2.20) with respect to β leads to

$$\beta = \arcsin\left(\frac{b}{a}\sin(\alpha)\right) = \arcsin\left(\frac{311.13\text{ V}}{324.68\text{ V}}\sin(120°)\right) = \arcsin(0.8298) = 56.1°. \tag{7.2.21}$$

Using the result for β leads to

$$\gamma = 180° - \alpha - \beta = 180° - 120° - 56.1° = 3.9°. \tag{7.2.22}$$

In Sol.-Fig. 7.2.6 the triangle is depicted. In a symmetrical three-phase system, the active power is:

$$P = \sqrt{3}U_{\mathrm{L-L}}I_{\mathrm{L}}\cos(\varphi). \tag{7.2.23}$$

$U_{\mathrm{L-L}}$ corresponds to the effective value of the line-to-line voltage and I_{L} is the effective value of the line current and φ is the phase angle between voltage and current. For this case $U_{\mathrm{L-L}}$ is calculated by

$$U_{\mathrm{L-L}} = \sqrt{3}\frac{\hat{u}_{2\mathrm{a}}^{(1)}}{\sqrt{2}} = \sqrt{\frac{3}{2}}\frac{2U_1}{\pi} = \sqrt{3}\sqrt{2}\frac{U_1}{\pi} = \sqrt{3}\sqrt{2}\cdot\frac{510\text{ V}}{\pi} = 397.6\text{ V}. \tag{7.2.24}$$

The line current I_{L} is obtained by

$$I_{\mathrm{L}} = \frac{\hat{i}_{2\mathrm{a}}^{(1)}}{\sqrt{2}} = \frac{12.37\text{ A}}{\sqrt{2}} = 8.75\text{ A}. \tag{7.2.25}$$

The angle φ results in

$$\begin{aligned}\varphi &= 30° + \gamma = 30° + 3.9° = 33.9° \\ \cos(\varphi) &= \cos(33.9°) = 0.83.\end{aligned} \tag{7.2.26}$$

Using (7.2.24), (7.2.25) and (7.2.26) in (7.2.23) leads to

$$P = \sqrt{3}U_{\mathrm{L-L}}I_{\mathrm{L}}\cos(\varphi) = \sqrt{3}397.6\text{ V}\cdot 8.75\text{ A}\cdot 0.83 = 5\text{ kW}. \tag{7.2.27}$$