Exercise 01: Step-down converter and power loss calculation

Task 1.1: Step-down converter without output filter

An ideal switching transistor is used for loss-free and stepless control of a car's rear window heating. By varying the duty cycle of the transistor the average value of the heating power can be adjusted. The voltage in the car's electrical system is assumed to be constant with $U_1 = 14$ V. The heater is dimensioned in such a way that at its nominal voltage $U_{2N} = 14$ V it consumes a power of $P_{LN} = 500$ W and can be modeled with an ohmic resistor. This circuit is shown in Fig. 1.1.1.

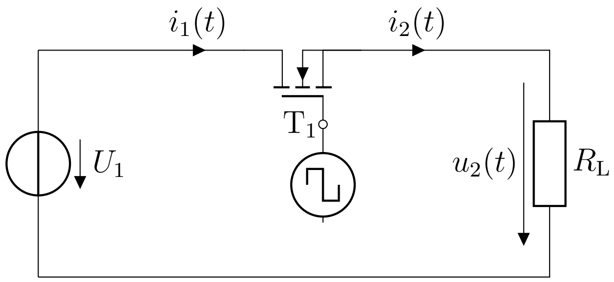


Figure 1.1.1: Circuit with one transistor and one load resistor.

1.1.1 Draw the qualitative current $i_2(t)$ and voltage $u_2(t)$ curves at the load resistor for some switching periods T_s.

1.1.2 Draw the instantaneous power at the load resistor.

1.1.3 Derive the relationships for the mean voltage $\overline{u}_2$, the mean current $\overline{i}_2$ and the mean power $\overline{p}_2$.

1.1.4 How large should the duty cycle D be selected so that an average voltage of $\overline{u}_2 = 8$ V is applied to the heater? What is the mean value of the current $\overline{i}_2$? What power $\overline{p}_2$ is converted into heat?

1.1.5 When starting the engine, the heater may draw a maximum average current $\overline{i}_{2,s} = 10$ A from the vehicle electrical system. With which duty cycle D should the transistor be switched in this case? What is the average voltage $\overline{u}_2$ at the heater? What power $\overline{p}_2$ is converted into heat?

1.1.6 During the journey, the heat output should be $\overline{p}_{2,f} = 200$ W. How is the duty cycle D set? What are the mean values of the current $\overline{i}_2$ and the voltage $\overline{u}_2$?

Task 1.2: Step-down converter with output filter

A step-down converter is used to charge a mobile phone from the vehicle electrical system with the vehicle electrical system voltage $U_1 = 13.5$ V. The input voltage of the mobile phone is $U_2 = 4.5$ V. Consider both voltages as constant, the inductance of the coil is $L = 10$ µH and the switching frequency is $f_s = 100$ kHz. All components are considered ideal.

1.2.1 Draw the equivalent circuits for the two switching states.

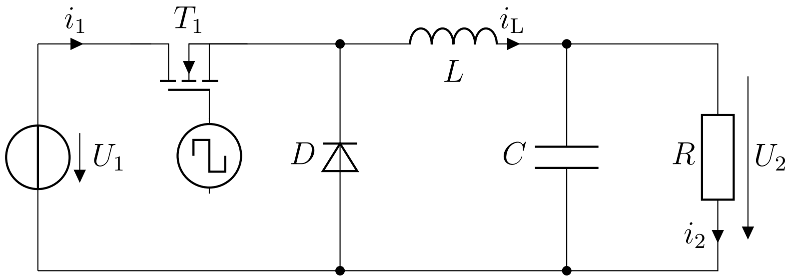


Figure 1.2.1: Circuit with one transistor, filter and one load resistor.

1.2.2 At what duty cycle D should the buck converter be operated?

1.2.3 Sketch the voltage and current signals in the components.

1.2.4 How large is the current ripple Δi_L of the coil current in boundary conduction mode (BCM) operation?

1.2.5 How large is the worst-case current ripple?

1.2.6 When starting the engine, the input voltage drops to $U_{1,\text{min}} = 10$ V. The voltage regulator of the buck converter changes the duty cycle so that the output voltage $U_2 = 4.5$ V is kept stable. What duty cycle D is set?

1.2.7 Calculate the average and ripple current in BCM mode considering $U_{1,\text{min}}$ at the input.

Task 1.3: Power losses within the step-down converter

The power loss of a buck converter is to be analyzed. The inductance is so large that the current ripple in the output current can be neglected, i.e., $i_2(t) = I_2 = \text{const}$. Furthermore, the component values are given in Tab. 1.3.1 and the currents and voltages of the switch-on and switch-off processes in Fig. 1.3.2 and Fig. 1.3.3.

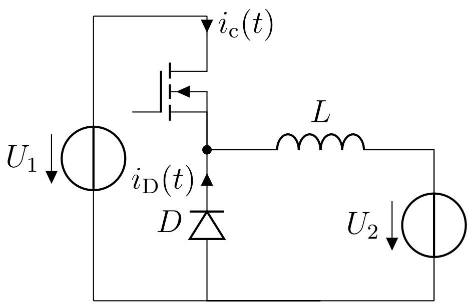


Figure 1.3.1: Buck converter with one transistor and one diode.

1.3.1 Calculate the switch-on and switch-off loss work $E_{\text{on,T}}$ and $E_{\text{off,T}}$ of the IGBT.

General parameters:		**Diode:**	
Input voltage:	$U_1 = 600$ V	Forward voltage:	$u_\mathrm{F} = 2.7$ V
Output current:	$I_2 = 30$ A	Switch-on losses:	$E_\mathrm{on,D} = 52$ µJ
Switching frequency:	$f_\mathrm{s} = 25$ kHz	Switch-off losses:	$E_\mathrm{off,D} = 240$ µJ
IGBT:		**Inductance:**	
Collector-emitter voltage:	$u_\mathrm{on,CE} = 2.5$ V	Copper resistance:	$R_\mathrm{Cu} = 45$ mΩ
		Iron losses:	$P_\mathrm{l,Fe} = 13$ W

Table 1.3.1: Parameters of the circuit.

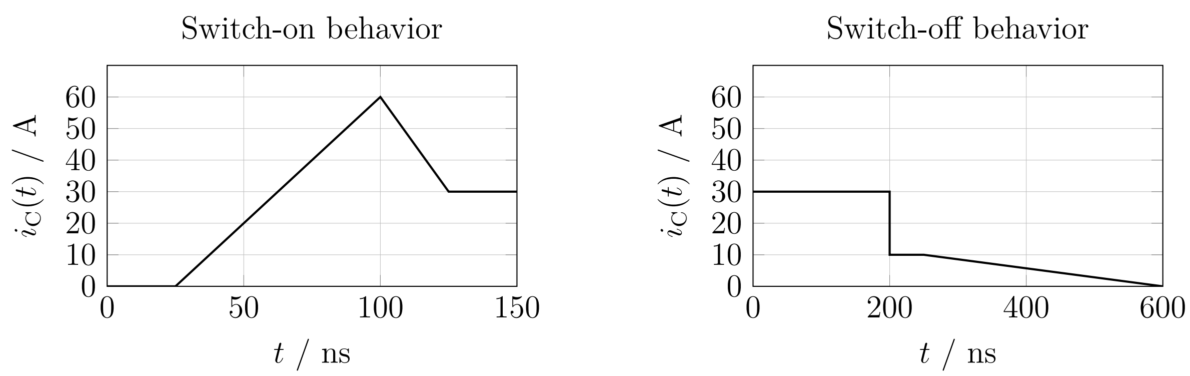


Figure 1.3.2: Switch-on behavior and switch-off behavior of $i_\mathrm{C}(t)$.

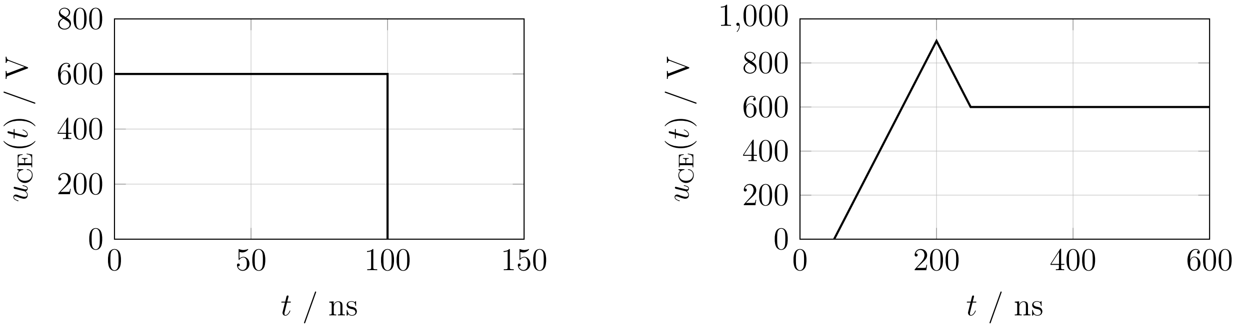


Figure 1.3.3: Switch-on behavior and switch-off behavior of $u_\mathrm{CE}(t)$.

1.3.2 Calculate the (average) switching power loss in the IGBT $P_\mathrm{l,sw,T}$ and in the diode $P_\mathrm{l,sw,D}$.

1.3.3 Calculate the (average) conduction losses in the IGBT $P_\mathrm{l,cond,T}(D)$ and in the diode $P_\mathrm{l,cond,D}(D)$ as a function of the duty cycle D.

1.3.4 Calculate the efficiency $\eta(U_2)$ as a function of the output voltage.

1.3.5 The circuit is to be evaluated at $U_2 = 300$ V. Calculate the total power loss in the IGBT $P_\mathrm{l,T}$ and in the diode $P_\mathrm{l,D}$.

Exercise 02: Step-up and (synchronous) buck-boost converters

Task 2.1: Boost converter with no losses

The boost converter which is shown in Fig. 2.1.1 supplies a resistive load with a rated voltage of $U_2 = 60$ V. The converter operates in steady state and all losses of the boost converter are neglected.

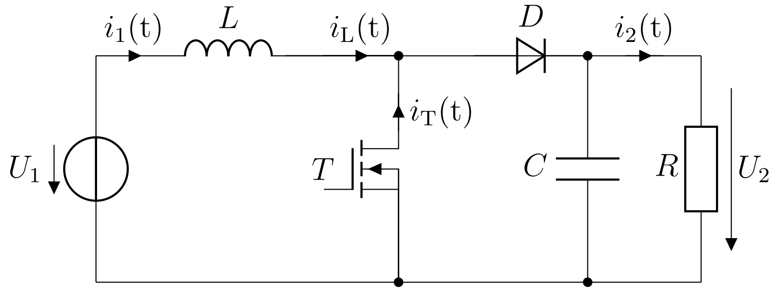


Figure 2.1.1: Boost converter with no losses.

The specification of the boost converter is given in Tab. 2.1.1.

Input voltage:	$U_1 = 12$ V	Output voltage:	$U_2 = 60$ V
Output current:	$I_2 = 2$ A	Minimal output power:	$P_{2,\mathrm{min}} = 10$ W
Output voltage ripple:	$\triangle u_2 = 120$ mV	Switching frequency:	$f_\mathrm{s} = 100$ kHz

Table 2.1.1: Parameters of the boost converter.

2.1.1 Derive the duty cycle D which leads to the specific output voltage of $U_2 = 60$ V.

2.1.2 Determine the average input current $\bar{i}_\mathrm{L}$.

2.1.3 Define a suitable inductance for the coil L, so that the boost converter is operating in boundary conduction mode (BCM) when supplying the minimum output power $P_{2,\mathrm{min}}$. Determine the maximal switch-off current i_T of the transistor T for the rated output current $I_2 = 2$ A.

2.1.4 Calculate a suitable capacitance to meet the output voltage ripple specification. Determine the current stress of the capacitor $I_{\mathrm{C,RMS}}$.

Task 2.2: Boost converter with losses

Next, the impact of power losses on the above's converter behavior is investigated. For the following points an ideally smoothed input current and a ripple-free output voltage are assumed. The boost converter with losses is shown in Fig. 2.2.1.

2.2.1 From now on, the influence of the resistor R_L is considered. Derive the efficiency η and voltage ratio U_1/U_2 of the boost converter in dependence on the duty cycle D and the resistance ratio $\alpha = R_\mathrm{L}/R$. Sketch both functions, η and U_2/U_1 over the duty cycle D and analyze the findings.

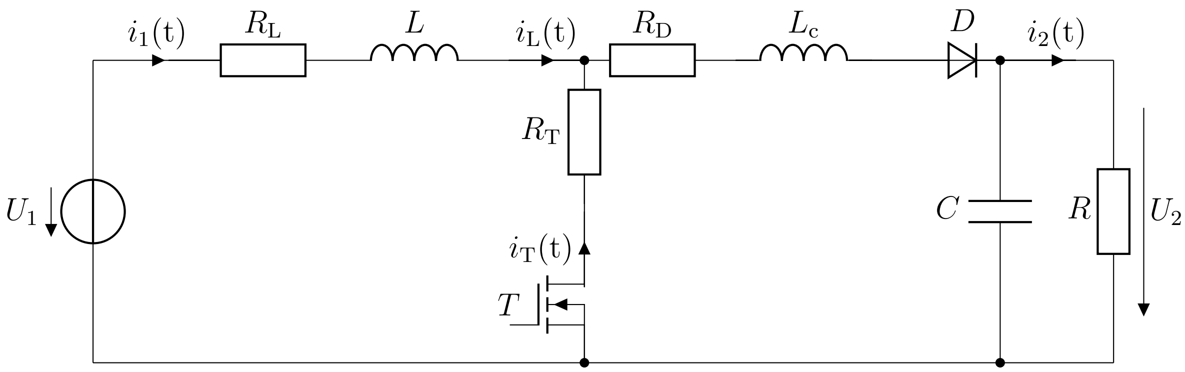


Figure 2.2.1: Boost converter with losses.

2.2.2 For $R_\mathrm{L} = 0.2\ \Omega$, determine the required duty cycle value for the given input and output voltages.

2.2.3 Calculate the efficiencies $\eta_1(D_1)$ and $\eta_2(D_2)$ of the boost converter for an output current of 2 A and a coil resistance of $R_\mathrm{L} = 0.2\ \Omega$.

2.2.4 In addition, consider the conduction losses of the diode D and the transistor T. Assume an equivalent resistance of $R_\mathrm{D} = 0.5\ \mathrm{m}\Omega$ and forward voltage of $U_\mathrm{D} = 1$ V for the diode and an equivalent resistance of $R_\mathrm{DS,on} = 30\ \mathrm{m}\Omega$ for the transistor. Determine the required duty cycle value when the conduction losses are considered.

2.2.5 Beside the conduction losses, also switching losses need to be considered in practice. In Fig. 2.2.2 the voltage and current waveforms are visualized for the turn-off event of a diode (reverse recovery effect). Therefore, calculate the turn-off losses for a fast diode with a commutation inductivity loop of $L_\mathrm{c} = 500$ nH. The peak reverse recovery current is $\hat{i}_\mathrm{rr} = 4$ A and the reverse recovery time is $t_\mathrm{rr} = t_2 - t_0 = 46.6$ ns.

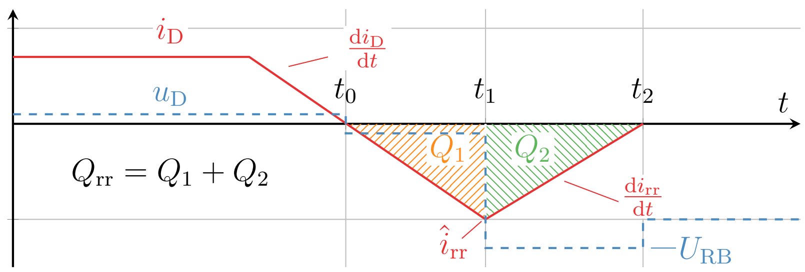


Figure 2.2.2: Turn-off behavior of a fast silicon diode.

2.2.6 Determine the turn-off switching losses of a normal silicon diode with a reverse recovery work $Q_\mathrm{rr} = 16\ \mu\mathrm{C}$ and a rate of current rise $\frac{\mathrm{d}i_\mathrm{rr}}{\mathrm{d}t} = 40\ \frac{\mathrm{A}}{\mu\mathrm{s}}$. Compare the result with the previous subtask.

Task 2.3: Buck-boost converter

A wide input-to-output voltage range can be realized by the cascade of buck and boost converters with a common inductance. With this topology the output voltage can be adjusted to a value which is higher or lower than the input voltage.

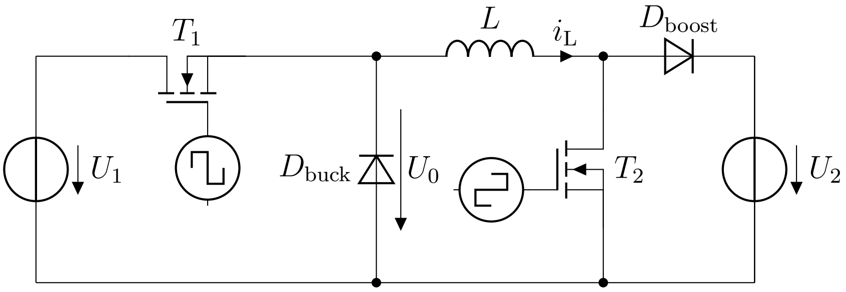


Figure 2.3.1: Buck-boost converter circuit.

Input voltage:	$U_1 = 320$ V to 720 V	Output voltage:	$U_2 = 400$ V
Output power:	$P_{out,min} = 5000$ W	Switching frequency:	$f_s = 25$ kHz

Table 2.3.1: Parameters of the circuit.

The output voltage is kept at the specified constant value by adjusting the duty cycles D_1 (of transistor T_1) and D_2 (of transistor T_2) using a control system. Both transistors operate at the same switching frequency. The ripple of the output voltage and of the current in the inductor can be ignored. The current in L is continuous. Initially, both transistors operate with the same duty cycle $D_1 = D_2 = D$ and their switching patterns are synchronized.

2.3.1 Calculate the duty cycle of the transistors T_1 and T_2 depending on the voltage transformation ratio U_2/U_1.

2.3.2 Calculate I_L depending on D. Plot D and I_L against U_1 and enter the numerical values for $U_1 = 320$ V, $U_1 = 400$ V and $U_1 = 720$ V.

Both transistors should now be able to have different duty cycles. Assume that the transistors are switched on at the same time.

2.3.3 Graphically represent the time profiles of the voltage at L for $U_1 = 320$ V and $D_1 = 0.9$ and for $U_1 = 720$ V and $D_2 = 0.1$ for one pulse period each.

2.3.4 Calculate the voltage transformation ratio as a function of D_1 and D_2.

2.3.5 Express the current I_L as a function of the specified operating parameters (U_1, U_2, P_2) and as a function of D_1 and D_2.

2.3.6 Are the calculated relationships valid if T_1 and T_2 do not switch synchronously or operate with different clock frequencies?

If the transistors T_1 and T_2 are switched on, a constant voltage drop $U_\mathrm{F} = 2.5$ V occurs at the transistors regardless of the current. All other components are considered ideal and loss-free.

2.3.7 How should D_1 and D_2 be selected so that the losses of the overall system are minimal? The relationships calculated under subtask 3.4 and 3.5 can be used for the voltage transformation ratio and the value of I_L.

2.3.8 Plot D_1 and D_2 and the efficiency over U_1 and give numerical values for $U_1 = 320$ V, $U_1 = 400$ V and $U_1 = 720$ V.

2.3.9 Calculate the efficiency for the three operating points in subtask 3.2.

2.3.10 How high is the maximum efficiency gain and at which operating point does it occur? Give an explanation for the observed finding.

Exercise 03: Combined step-up / step-down converters

Task 3.1: Inverting buck-boost converter

An inverting buck-boost converter (see Fig. 3.1.1) is used to generate the negative supply voltage of a control electronic unit. The input voltage is specified as $U_1 = 18$ V, the output voltage is regulated to $U_2 = 12$ V. The output power can vary in the range $P_2 = 2$ W … 15 W.

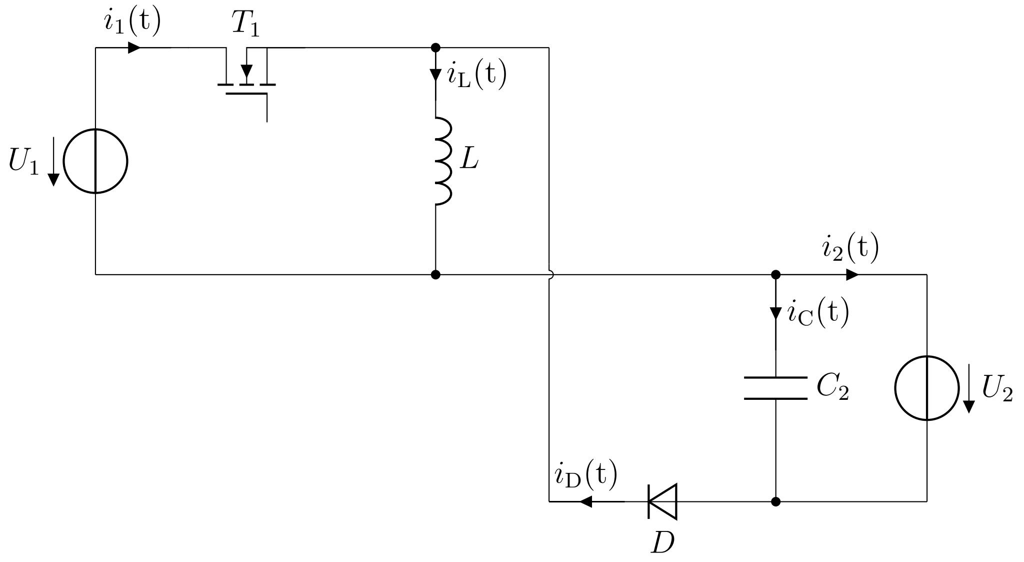


Figure 3.1.1: Inverting buck-boost converter.

3.1.1 The system should operate at boundary conduction mode (BCM) throughout the entire output power range. How should the inductance be selected so that the switching frequency is always above the hearing threshold $f_s = 20$ kHz?

3.1.2 In what value range does the switching frequency f_s vary considering the inductance choice from the previous subtask and the given output power range?

3.1.3 What is the peak value $\hat{i}_1 = \max\{i_1(t)\}$ of the transistor current?

3.1.4 How does the duty cycle D change with the output power? Calculate the duty cycle values and the transistor switch-on times $T_{on} = DT_s$ for minimum and maximum output power.

3.1.5 Sketch the course of the inductor current $i_L(t)$ for minimum and maximum output power.

3.1.6 At which operating point does the maximum output voltage ripple Δu_2 occur (assumption: the load draws a constant current)?

3.1.7 How high should the output capacitance be selected to ensure $\Delta u_2 < 0.02 \cdot U_2$?

3.1.8 What is the maximum effective value $i_{C,RMS}$ of the output capacitor current?

3.1.9 Sketch the curves of the voltage $u_{\mathrm{T}}(t)$ at the power transistor and the current $i_{\mathrm{D}}(t)$ in the output diode for $P_2 = 2$ W. What is the maximum blocking voltage of the transistor?

Task 3.2: Boost-buck converter and SEPIC topology

The supply of a plasma coating system is realized by a boost converter followed by a buck converter according Fig. 3.2.1 (with common capacitance). The converter is connected to a voltage U_1 and provides a variable output voltage U_2. The parameters are displayed in Tab. 3.2.1.

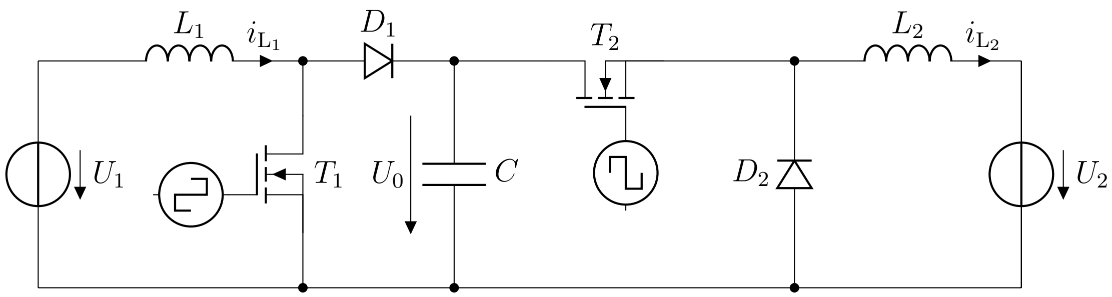


Figure 3.2.1: Boost-buck converter circuit.

Input voltage:	$U_1 = 380$ V	Output voltage:	$U_2 = 285$ V to 450 V
Output power:	$P_2 = 3000$ W	Switching frequency:	$f_\mathrm{s} = 50$ kHz
P_2 is constant (unless otherwise stated)			

Table 3.2.1: Parameter of the boost-buck converter circuit.

The output voltage is set to the specified value by adjusting the duty cycles D_1 (of transistor T_1) and D_2 (of transistor T_2) using a control system. Both transistors operate at the same switching frequency. The switching frequency fluctuation of the intermediate circuit voltage and the currents in the inductors can be neglected unless otherwise stated. The currents in L_1 and L_2 show a continuous course. Both transistors are operated with the same duty cycle $D_1 = D_2 = D$.

3.2.1 Calculate the duty cycle D range to achieve the stated output voltage U_2 range.

3.2.2 Which intermediate circuit voltage U_0 results depending on U_2?

3.2.3 Plot D and U_0 against U_2 and calculate the numerical values for $U_2 = 285$ V, $U_2 = 380$ V and $U_2 = 450$ V.

3.2.4 What blocking voltage ratings must the transistors T_1 and T_2 and the diodes D_1 and D_2 have?

The converter operation continues with $D_1 = D_2 = D$. The input and output inductances have the same value $L_1 = L_2 = L = 0.5$ mH.

3.2.5 Derive the input and output current ripple Δi_{L_1} and Δi_{L_2} depending on the duty cycle.

3.2.6 To what minimum value can the output power be reduced while still ensuring continuous operation across the entire output voltage range (i.e., continuous current flow in L_1 and L_2)?

3.2.7 At which value of the output voltage range is this limit reached first on the input side and at which value is it reached first on the output side?

For comparison reasons, the single ended primary inductance converter (SEPIC) in Fig. 3.2.2 shall be considered under similar conditions as an alternative circuit.

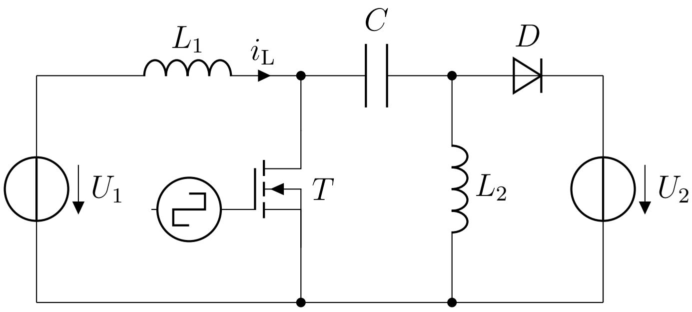


Figure 3.2.2: Single ended primary inductance converter circuit.

3.2.8 What blocking voltage ratings must the transistors T_1 and the diode D have?

3.2.9 Derive the input and output current ripple Δi_{L_1} and Δi_{L_2} depending on the duty cycle.

3.2.10 To what minimum value can the output power be reduced while still ensuring continuous operation across the entire output voltage range? (i.e., continuous current flow in L_1 and L_2)?

3.2.11 Describe the advantages and disadvantages of the SEPIC topology compared to the boost-buck converter. Consider the necessary components and the quality of the output voltage.

Exercise 04: Isolated DC-DC converters

Task 4.1: Flyback converter

A flyback converter with an input voltage range $U_1 = 300\ \text{V} \ldots 900\ \text{V}$ is used to supply a control electronics unit. The converter delivers a rated output power of $P_2 = 30$ W at a regulated (constant) output voltage of $U_2 = 15$ V. The flyback converter is operated in discontinuous conduction mode with a constant switching frequency of $f_\text{s} = 50$ kHz. The turns ratio of the transformer is $N_1/N_2 = 60/12$, the magnetizing inductance on the primary side is $L_\text{m} = 760$ µH. The coupling between the primary and secondary windings is ideal and the converter operates in steady state.

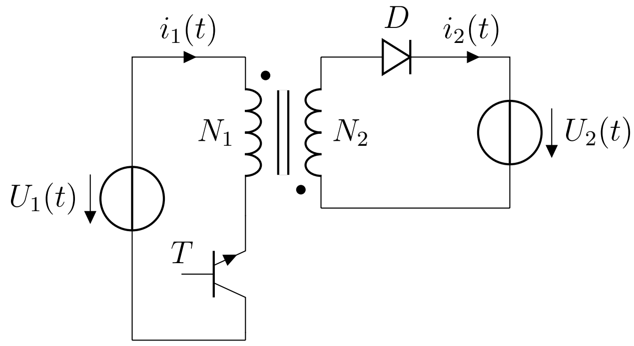


Figure 4.1.1: Flyback converter topology.

Input voltage:	$U_1 = 300\ \text{V} \ldots 900\ \text{V}$	Output voltage:	$U_2 = 15$ V
Output power:	$P_2 = 30$ W	Transformation ratio:	$N_1/N_2 = 60/12$
Magn. inductance:	$L_\text{m} = 760$ µH	Switching frequency:	$f_\text{s} = 50$ kHz

Table 4.1.1: Parameters of the flyback converter.

4.1.1 The input voltage is $U_1 = 760$ V at rated output power. What is the peak value $\hat{i}_1$ of the primary current i_1? What is the peak value $\hat{i}_2$ of the secondary current i_2? Calculate the duty cycle of the transistor for this operating point.

4.1.2 The input voltage is $U_1 = 382$ V at nominal load. Calculate and sketch the following voltage and current curves for this operating point over two cycle periods: $u_\text{T}(t), u_\text{s}(t), i_2(t), i_1(t)$. Here, $u_\text{T}(t)$ is the transistor voltage and $u_\text{s}(t)$ is the voltage on the secondary side of the transformer.

4.1.3 Determine the mean value $\bar{i}_\text{T}$ and the RMS current I_T through the transistor. Also, determine the mean value $\bar{i}_\text{D}$ and the RMS current I_D through the diode. What is the maximum reverse voltage $u_\text{T,max}$ of the transistor and $u_\text{D,max}$ of the diode? Consider the same operation conditions as in the previous subtask.

4.1.4 How much energy is transferred from the input to the output per switching period ΔE and what is the resulting average power P (consider the same operation conditions as in the previous subtask)? What happens if there is no ideal voltage source on the output side but an unloaded capacitor and the circuit is operated with $D > 0$?

Task 4.2: Forward converter with asymmetric half-bridge

The schematic of a forward converter with an asymmetric half-bridge is shown in Fig. 4.2.1. For the calculations the diodes and transistors are considered as ideal components.

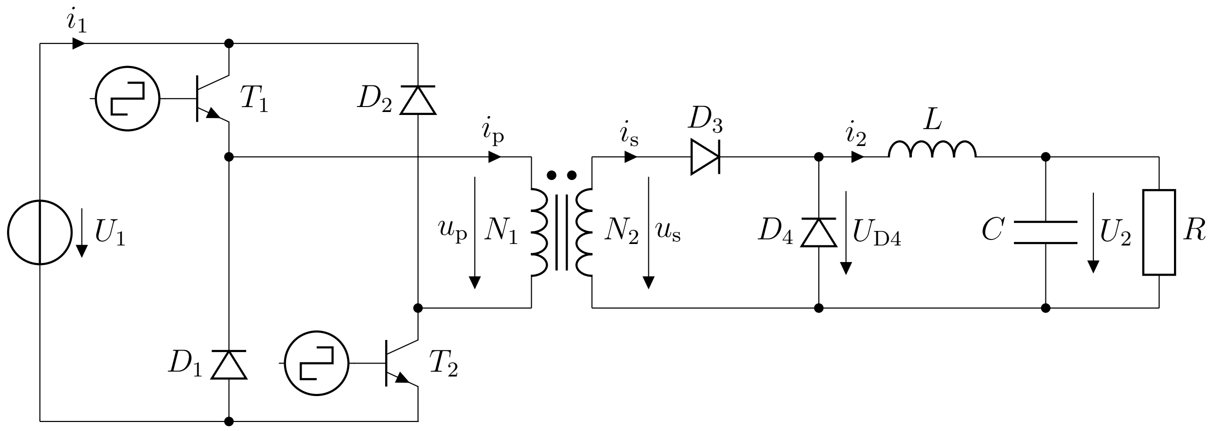


Figure 4.2.1: Forward converter with asymmetric half-bridge.

The parameters are listed in Fig. 4.2.1.

Input voltage:	$U_1 = 325$ V	Output voltage:	$U_2 = 15$ V
Output power:	$P_2 = 50$ W	Switching frequency:	$f_s = 50$ kHz
Turns ratio:	$N_1/N_2 = 10$	Magnetizing inductance:	$L_m = 2$ mH

Table 4.2.1: Parameter overview of the circuit.

The leakage inductance, the resistive losses, and the core losses of the transformer are negligible. The converter operates in steady-state conditions. Both transistors are controlled by the same signal.

4.2.1 At what duty cycle D does the circuit operate?

4.2.2 Calculate the average currents $\bar{i}_2$ and $\bar{i}_1$ over a switching cycle assuming ideal filtering of i_2.

4.2.3 Calculate the peak value $\hat{i}_m$ of the magnetizing current i_m.

4.2.4 Sketch the signals u_p, i_m, i_p and i_1 considering the switching-induced ripples.

4.2.5 Calculate the minimal necessary input voltage U_1, if $U_2 = 20$ V shall being constant.

4.2.6 Determine L such that the ripple current Δi_2 is 10 % of the average output current $\bar{i}_2$.

Task 4.3: Singled-ended forward converter (demagnetization winding)

The power supply of a data processing system shall be realized by a singled-ended forward converter.

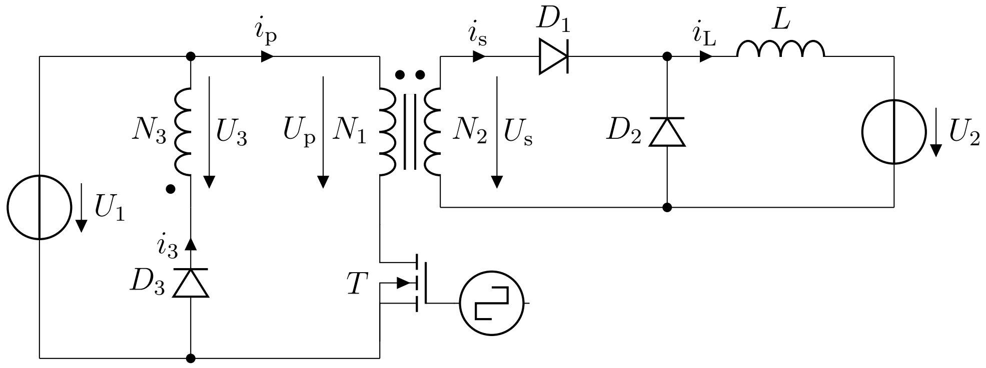


Figure 4.3.1: Single ended forward converter circuit.

The parameters are listed in Tab. 4.3.1. The output inductance L is dimensioned so that the current i_L exhibits a continuous curve shape. The transformer's leakage inductance can be neglected.

Input voltage:	$U_1 = 240\ \mathrm{V} \ldots 360\ \mathrm{V}$	Output voltage:	$U_2 = 5\ \mathrm{V}$
Output power:	$P_2 = 125\ \mathrm{W}$	Switching frequency:	$f_\mathrm{s} = 48\ \mathrm{kHz}$
Forward voltage of D_1:	$U_\mathrm{D1,f} = 0.4\ \mathrm{V}$	Forward voltage of D_2:	$U_\mathrm{D2,f} = 0\ \mathrm{V}$

Table 4.3.1: Parameters of the circuit.

4.3.1 Calculate the turns ratio N_3/N_1 limiting the maximum transistor blocking voltage to 600 V.

4.3.2 What is the maximum permissible duty cycle of the power transistor in this case?

4.3.3 What turns ratio N_1/N_2 should be chosen to achieve the required secondary voltage?

4.3.4 Does the duty cycle need to be adjusted when the output power changes? Over what range must the duty cycle be adjustable, considering the input voltage range?

4.3.5 What are the resulting maximum blocking voltages of the diodes D_1 and D_2?

4.3.6 Determine the magnetizing inductance L_m to ensure that the peak value of the magnetizing current remains below 10 % of the $\bar{i}'_\mathrm{L}$, which corresponds to the average current $\bar{i}_\mathrm{L}$ through the output inductance translated to the primary side at a nominal load of $P_2 = 125$ W.

4.3.7 Sketch the signals of the voltage across the power transistor, the current through the demagnetization winding, and the current through the freewheeling diode D_2 for $U_1 = 240$ V and $U_1 = 360$ V.

4.3.8 Calculate the peak magnetizing current for each case assuming a constant output current.

4.3.9 Could a higher power be transferred by doubling the switching frequency of the converter?

Exercise 05: Rectifiers

Task 5.1: B2U topology with capacitive filtering

An uncontrolled single-phase, two-pulse rectifier circuit with capacitive filtering is shown in Fig. 5.1.1. All components, including the diodes, are assumed to be ideal. On the input side, the single-phase AC supply with voltage $u_1(t)$ is connected, while on the output side, a smoothing capacitor C and a constant current load I_0 are present.

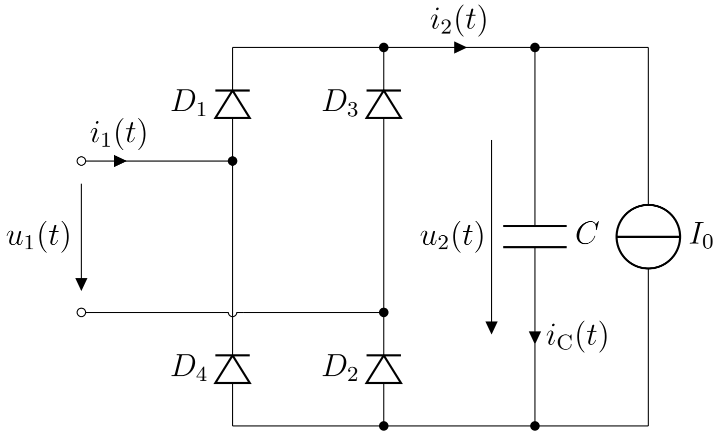


Figure 5.1.1: B2U rectifier with capacitive output filtering.

Input voltage:	$u_1(t) = 156\text{ V} \cdot \sin(\omega t)$	Load current:	$I_0 = 7.5\text{ A}$
Filter capacitance:	$C = 330\ \mu\text{F}$	Frequency:	$f = 60\text{ Hz}$

Table 5.1.1: Parameters of the B2U rectifier.

The angle α represents the phase angle range between zero crossing of the supply voltage and the phase angle at which all four diodes are blocked, meaning the capacitor discharges through the load. The angle β represents the phase angle range between α and the phase angle at which two of the four diodes begin to conduct, i.e., when the capacitor is recharged from the mains supply. A steady-state operation is assumed for this task.

5.1.1 Calculate the two angles α and β. Note: For the calculation of β you can use the following simple approximation: $\sin(x) \approx x$. (This approximation is sufficiently accurate within a range of approximately $x = \pm 25°$.)

5.1.2 Sketch the capacitor voltage $u_2(\omega t)$ considering $\omega t \in [0, ..., 2\pi]$ taking into account the previously calculated angles α and β.

5.1.3 Calculate the currents $i_1(\omega t)$ and $i_C(\omega t)$ and add them to the previous plot.

5.1.4 Assume the smoothing capacitor is very large, i.e., $C \to \infty$. What is the average active power P_0 absorbed by the current source? What will P_0 be if $C = 330$ µF?

5.1.5 What is the minimum blocking voltage ratings of the diodes to ensure that the rectifiers is not damaged?

Task 5.2: PFC rectifier

Due to the constantly increasing load on the grid with harmonics as a result of the use of power converters, the regulations regarding the permissible harmonic content of the current consumption of electrical consumers are being tightened. It is therefore necessary, e.g. for the rectification of single-phase AC mains voltage, to design power converters with a high power factor. A variant of a PFC rectifier circuit is shown in Fig. 5.2.1. The prerequisite for the use of the boost converter is: $u_2 = U_2 > u'(t)$. The boost converter is operated with a pulse width modulated (PWM)-based controller for which the switching frequency f_T has a constant value $f_\mathrm{T} = 20$ kHz. CCM is assumed as the operating mode.

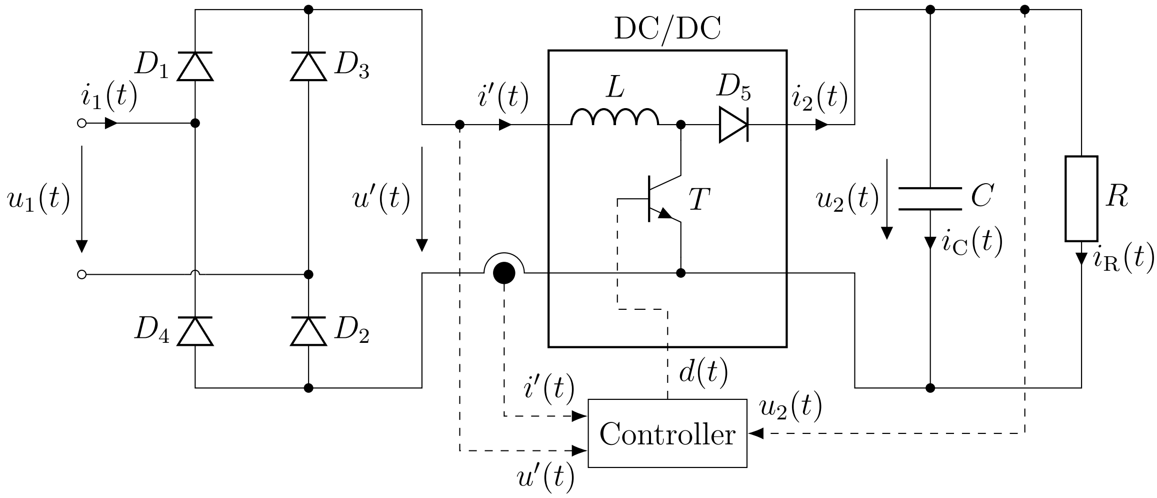


Figure 5.2.1: PFC rectifier with single-phase diode bridge and a cascaded DC/DC boost converter.

Input voltage:	$u_1(t) = \hat{u}_1 \sin(\omega t) = \sqrt{2} \cdot 230\ \mathrm{V} \cdot \sin(\omega t)$	Output voltage:	$u_2(t) = 400$ V
Output power:	$P_2 = 4$ kW	Grid frequency:	$f = 50$ Hz
Inductance:	$L = 570$ µH	Switching frequency:	$f_\mathrm{s} = 20$ kHz

Table 5.2.1: Parameters of the PFC rectifier.

5.2.1 Specify the voltage transformation ratio $m(t) = \frac{u_2(t)}{u'(t)}$ as a function of the duty cycle $d(t)$.

5.2.2 Specify the conduction time of the transistor and the diode as a function of the transformation ratio M and the time t, with the assumption $u_2(t) \approx U_2$.

5.2.3 Calculate the maximum amplitude of the switching-induced ripple of the mains current i_1 for the specified operating point. Note: Consider the conductive state of T and set the voltage across the inductance as a function of the phase angle (ωt) and the conduction time of the transistor according to the previous subtask.

5.2.4 Complete the current curve for a switching frequency $f_{s2} = 2$ kHz and an inductance $L = 5$ mH in Fig. 5.2.2. Note: At the time $t = 0$ is $i' = 0$. The switch-on and switch-off times are determined by the control signal of the transistor T and are summarized for the first 4 switching times in Tab. 5.2.2.

	$i = 1$	2	3	4
$T_{\mathrm{i,OFF}}$	490 µs	960 µs	1432 µs	1904 µs
$T_{\mathrm{i,ON}}$	506 µs	1040 µs	1573 µs	2104 µs

Table 5.2.2: Switching times $T_{\mathrm{i,OFF}}$ and $T_{\mathrm{i,ON}}$ for different i-values.

5.2.5 Approximately sketch the envelope of the current ripple in Fig. 5.2.2. Sketch the voltage across the inductor in Fig. 5.2.3 and enter the average value of the voltage as an approximation. How would the switch-on/switch-off ratio of the transistor have to be changed before and after the current peak in order to bring the average actual current value closer to the current setpoint?

5.2.6 Dimension the output capacitance C in a way that the amplitude of the output voltage ripple is $\Delta u_2 < 0.05\hat{u}_2$. Note: assume an idealized lossless converter, with the assumption $u_2(t) \approx U_2$.

5.2.7 Calculate the RMS value of the current through the capacitor I_{C}. Note: the mean value of the current through the capacitor is $\bar{i}_{\mathrm{C}} = 0$!

5.2.8 The current carrying capacity of the capacitor is $\frac{10\ \mathrm{A}}{1\ \mathrm{mF}}$. How large must its capacitance be selected to stay below this threshold with $I_{\mathrm{C}} = 10.44$ A? Is the permissible output voltage fluctuation or is it the current carrying capacity that determines the capacitance?

5.2.9 What is the current load (effective and average value) of the mains diodes?

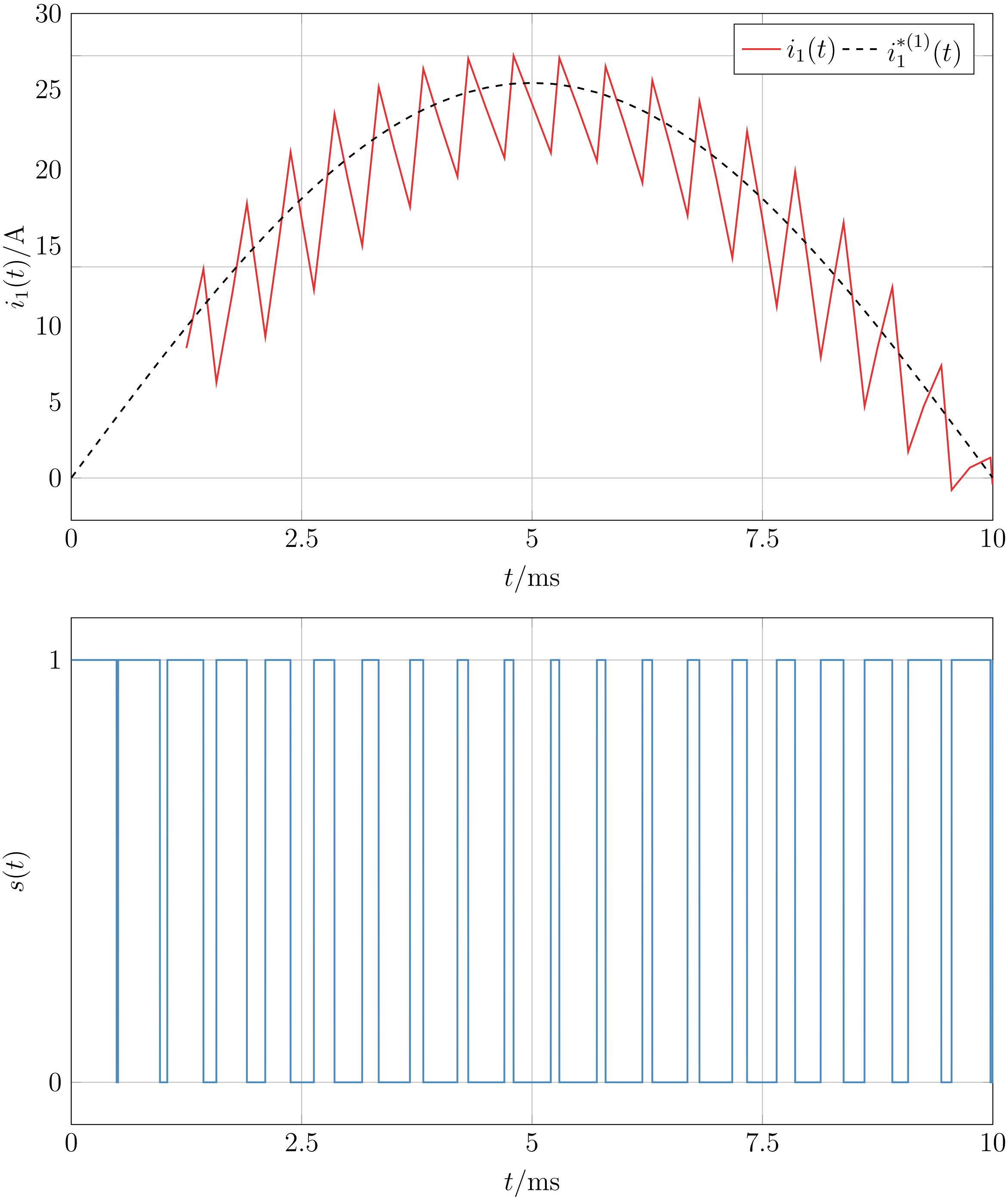


Figure 5.2.2: Incomplete representation of the mains current $i_1(t)$ and control signal $s(t)$ for power transistor T.

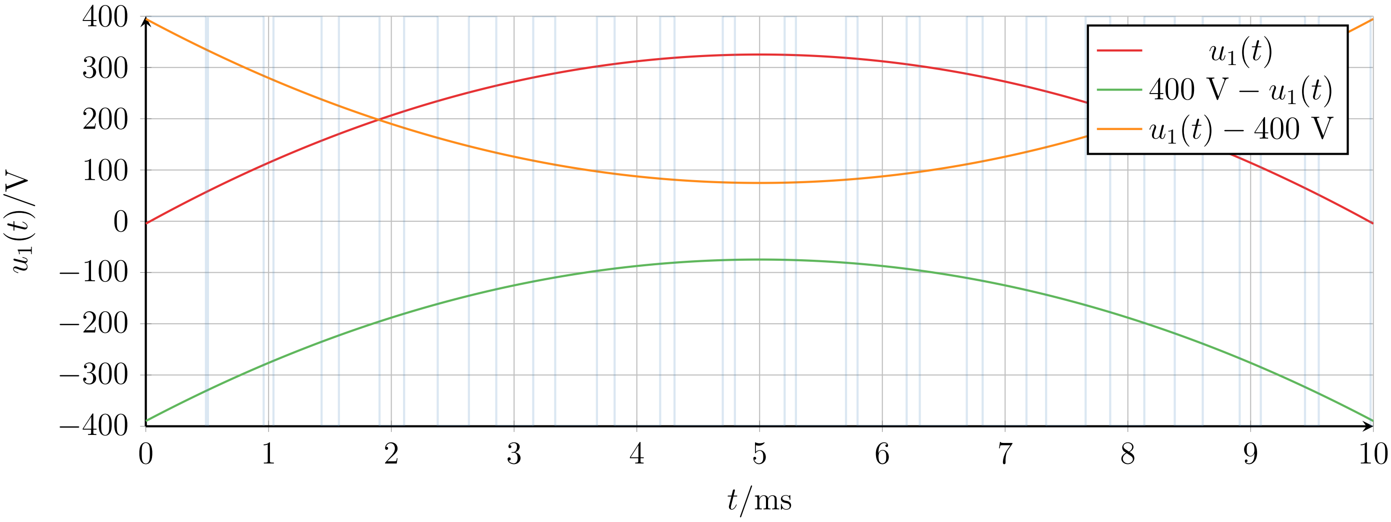
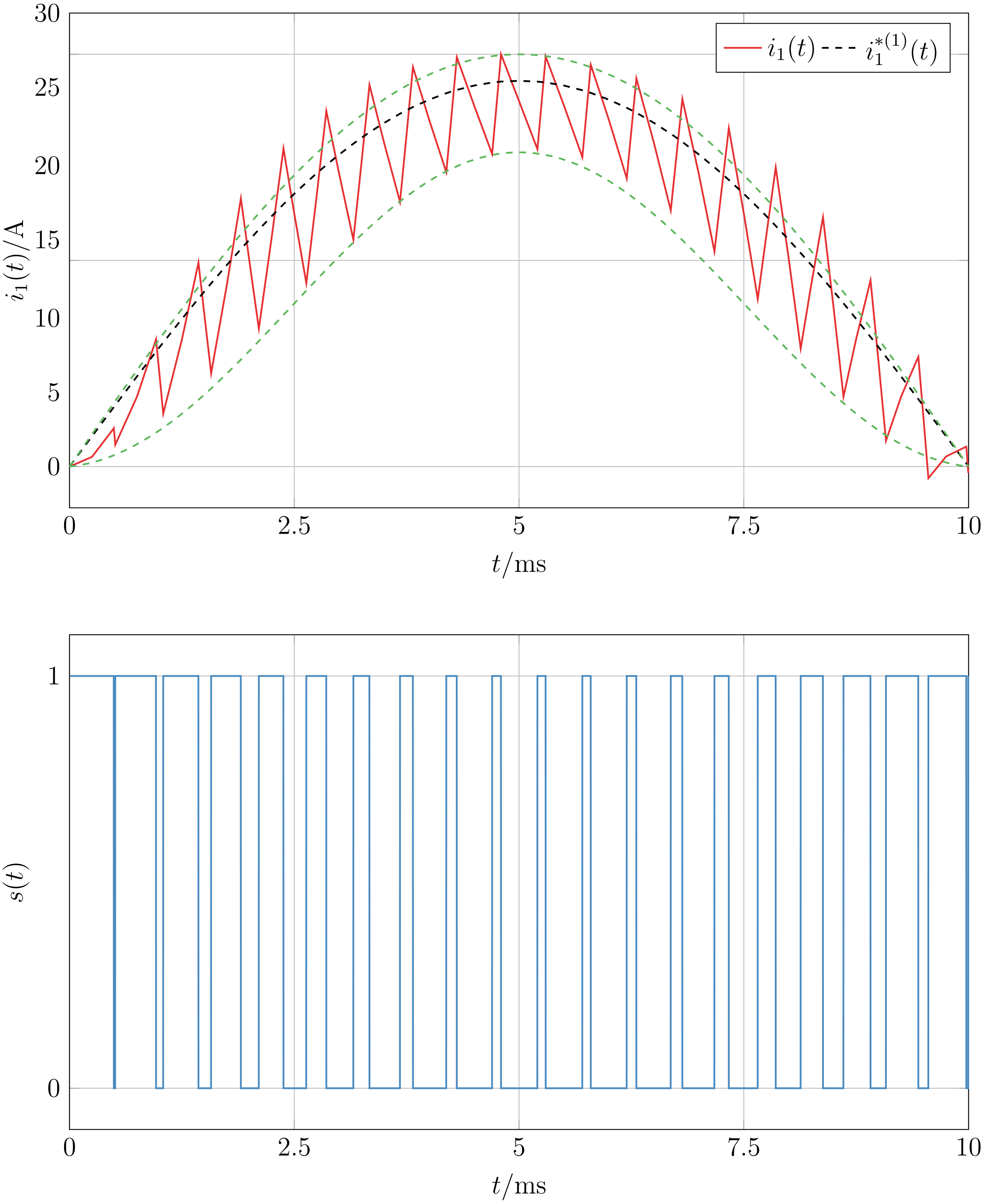
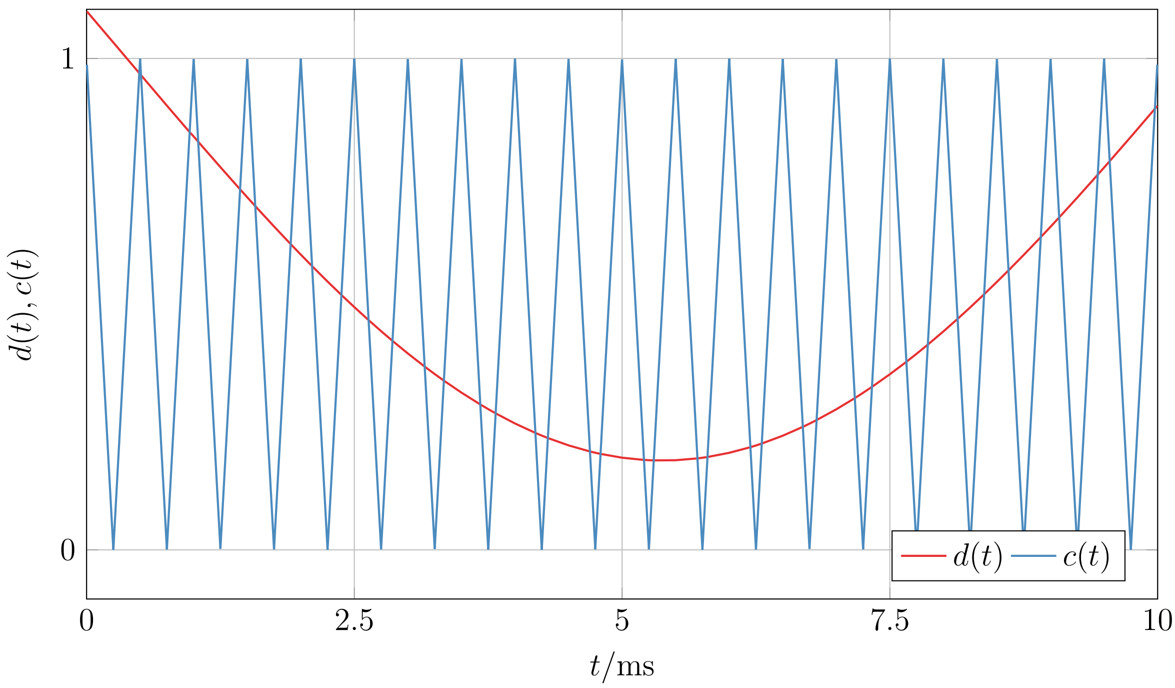


Figure 5.2.3: Voltage curve $u_1(t)$ with signal for transistor T.



Solution Figure 5.2.1: Overall representation of the mains current i' and transistor T control signal.



Solution Figure 5.2.2: Reference signal $c(t)$ and duty cycle signal d(t).

Exercise 06: Controlled rectifiers

Task 6.1: M3C converter at an RL-load

A controlled three-pulse midpoint circuit feeds an ohmic-inductive load. The load inductance L is infinitely large such that a pure direct current I_2 is taken from the converter. The load resistance is $R = 5\ \Omega$. The converter's ideal transformer is connected to the symmetrical three-phase grid with $U_\mathrm{N} = 230$ V (effective value of phase voltage) and $U_\mathrm{N,LL} = 400$ V (line-to-line voltage). The secondary side phase voltages point an effective value of $U_{1,i} = 230\ \mathrm{V}, \forall i = \mathrm{a, b, c}$. The thyristors and commutation can be assumed to be ideal.

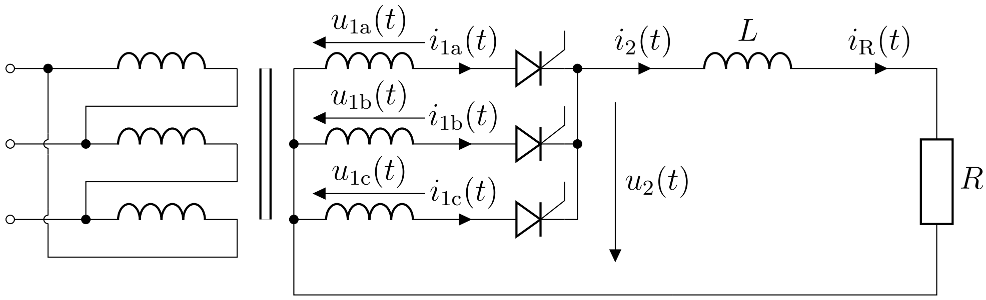


Figure 6.1.1: M3C topology with an input three-phase transformer and an RL-load.

6.1.1 Calculate the firing angle α such that an active power of $P = 6$ kW is delivered to the load. How big is the resulting load current $i_2(t) = I_2$?

6.1.2 Draw the normalized control characteristic curve ${U_2(\alpha)}/{U_2(\alpha=0)}$ and mark the operating point $P = 6$ kW at $R = 5\ \Omega$.

6.1.3 Draw the curve of the converter's output voltage $u_2(t)$ for the calculated control angle α, from subtask 6.1.1, within a plot of the three-phase transformer's secondary voltages.

6.1.4 Calculate the effective value $I_{1a}^{(1)}$ of the fundamental current component $i_{1a}^{(1)}(t)$ and add the latter to the previous plot. How big is the phase shift φ_{1a} between $u_{1a}(t)$ and $i_{1a}^{(1)}(t)$?

6.1.5 Calculate the fundamental reactive power $Q^{(1)}$ drawn by the converter from the grid.

Task 6.2: B6C converter at a motor load

In a lifting drive, a permanent magnet DC motor is supplied by a B6C converter circuit. The B6C-topology is connected to the three-phase grid. With the assumption of $L \to \infty$ the motor operates with constant nominal current and constant nominal voltage when lifting as well as lowering the load. This corresponds to a terminal voltage of $u_\mathrm{mot,up}(t) = U_\mathrm{mot}$ when lifting the load and $u_\mathrm{mot,down}(t) = -U_\mathrm{mot}$ when lowering it. In order to generate the necessary torque, the motor absorbs the current $i_\mathrm{mot}(t) = I_\mathrm{mot}$.

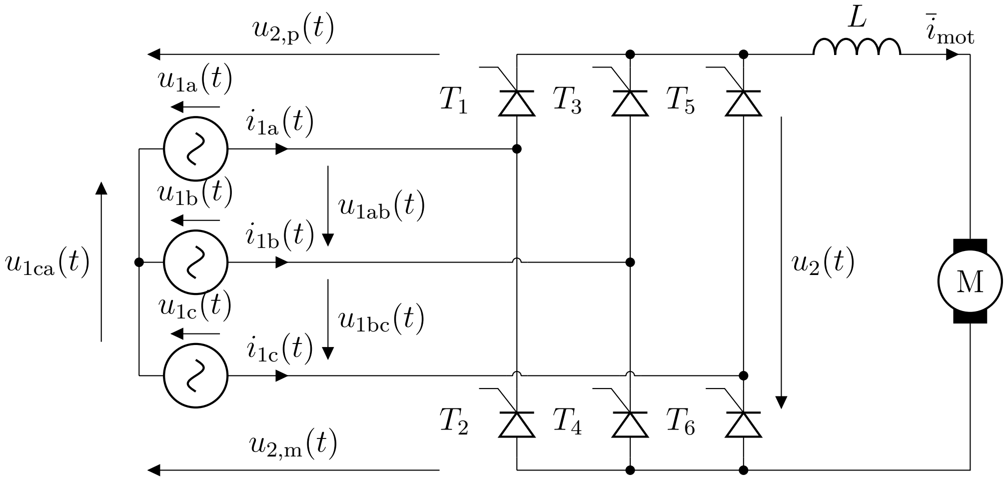


Figure 6.2.1: B6C converter at a motor load.

Input voltages ($i = \mathrm{a, b, c}$):	$U_{1,i} = 230$ V (phase voltage)
	$U_{1,\mathrm{LL},i} = 400$ V (line-to-line voltage)
Nom. motor current:	$I_{\mathrm{mot}} = 20$ A
Nom. motor voltage:	$U_{\mathrm{mot}} = 466$ V
Grid frequency:	$f = 50$ Hz

Table 6.2.1: Parameters of the lifting drive with B6C converter.

6.2.1 Calculate the firing angle α_{up} required for lifting and the firing angle α_{down} for lowering the load to operate the motor at rated voltage.

6.2.2 Sketch following signals for the two calculated firing angles α_{up} and α_{down}:

- The output voltage $u_{2,\mathrm{p}}(t)$ and $u_{2,\mathrm{m}}(t)$ of the two partial converters (reference point is neutral) and shade the effective voltage-time area for the two cases,
- The output voltage $u_2(t)$ and the mean voltage $\overline{u}_2$,
- The current $i_{1\mathrm{a}}(t)$ and it's fundamental $i_{1\mathrm{a}}^{(1)}(t)$,
- The voltage of thyristor $u_{\mathrm{T1}}(t)$,
- Indicate which thyristors are conducting during a pulse interval.

6.2.3 Calculate the active power P, the fundamental reactive power $Q^{(1)}$ and the fundamental apparent power $S^{(1)}$ for the two considered operating points. Represent P, $Q^{(1)}$ and $S^{(1)}$ in the complex plane.

6.2.4 Calculate the fundamental current ratio $g = \frac{I^{(1)}}{I}$ and the power factor λ for the two considered operating points.

Exercise 07: Transistor-based AC-DC converters

Task 7.1: Single-phase AC-DC converter

The following ideal single-phase converter

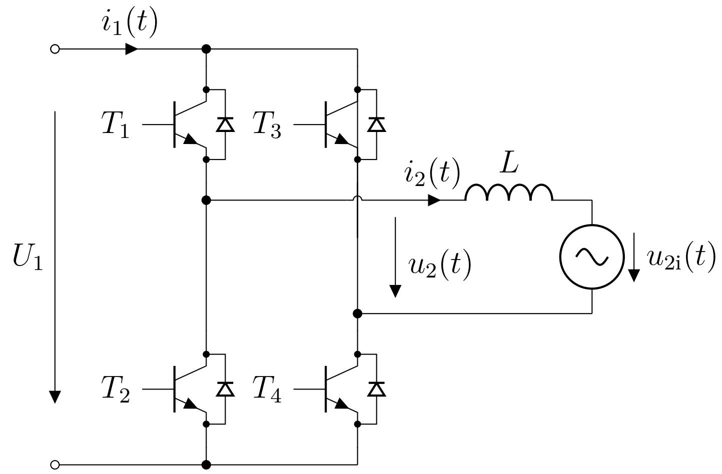


Figure 7.1.1: Single-phase AC-DC converter.

configured in a bridge topology and supplies a load consisting of an inductor and an internal load voltage. The converter consists of four transistors arranged in full bridge configuration.

Input DC voltage:	$U_1 = 200$ V
Inductance:	$L = 4.8$ mH
Internal load voltage:	$u_{2i}(t) = 150 \sin(\omega t - \frac{\pi}{6})$
Reference angular frequency:	$\omega_2 = 2\pi \cdot 50$ Hz

Table 7.1.1: Parameters of the single-phase AC-DC converter.

The converter is modulated using PWM with a modulation index of $m = 0.75$. Assuming ideal operation of the switching components, perform the following tasks:

7.1.1 Draw the converter's output voltage $u_2(t)$ belonging to Fig. 7.1.2. and its fundamental component $u_2^{(1)}(t)$. How large is the phase difference φ_{2i} of the voltage fundamental component $u_2^{(1)}(t)$ compared to the internal load voltage $u_{2i}(t)$?

7.1.2 Calculate the amplitude $\hat{i}_2^{(1)}$ and the phase angle $\varphi^{(1)}$ of the current fundamental component $i_2^{(1)}(t)$ compared to $u_{2i}(t)$, and draw $u_2^{(1)}(t)$ as well as $i_2^{(1)}(t)$ in Fig. 7.1.3.

7.1.3 In Fig. 7.1.4, draw the voltage harmonics $u_2^{(h)}(t)$. Try to sketch, approximately, the current harmonics $i_2^{(h)}(t)$ by counting the voltage time area squares. One square corresponds to a voltage time area of 25.9 mVs. The starting point is marked with **x**.

Hint: The current harmonics $i_2^{(h)}(t)$ are free of any bias.

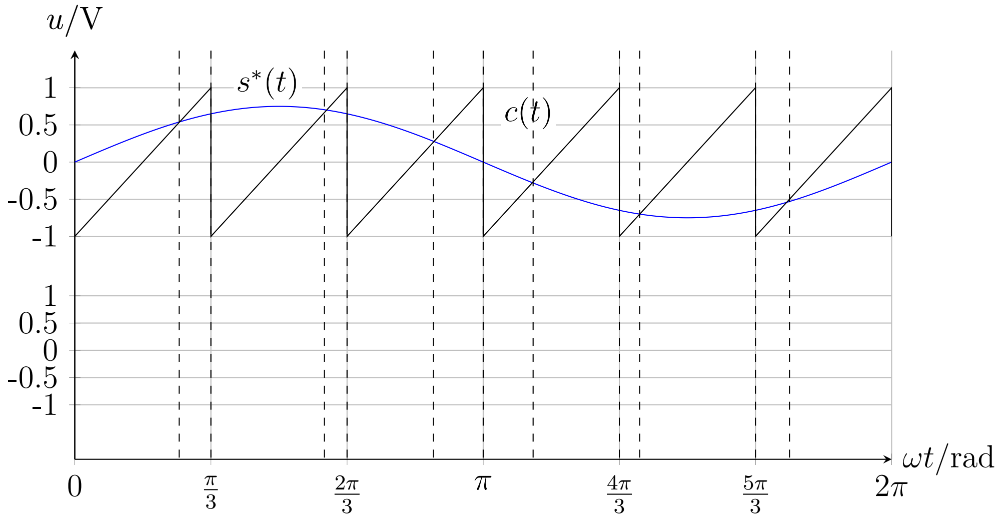


Figure 7.1.2: Output voltage $u_2(t)$ and fundamental voltage component $u_2^{(1)}(t)$.

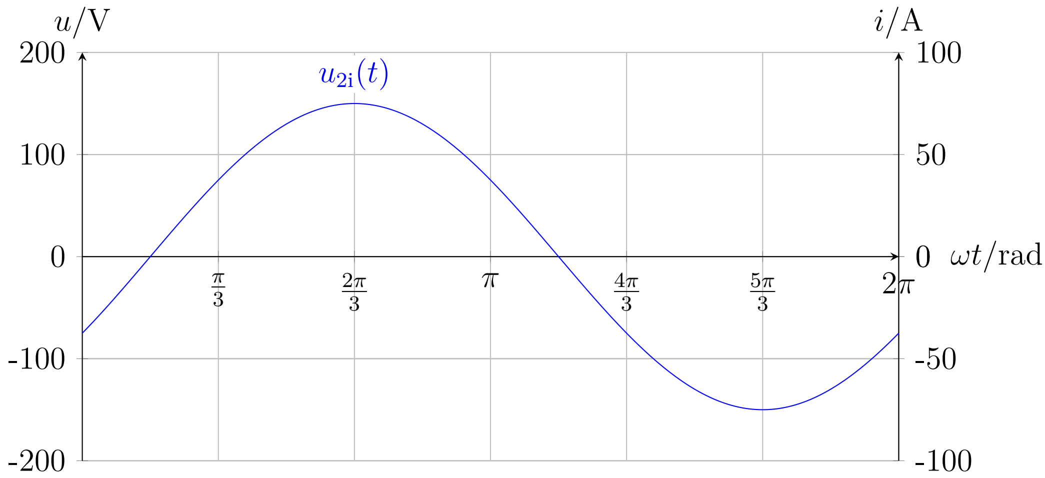


Figure 7.1.3: Internal load voltage $u_{2\mathrm{i}}(t)$, fundamental voltage and current components $u_2^{(1)}(t)$ and $i_2^{(1)}(t)$, respectively.

7.1.4 Draw the converter's output current $i_2(t)$, its fundamental component $i_2^{(1)}(t)$ and its harmonics $i_2^{(\mathrm{h})}(t)$ in Fig. 7.1.5.

7.1.5 Mark the input current $i_1(t)$ of the AC-DC converter in Fig. 7.1.5.

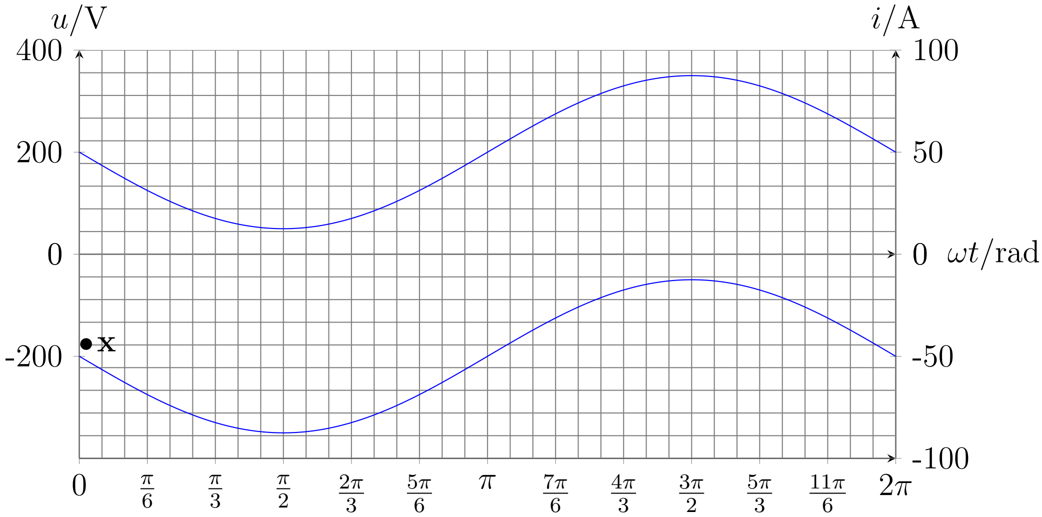


Figure 7.1.4: Harmonics of the output voltage and current.

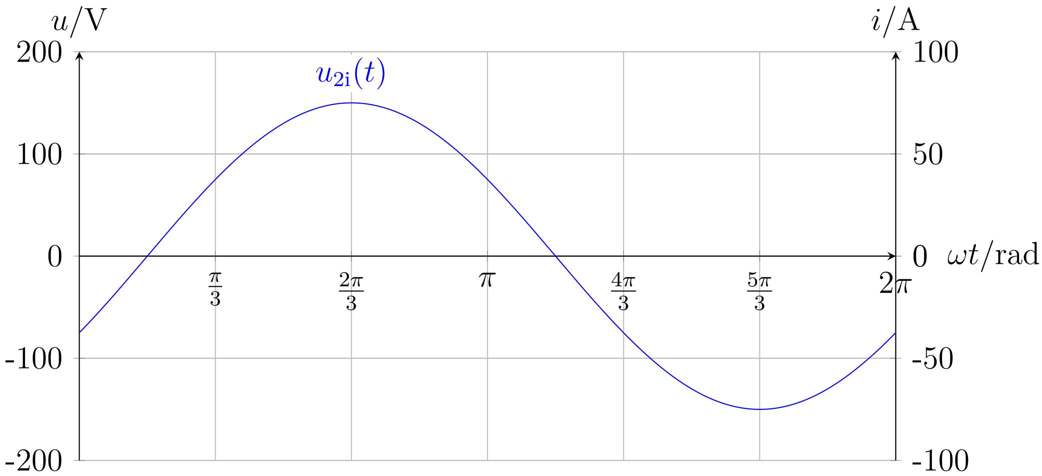


Figure 7.1.5: Output current $i_2(t)$, its fundamental component $i_2^{(1)}(t)$, its harmonics $i_2^{(\mathrm{h})}(t)$, and input current $i_1(t)$.

Task 7.2: Symmetrical 3-phase rectifier

A rectifier in three-phase bridge topology shall supply a symmetrical three-phase load in star connection. The load is represented by an inductance and a sinusoidal internal (or inner) voltage per phase. The inverter is operated with the fundamental frequency modulation (also known as six-step mode) and the switching elements are considered as ideal. The schematic is depicted in Fig. 7.2.1.

7.2.1 Create a table with all possible switching states for fundamental frequency modulation. Use the following notation:

$$\{s_\mathrm{a}(t), s_\mathrm{b}(t), s_\mathrm{c}(t)\} = \begin{cases} s_i(t) = +1 & \text{upper position,} \\ s_i(t) = -1 & \text{lower position.} \end{cases}$$

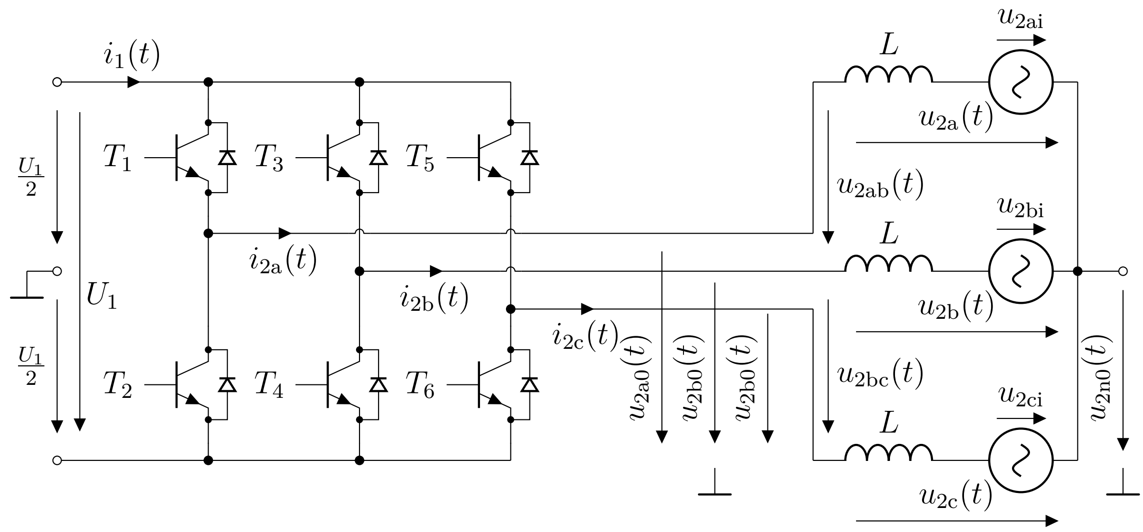


Figure 7.2.1: Three-phase inverter in six-step mode.

Input voltages:	$U_1 = 510$ V
Internal voltages:	$u_{2ai}(t) = \sqrt{2} \cdot 220\text{ V} \cdot \sin(\omega_1 t)$
Angular load frequency:	$\omega_1 = 2\pi \cdot 30\ \frac{1}{\mathrm{s}}$
11 Inductance per phase:	$L = 10$ mH
Phase angle between $u_{2ai}(t)$ and $i_{2ai}^{(1)}(t)$	$\varphi_{2a}^{(1)} = 30°$

Table 7.2.1: Parameters of three-phase inverter in six-step mode.

Sketch the switching states in the correct chronological order for one period. Calculate and sketch the voltages $u_{2a0}(t)$, $u_{2b0}(t)$ and $u_{2c0}(t)$ depending on these switching states.

7.2.2 The internal voltages $u_{2ai}(t)$, $u_{2bi}(t)$ and $u_{2ci}(t)$ are from a symmetrical voltage system, i.e., the following is always applicable: $u_{2ai}(t) + u_{2bi}(t) + u_{2ci}(t) = 0$ V. Show that this equation is also applicable for the voltages $u_{2a}(t)$, $u_{2b}(t)$ and $u_{2c}(t)$ under the same conditions.

7.2.3 Calculate and sketch the voltages $u_{2ab}(t)$, $u_{2bc}(t)$, $u_{2a}(t)$ and the star-to-ground voltage $u_{2n0}(t)$ depending on the switching states.

7.2.4 Decompose the voltage $u_{2a}(t)$ into a Fourier series and sketch the spectral lines related to the amplitude of the fundamental signal up to order $k = 13$.
Hint:

$$b_k = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} f(x)\sin(kx)\mathrm{d}x \quad k = \text{odd}.$$

The formula above applies to the Fourier coefficients of an odd and alternating function.

7.2.5 Based on subtask 7.2.4, calculate the fundamental amplitude $\hat{i}_{\mathrm{a}}^{(1)}$ using a vector diagram and complex alternating current calculations. From this, determine the total active power fed to the load.