Electrical Machines and Drives
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Universität Siegen / IAS Interconnected Automation Systems

Exercise 01: Semiconductor properties

Task 1.1: Properties of germanium as semiconductor

The following problems explore fundamental semiconductor properties of germanium. Starting with atomic concentration and intrinsic resistivity, the tasks gradually introduce doping effects and compare the electrical behavior of intrinsic and extrinsic materials under simplified assumptions. Take the values from Tab. 1.1.1.

Intrinsic concentration n_i:	$2.5 \cdot 10^{13}$ $1/\mathrm{cm}^3$
Mobility μ_p:	1800 $\frac{\mathrm{cm}^2}{\mathrm{Vs}}$
Mobility μ_n:	3800 $\frac{\mathrm{cm}^2}{\mathrm{Vs}}$
Atomic weight a_ger:	72.6 $\frac{\mathrm{g}}{\mathrm{mol}}$
Material density D_ger:	5.32 g/cm^3

Table 1.1.1: key figures of germanium at 300 K.

1.1.1 Using Avogadro's number $(N_{Av} = 6.02 \cdot 1023 \ \mathrm{mol}^{-1})$, calculate the concentration of atoms in germanium.

Answer:

A quantity of any substance equal to its molecular weight in grams is a mole. So the concentration is calculated by

$$C_{Ger} = \frac{N_\mathrm{Av}}{a_\mathrm{ger}} \cdot D_\mathrm{ger} = \frac{6.02 \cdot 1023 \ \mathrm{mol}^{-1}}{72.6 \ \frac{\mathrm{g}}{\mathrm{mol}}} \cdot 5.32 \ \mathrm{g}/\mathrm{cm}^3 = 4.41 \cdot 10^{22} \ \mathrm{atoms}/\mathrm{cm}^3. \tag{1.1.1}$$

1.1.2 Calculate the resistivity of intrinsic germanium at 300 K.

Answer:

Using the equation $n = p = n_\mathrm{i}$ the conductivity of the intrinsic germanium is calculated by

$$\sigma = n_\mathrm{i}q(\mu_n + \mu_p) = 2.5 \cdot 10^{13} \ 1/\mathrm{cm}^3 \cdot 1.60 \cdot 10^{-19} \ \mathrm{C} \cdot \left(3800 \ \mathrm{cm}^2/\mathrm{Vs} + 1800 \ \mathrm{cm}^2/\mathrm{Vs}\right)$$
$$= 0.00224 \ \frac{1}{\Omega\mathrm{cm}}. \tag{1.1.2}$$

The resistivity results by the reciprocal value:

$$\rho = \frac{1}{\sigma} = \frac{1}{0.00224 \ 1/\Omega\mathrm{cm}} = 44.6 \ \Omega\mathrm{cm}. \tag{1.1.3}$$

1.1.3 If a donor-type impurity is added to the extent of 1 part in 10^8 germanium atoms, find the resistivity.

Answer:

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If there is 1 donor atom per 10^8 germanium atoms, then $N_D = 4.41 \cdot 10^{14}$ 1/cm³. Using the fact $n \approx N_D$ leads to:

$$p = \frac{n_i^2}{N_D} = \frac{\left(2.5 \cdot 10^{13} \text{ 1/cm}^3\right)^2}{4.41 \cdot 10^{14} \text{ 1/cm}^3} = 1.42 \cdot 10^{12} \text{ holes/cm}^3. \tag{1.1.4}$$

Since $n \gg p$, we can neglect p in calculating the conductivity. This leads to:

$$\sigma = nq\mu_n = 4.41 \cdot 10^{14} \text{ 1/cm}^3 \cdot 1.60 \cdot 10^{-19} \text{ C} \cdot 3800 \text{ cm}^2/\text{Vs} = 0.268 \, \frac{1}{\Omega\text{cm}}. \tag{1.1.5}$$

The resistivity results by the reciprocal value:

$$\rho = \frac{1}{\sigma} = \frac{1}{0.268 \text{ 1/}\Omega\text{cm}} = 3.72 \, \Omega\text{cm}. \tag{1.1.6}$$

1.1.4 If germanium were a monovalent metal, find the ratio of its conductivity to that of the n-type semiconductor in previous subtask.

Answer:

If each atom contributed one free electron to the material the number of free electrons are $n = 4.41 \cdot 10^{22}$. This leads to:

$$\sigma = nq\mu_n = 4.41 \cdot 10^{22} \text{ 1/cm}^3 \cdot 1.60 \cdot 10^{-19} \text{ C} \cdot 3800 \text{ cm}^2/\text{Vs} = 2.58 \cdot 10^7 \, \frac{1}{\Omega\text{cm}}. \tag{1.1.7}$$

The ratio yields:

$$\frac{\sigma_{metal}}{\sigma_{\text{semi}}} = \frac{2.58 \cdot 10^7 \, \frac{1}{\Omega\text{cm}^3}}{0.268 \text{ 1/}\Omega\text{cm}} \approx 10^8. \tag{1.1.8}$$

Task 1.2: Energy level and silicon semiconductor behaviour.

1.2.1 Calculate the frequency of revolution for an electron in the ground state of hydrogen. For comparison, find the frequency emitted when an electron with the mass $m_e = 9.11 \cdot 10^{-31}$ kg of falls from the state $n = 2$ to the ground state $n = 1$. Take in account, that $W_{\text{kin}}(n) \propto \frac{1}{n^2}$ and the ground state orbit r corresponds to 0.0529 nm.

Answer:

The kinetic energy of this electron for ground state is calculated by

$$W_{\text{kin}} = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e r\omega^2. \tag{1.2.1}$$

If the electron with the mass m_e orbits a proton in a circular path with radius r and speed v, the centrifugal force (from circular motion) is balanced by the electrostatic attraction (Coulomb force) between the electron and the proton. This balance leads to the equation:

$$m_e r\omega^2 = \frac{q_e^2}{4\pi\epsilon_0} \tag{1.2.2}$$

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Using (1.2.1) and (1.2.2) leads to

$$W_{\text{kin}} = \frac{1}{8\pi\epsilon_0} \cdot \frac{q_e^2}{rn^2} = \frac{1}{8\pi \cdot 8.854 \cdot 10^{-12} \frac{F}{m}} \cdot \frac{(1.60 \cdot 10^{-19} \text{ C})^2}{0.0529 \text{ nm}} = 2.17 \cdot 10^{-18} \text{ J} \tag{1.2.3}$$

Solving (1.2.1) for ω yields

$$\omega = \sqrt{\frac{2 \cdot W_{\text{kin}}}{m_e r}} = \sqrt{\frac{2 \cdot 2.17 \cdot 10^{-18} \text{ J}}{9.11 \cdot 10^{-31} \text{ kg} \cdot 0.0529 \text{ nm}}} = 3 \cdot 10^{11} \text{ 1/s.}$$
$$f = 47.8 \text{ GHz.} \tag{1.2.4}$$

The total energy for ground state is calculated by

$$W_{\text{n}=1} = -\frac{q_e^2}{8\pi\epsilon_0 r} == -\frac{1.60 \cdot 10^{-19} \text{ C}^2}{8\pi \cdot 8.854 \cdot 10^{-12} \frac{F}{m} \cdot 0.0529 \text{ nm}} = -2.17 \cdot 10^{-18} \text{ J} \tag{1.2.5}$$

The energy difference a transition from $n = 2$ to $n = 1$ is calculated by

$$\Delta W = W_{\text{n}=1} \cdot \left(\frac{1}{2^2} - 1\right) = -2.17 \cdot 10^{-18} \text{ J} \cdot (-0.75) = 1.63 \cdot 10^{-18} \text{ J} \tag{1.2.6}$$

This corresponds to a frequency of

$$f = \frac{\Delta W}{h} = \frac{1.63 \cdot 10^{-18} \text{ J}}{6.63 \cdot 10^{-34} \frac{J}{s}} = 2.5 \cdot 10^{15} \text{ Hz} \tag{1.2.7}$$

1.2.2 Find the room-temperature resistivity of an n-type silicon doped with 10^{16} phosphorus atoms/cm³.

<u>Answer:</u>
At room temperature we assume that all donors are ionized. This leads to:

$$n \approx N_{\text{D}} = 10^{16} \text{ 1/cm}^3. \tag{1.2.8}$$

The conductivity is calculated by

$$\sigma = nq\mu_n = 10^{16} \text{ 1/cm}^3 \cdot 1.60 \cdot 10^{-19} \text{ C} \cdot 1300 \text{ cm}^2/\text{Vs} = 20.8 \frac{1}{\Omega\text{cm}}. \tag{1.2.9}$$

The resistivity results by the reciprocal value:

$$\rho = \frac{1}{\sigma} = \frac{1}{20.8 \text{ }\Omega\text{cm}^{-1}} = 0.48 \text{ }\Omega\text{cm.} \tag{1.2.10}$$

1.2.3 A sample of Si is doped with 10^{16} phosphorus atoms/cm³. Find the Hall voltage in a sample with $l_{\text{w}} = 500$ µm, $A = 2.5 \cdot 10^{-3}$ cm², $I = 1$ mA, and $B_{\text{z}} = 10^{-4} \frac{\text{Wb}}{\text{cm}^2}$.

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Answer:

The Hall coefficient is calculated by

$$R_{\mathrm{H}} = \frac{1}{qn} = \frac{1}{1.60 \cdot 10^{-19} \text{ C} \cdot 10^{16} \frac{1}{\text{cm}^3}} = 625 \text{ cm}^3/\text{C}. \tag{1.2.11}$$

The Hall voltage results from

$$U_{\mathrm{H}} = R_{\mathrm{H}} \frac{I}{A} B_{\mathrm{z}} l_{\mathrm{w}} = 625 \text{ cm}^3/\text{C} \frac{1 \text{ mA}}{2.5 \cdot 10^{-3} \text{ cm}^2} 10^{-4} \frac{\text{Wb}}{\text{cm}^2} \cdot 500 \text{ μm} = -1.25 \text{ mV}. \tag{1.2.12}$$

1.2.4 Assume that, in an n-type semiconductor at $T = 300$ K, the electron concentration varies linearly from 10^{18} 1/cm^3 to $7 \cdot 10^{17}$ 1/cm^3 over a distance of 0.1 cm. Calculate the diffusion current density if the electron diffusion coefficient is $D_n = 22.5 \frac{\text{cm}^2}{\text{s}}$.

Answer:

The diffusion current density is given by

$$
\begin{aligned}
J_{\mathrm{n,diff}} &= qD_{\mathrm{n}} \frac{\mathrm{d}n}{\mathrm{d}x} \approx qD_{\mathrm{n}} \frac{\Delta n}{\Delta x} \\
&= 1.60 \cdot 10^{-19} \text{ C} \cdot 22.5 \frac{\text{cm}^2}{\text{s}} \cdot \frac{10^{18} \frac{1}{\text{cm}^3} - 7 \cdot 10^{17} \frac{1}{\text{cm}^3}}{0.1 \text{ cm}} = 10.8 \frac{\text{A}}{\text{cm}^2}.
\end{aligned}
\tag{1.2.13}
$$

1.2.5 Minority carriers (holes) are injected into a homogeneous n-type semiconductor sample at one point. An electric field of 50 V/cm is applied across the sample, and the field moves these minority carriers a distance of 1 cm in 100 μs. Find the drift velocity and the diffusivity of the minority carriers. The temperature is 300 K.

Answer:

The drift velocity is simply calculated by the distance traveled per unit time.

$$v_{\mathrm{p}} = \frac{l_{\mathrm{d}}}{t} = \frac{1 \text{ cm}}{100 \text{ μs}} = 10^4 \frac{\text{cm}}{\text{s}}. \tag{1.2.14}$$

The mobility results from the quotient of drift velocity and the electrical field along parallel to the drift direction.

$$\mu_{\mathrm{p}} = \frac{v_{\mathrm{p}}}{\vec{E}} = \frac{10^4 \frac{\text{cm}}{\text{s}}}{50 \text{ V/cm}} = 200 \frac{\text{cm}^2}{\text{Vs}}. \tag{1.2.15}$$

This leads to the diffusivity of the minority carriers:

$$D_{\mathrm{p}} = \frac{kT}{q} \mu_{\mathrm{p}} = \frac{1.38 \cdot 10^{-19} \frac{\text{J}}{\text{K}} \cdot 300 \text{ K}}{1.60 \cdot 10^{-19} \text{ C}} \cdot 200 \frac{\text{cm}^2}{\text{Vs}} = 5.18 \frac{\text{cm}^2}{\text{s}}. \tag{1.2.16}$$

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Task 1.3: Quality assessment at doping of silicon semiconductor

A silicon semiconductor is to be doped for a specific application. To assess the doping quality, the resistivity of a sample is measured before and after the process. The assessment is performed at a temperature of $T = 300$ K.

Intrinsic concentration n_i:	$1.5 \cdot 10^{10}$ cm^3
Mobility μ_p:	$480 \frac{\mathrm{cm}^2}{\mathrm{Vs}}$
Mobility μ_n:	$1350 \frac{\mathrm{cm}^2}{\mathrm{Vs}}$

Table 1.3.1: key figures of silicon.

1.3.1 Calculate the expected resistivity of silicon before doping.

Answer:

The number of holes are equal to the numbers of free electrons according $n_0 = p_0 = n_i$. So the total conductivity is calculated by

$$
\begin{aligned}
\sigma &= n_\mathrm{i} q (\mu_\mathrm{p} + \mu_\mathrm{n}) \\
&= 1.5 \cdot 10^{10} \text{ cm}^3 \cdot 1.60 \cdot 10^{-19} \text{ C} \cdot \left(480 \, \frac{\mathrm{cm}^2}{\mathrm{Vs}} + 1350 \, \frac{\mathrm{cm}^2}{\mathrm{Vs}} \right) = 4.39 \cdot 10^{-6} \, \frac{1}{\Omega \mathrm{cm}}.
\end{aligned} \tag{1.3.1}
$$

The resistivity results by the reciprocal value:

$$
\rho = \frac{1}{\sigma} = \frac{1}{4.39 \cdot 10^{-6} \, \Omega \mathrm{cm}^{-1}} = 2.28 \cdot 10^5 \, \Omega \mathrm{cm}. \tag{1.3.2}
$$

1.3.2 Calculate the expected resistivity of silicon at $T = 300$ K after doping with arsenic impurities with concentration $N_d = 2 \cdot 10^{16}$ cm^3.

Answer:

The concentration of majority carriers is so large that only they will be considered in the following calculation. With $n_0 \approx N_\mathrm{d}$ the conductivity is calculated by:

$$
\sigma \approx N_\mathrm{d} q \mu_\mathrm{n} \approx 2 \cdot 10^{16} \text{ cm}^3 \cdot 1.60 \cdot 10^{-19} \text{ C} \cdot 1350 \, \frac{\mathrm{cm}^2}{\mathrm{Vs}} = 2.3 \, \frac{1}{\Omega \mathrm{cm}}. \tag{1.3.3}
$$

The resistivity results by the reciprocal value:

$$
\rho = \frac{1}{\sigma} = \frac{1}{2.3 \, \Omega \mathrm{cm}^{3-1}} = 0.31 \, \Omega \mathrm{cm}. \tag{1.3.4}
$$

Task 1.4: Hall sensor for magnetic field measurement

A bar of type p silicon, of thickness $d = 0.5$ mm, with impurity concentration $N_a = 10^{14}$ cm^3, is used as a Hall sensor. See also Fig. 1.4.1.

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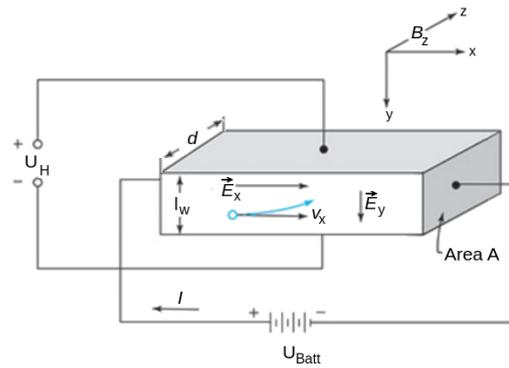Universität Siegen / IAS Interconnected Automation Systems

Figure 1.4.1: Silicon semiconductor bar as Hall-sensor.

1.4.1 Calculate the Hall voltage for a probe current of 100 mA and a magnetic field perpendicular to the plane of $B = 0.1$ T.

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Answer:

The Hall voltage is given by

$$U_\mathrm{H} = \vec{E_\mathrm{y}} l_\mathrm{w} \quad \text{and} \quad J = \frac{I}{l_\mathrm{w} d}. \tag{1.4.1}$$

with l_w corresponds to the width and d to the thickness of the bar. J corresponds to the current density. Since $N_\mathrm{a} \gg n_\mathrm{i}$ which leads to $p \approx N_\mathrm{a} \gg n$, so that the current is dominated by holes. The electrical field in y-direction results in

$$\vec{E_\mathrm{y}} = \frac{J_\mathrm{p}}{qp} B_\mathrm{z} \tag{1.4.2}$$

Using (1.4.1) this leads to

$$U_\mathrm{H} = \frac{J_\mathrm{p}}{qp} B_\mathrm{z} l_\mathrm{w} = \frac{I \cdot B_\mathrm{z}}{qpd} = \frac{100 \ \mathrm{mA} \cdot 0.1 \ \mathrm{T}}{1.60 \cdot 10^{-19} \ \mathrm{C} \cdot 10^{14} \ \frac{1}{\mathrm{cm}^3} \cdot 0.1 \ \mathrm{mm}} = 1.25 \ \mathrm{V} \tag{1.4.3}$$

Remark: This example shows that for magnetic fields with intensities typical of those used in laboratories, the Hall voltage has a relatively high value for electronic circuits. This does not happen in metals, because the concentration of free electrons $\approx 10^{22} \ \frac{1}{\mathrm{cm}^3}$ is much larger than in semiconductors, and thus the voltage is quite small.