Exam

Electrical Machines and Drives

Winter 2025/26

First name: Last name:

Matriculation number: Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 120 minutes.

Evaluation:

Task	1	2	3	4	$\sum$
Maximum score	10	12	7	13	42
Achieved score					

Task 1: Design of a singe phase transformer **[10 Points]**

An (older) smartphone charger is constructed as shown in Fig. 1. The input (grid) voltage at the transformer is $U_1 = 230$ V. The output voltage of the charger should have an average voltage of $U_{\mathrm{out}} = 5$ V. The rectifier causes an average voltage drop of $U_{\mathrm{rectifier}} = 0.7$ V between U_2 and U_{out}. The transformer has a rated apparent power of 100 VA.

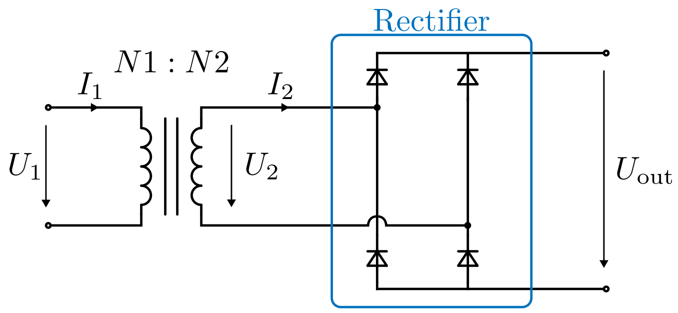


Fig. 1: Simplified sketch of a smartphone charger.

1.1 Calculate the secondary voltage, the turns ratio as well as the primary and secondary current of the transformer for rated and idealized conditions. [2 Points]

Hint: if and only if you are not able to solve this task, use $I_1 = 0.6$ A, $I_2 = 21$ A and ü $= 35$ as substitute results for the subsequent tasks.

Answer:

The secondary voltage is calculated by

$$U_2 = U_{\mathrm{out}} + U_{\mathrm{rectifier}} = 5\ \mathrm{V} + 0.7\ \mathrm{V} = 5.7\ \mathrm{V},$$

resulting in the transformation ratio of:

$$\ddot{u} = \frac{N_1}{N_2} = \frac{U_1}{U_2} = 40.35.$$

The primary current is given by

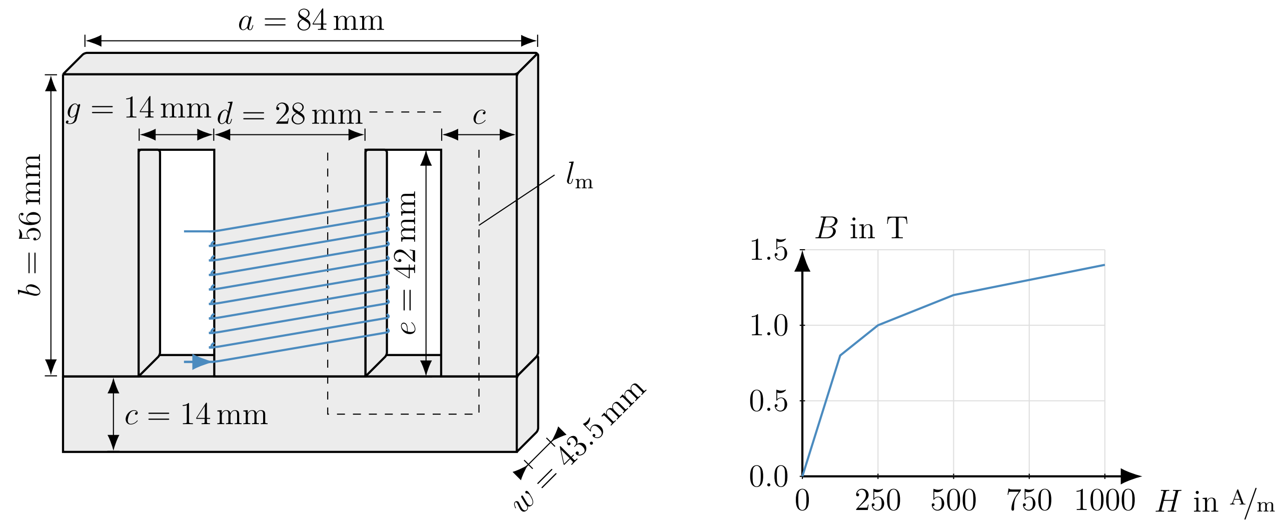
$$I_1 = \frac{S}{U_1} = 0.43\ \mathrm{A},$$

and, the secondary current is calculated by:

$$I_2 = \frac{S}{U_2} = 17.54\ \mathrm{A}.$$

1.2 The transformer's geometry and design is to be analyzed in more detail. The laminated core is specified as EI 84/43.5 with a magnetic flux density of $B_{\mathrm{Fe}} = 1.2$ T in the middle yoke. There are no stray fluxes, and the output voltage remains constant when the transformer is loaded. Calculate the effective cross-sectional area A_{Fe} of the iron yoke, the magnetic flux ϕ_{Fe} and the magnetomotive force θ. The geometry and its dimensions are shown in Fig. 2. [3 Points]

Hint: if and only if you are not able to solve this task, use $\theta = 100$ A as a substitute result for the subsequent tasks.



(a) EI-84/43.5 core geometry. (b) Magnetization curve.

Fig. 2: EI-84/43.5 core and electrical steel magnetization curve.

Answer:

The cross sectional area is given with:

$$A_{\mathrm{Fe}} = dw = 0.0012 \text{ m}^2.$$

Thus, the magnetic flux is calculated as

$$\phi_{\mathrm{Fe}} = B_{\mathrm{fe}} A_{\mathrm{Fe}} = 0.00146 \text{ Wb},$$

and, the magnetic field can be taken from the magnetization curve, which is $H_{\mathrm{Fe}} = 500 \, \frac{\mathrm{A}}{\mathrm{m}}$. The average length of the magnetic path in the E-core is given by

$$l_{\mathrm{E}} = 2e + g + 2c = 0.126 \text{ m},$$

and for the I-core as

$$l_{\mathrm{I}} = 2c + g = 0.042 \text{ m}.$$

Thus, the average length of the magnetic path is given with

$$l_{\mathrm{m}} = l_{\mathrm{E}} + l_{\mathrm{I}} = 0.168 \text{ m},$$

therefore, the magnetomotive force is calculated as

$$\theta = H_{\mathrm{Fe}} l_{\mathrm{m}} = 84 \text{ A}.$$

1.3 Calculate the integer number of winding turns on the primary and secondary side. [1 Point]

Answer:
The primary winding turns are calculated by

$$N_1 = \frac{\theta}{I_1} = 193.2,$$

and, for the secondary side as

$$N_2 = \frac{N_1}{\ddot{\mathrm{u}}} = 4.78,$$

which results in $N_1 = 193$ and $N_2 = 5$ turns, respectively.

1.4 Determine the ohmic winding losses and wire diameters of the primary and secondary sides. First calculate the winding cross-sections assuming a current density of $J = 3 \text{ A/mm}^2$, an average turn length of $l_{\mathrm{w}} = 0.25$ m, and an electrical conductivity of $\kappa_{\mathrm{Cu}} = 50 \cdot 10^6 \frac{\mathrm{S}}{\mathrm{m}}$. [3 Points]

Answer:
The cross section is given by:

$$q_1 = \frac{I_1}{J} = 0.145 \cdot 10^{-6} \text{ m}^2,$$

and, $q_2 = 5.85 \cdot 10^-6 \text{ m}^2$, respectively. The resistance is calculated with

$$R_1 = \frac{1}{\kappa_{\mathrm{Cu}}} \frac{N_1}{q_1} l_{\mathrm{w}} = 6.66 \ \Omega,$$

and $R_2 = 0.004 \ \Omega$, respectively. The wire diameters are calculated by

$$d_1 = 2\sqrt{\frac{q_1}{\pi}} = 0.42 \text{ mm},$$

and $d_2 = 2.73$ mm. With the winding resistance, the cupper losses result in

$$P_1 = I_1^2 R_1 = 1.26 \text{ W},$$

and, $P_2 = 1.32$ W.

1.5 To also charge a laptop with a battery voltage of 12 V, which transformer type would allow one charger to provide both output voltages? [1 Point]
Hint: This task is independent of the previous tasks and can be solved qualitatively without reference to the previous results.

Answer:

In this case a tap transformer could be used, which has two taps on the secondary winding, allowing to select between the different output voltages. The required output voltage can be selected by a switch that connects the two windings to the rectifier.

Task 2: Working principle of a DC machine [12 Points]

Fig. 3 shows a simplified DC machine cross section. The symbols • and × indicate technical conductor current directions (out of / into the drawing plane). Assume that the drawn field lines represent the complete air-gap field, i.e., no additional field lines exist to the left and right of the shown field region. Thus, outside the depicted air-gap field region the flux density is assumed to be zero.

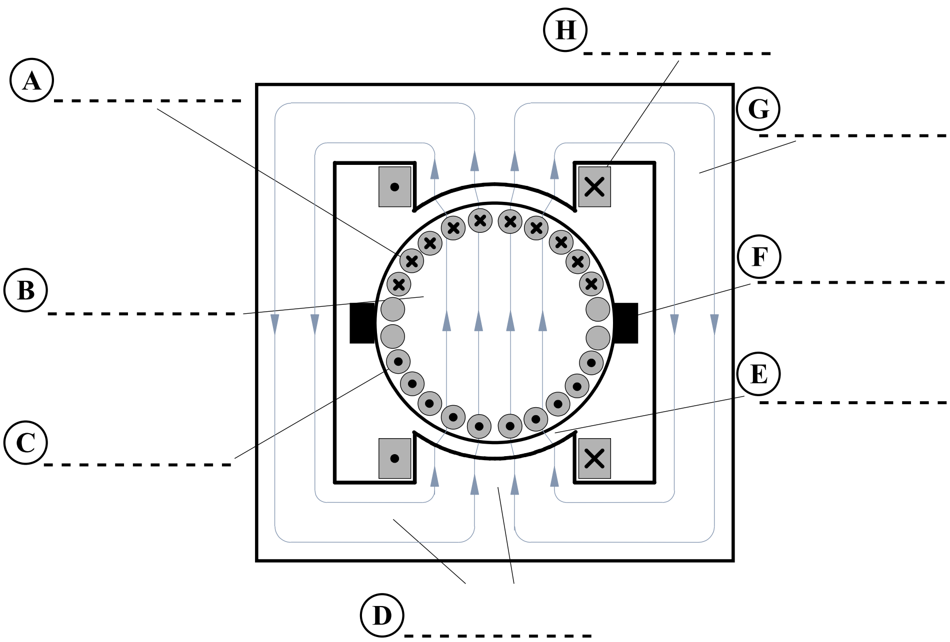


Fig. 3: DC machine cross section with missing component labels.

For the calculations, assume a homogeneous and block-shaped air-gap flux density $\hat{B}_\delta$ within the pole coverage. Use the data from Tab. 1 for the following calculations.

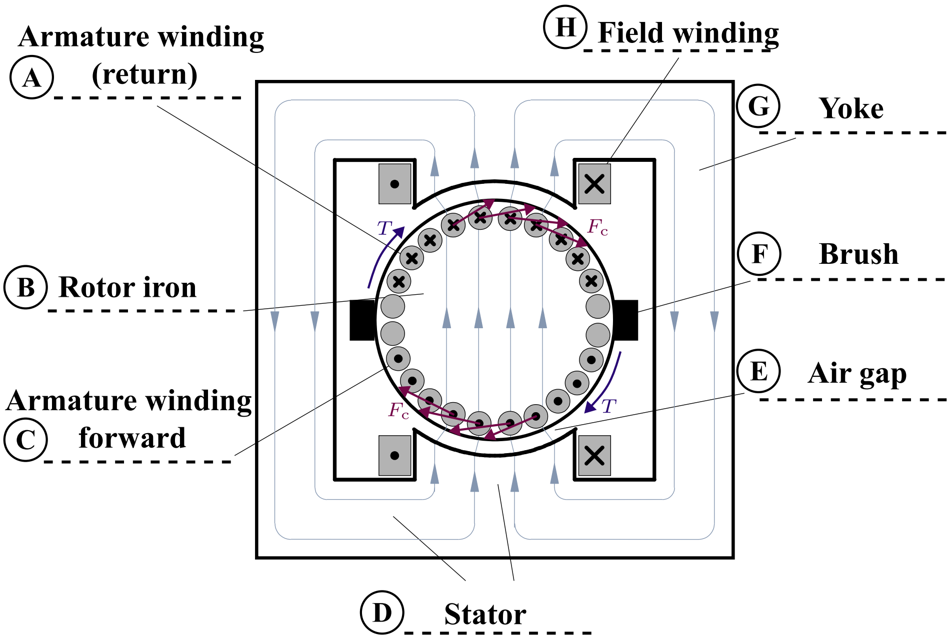
Tab. 1: Given DC machine parameters.

Symbol	Description	Value
$\hat{B}_\delta$	Air-gap flux density (block-shaped)	0.8 T
I_c	Conductor current (per conductor)	218.75 A
l_z	Axial length	0.25 m
d_a	Armature diameter	0.24 m
p	Pole pairs	1
α_p	Pole coverage factor	0.65
n	Nominal rotational speed	2500 $\frac{1}{\text{min}}$
N_{act}	Number of active conductors (4 forward + 4 return)	8
a	Number of parallel armature paths	1

2.1 Add the missing component labels (A–H) in Fig. 3. [2 Points]

Answer:

The correct component labels are shown in Sol.-Fig. 1.



Solution Fig. 1: DC machine cross section with component labels as well as torque and force indications.

2.2 In Fig. 3, draw the resulting Lorentz force on one exemplary forward conductor (•) and one return conductor (×). Also, indicate the resulting torque direction at the shaft. [2 Points]

Answer:
Apllying the Lorentz's force right hand law, the solution from Sol.-Fig. 1 is obtained. As the force is acting on the armature conductors in a clockwise direction, the resulting torque is also in clockwise direction.

2.3 Calculate the Lorentz force F_c acting on one active armature conductor assuming a perpendicular field penetration through the air gap. [1 Point]

Answer:

$$F_c = \hat{B}_\delta \, l_z \, I_c = 0.8 \text{ T} \cdot 0.25 \text{ m} \cdot 218.75 \text{ A} = 43.75 \text{ N}.$$

2.4 Calculate (i) the torque contribution T_c of one active conductor and (ii) the resulting shaft torque T from all active conductors. [2 Points]

Answer:

$$T_c = F_c \frac{d_a}{2} = 43.75 \text{ N} \cdot \frac{0.24 \text{ m}}{2} = 5.25 \text{ N m}.$$

With $N_{act} = 8$ active conductors (4 forward + 4 return),

$$T = N_{act} \, T_c = 8 \cdot 5.25 \text{ N m} = 42 \text{ N m}.$$

2.5 Compute the maximum flux linkage per armature coil $\hat{\phi}_\mathrm{c}$. [2 Points]

Answer:

The pole pitch for $p = 1$ is:

$$\tau_p = \frac{\pi d_a}{2} = \frac{\pi \cdot 0.24 \text{ m}}{2} = 0.377 \text{ m}.$$

The maximum flux linkage is given, when a conductor loop is covering the entire pole coverage area. Hence, the effective air-gap area is given by the pole coverage factor α_p times the pole pitch τ_p times the axial length l_z:

$$A_\delta = \alpha_p \, \tau_p \, l_z = 0.65 \cdot 0.377 \text{ m} \cdot 0.25 \text{ m} = 0.0613 \text{ m}^2.$$

The maximum flux linkage per conductor pair is then given by

$$\hat{\phi}_\mathrm{c} = \hat{B}_\delta \, A_\delta = 0.8 \text{ T} \cdot 0.0613 \text{ m}^2 = 0.049 \text{ Vs}.$$

2.6 Determine (i) the induced voltage magnitude $\hat{u}_\mathrm{i,c}$ per active conductor at the nominal speed and (ii) the total induced armature voltage magnitude $\hat{u}_\mathrm{i}$. For the latter assume that the voltage induction per conductor loop is roughly identical for all loops. [3 Points]

Answer:

The nominal angular speed is given by:

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \cdot 2500 \, \frac{1}{\text{min}}}{60 \, \frac{\text{s}}{\text{min}}} = 261.8 \text{ rad/s}.$$

Due to the block-shaped flux density distribution, the flux linkage is linearly dependent on the rotor angle. Hence, for a constant speed, the induced voltage is given by the rotor angle derivative (angular speed) times the flux linkage:

$$\hat{u}_\mathrm{i,c} = \omega \hat{\phi}_\mathrm{c} = 261.8 \text{ rad/s} \cdot 0.049 \text{ Vs} = 12.8 \text{ V}.$$

The total induced voltage results from the four active conductor loops (4 forward + 4 return) in series connection, i.e., $N_\mathrm{act}/2 = 4$ conductor loops:

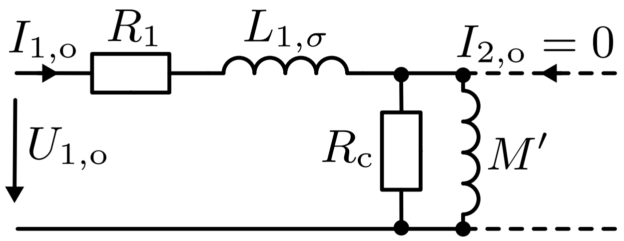
$$\hat{u}_\mathrm{i} = \frac{N_\mathrm{act}}{2} \hat{u}_\mathrm{i,c} = 4 \cdot 12.8 \text{ V} = 51.2 \text{ V}.$$

Task 3: Transformer model parameterization **[7 Points]**

The transformer designed in task 1 has been manufactured, and experimental tests must be carried out to determine its T-type equivalent circuit model parameters.
Hint: if you are not able to solve task 1, use ü = 35 as substitute results for the subsequent tasks.

3.1 Draw the equivalent circuit diagram for the open-circuit test (incl. labeling of components, voltages and currents). [1 Point]

Answer:
The ECD is shown in Sol.-Fig. 2.



Solution Fig. 2: ECD for the open-circuit test.

3.2 In the open-circuit test, $U_{1,\mathrm{o}} = 230$ V, $P_{1,\mathrm{o}} = 10$ W, and $S_{1,\mathrm{o}} = 15$ VA were measured. Determine the iron loss resistance R_c and the mutual inductances M' and M (referred to the primary side and unreferred). Neglect ohmic losses and stray inductances. [3 Points]

Answer:
Based on the made assumptions, the measured active input power is dissipated as iron losses yielding

$$R_\mathrm{c} = \frac{U_{1,\mathrm{o}}^2}{P_{1,\mathrm{o}}} = 5290\ \Omega.$$

Based on the measured input powers, the power factor for the open-circuit test is

$$\cos(\varphi_\mathrm{o}) = \frac{P_{1,\mathrm{o}}}{S_{1,\mathrm{o}}} = 0.67 \quad \Leftrightarrow \quad \varphi_\mathrm{o} = 84°.$$

The input current during the open-circuit test must have been

$$I_{1,\mathrm{o}} = \frac{S_{1,\mathrm{o}}}{U_{1,\mathrm{o}}} = 0.065\ \mathrm{A}.$$

The reactance of the open-circuit transformer is

$$X_\mathrm{o} = \frac{U_{1,\mathrm{o}}}{I_{1,\mathrm{o}}} \frac{1}{\sin(\varphi_\mathrm{o})} = Z_0 \frac{1}{\sin(\varphi_\mathrm{o})} = 4731.5\ \Omega.$$

This reactance must be identical to the reactance provided by the transformed mutual inductance as the stray inductance impact is neglected:

$$\omega M' = X_\mathrm{o} \quad \Leftrightarrow \quad M' = \frac{X_\mathrm{o}}{\omega} = 15.06\ \mathrm{H}.$$

Based on the estimated transformation ratio, the original mutual inductance yields

$$M = \frac{1}{\ddot{u}} M' = 0.37\ \mathrm{H}.$$

3.3 During a follow-up short-circuit test with $U_{1,\mathrm{s}} = 3.4$ V, $I_{1,\mathrm{s}} = 0.5$ A and $P_{1,\mathrm{s}} = 1.6$ W were measured. Determine $R_1 = R_2'$ and the untransformed R_2, as well as $L_{1,\sigma} = L_{2,\sigma}'$ and the untransformed $L_{2,\sigma}$. Neglect the mutual inductance. [3 Points]

<u>Answer:</u>
The short-circuit impedance is

$$Z_\mathrm{s} = \frac{U_{1,\mathrm{s}}}{I_{1,\mathrm{s}}} = 6.8\ \Omega,$$

while the corresponding power factor is

$$\cos(\varphi_\mathrm{s}) = \frac{P_{1,\mathrm{s}}}{U_{1,\mathrm{s}} I_{1,\mathrm{s}}} = 0.94 \quad \Leftrightarrow \quad \varphi_\mathrm{s} = 19.9^\circ.$$

Separating the real and imaginary part of the impedance, one receives

$$R_1 + R_2' = Z_\mathrm{s} \cos(\varphi_\mathrm{s}) = 6.4\ \Omega \quad \Longrightarrow \quad R_1 = R_2' = 3.2\ \Omega$$

and

$$(L_{1,\sigma} + L_{2,\sigma}')\omega = Z_\mathrm{s} \sin(\varphi_\mathrm{s}) = 721.9\ \Omega \quad \Longrightarrow \quad L_{1,\sigma} = L_{2,\sigma}' = 3.6\ \mathrm{mH}.$$

The untransformed secondary values result in

$$R_2 = \frac{1}{\ddot{u}^2} R_2' = 2\ \mathrm{m}\Omega, \quad L_{2,\sigma} = \frac{1}{\ddot{u}^2} L_{2,\sigma}' = 2.2\ \mathrm{\mu H}.$$

Task 4: Stationary operation of a permanent magnet synchronous machine [13 Points]

Fig. 4 shows the mechanical rotor angle $\varepsilon_\mathrm{r}(t)$ and the three-phase stator currents $i_\mathrm{s,a}(t)$, $i_\mathrm{s,b}(t)$, $i_\mathrm{s,c}(t)$ of a permanent magnet synchronous motor (PMSM). The rotor angle sensor is aligned for $\varepsilon_\mathrm{r} = 0$ with the a-phase winding current vector. Assume steady-state operation (constant speed) and a symmetric star-connected stator (zero-sequence-free currents). The PMSM parameters are given in Tab. 2. Neglect iron losses, friction/windage, and converter losses. Consider only stator copper losses via R_s.

Tab. 2: Given PMSM parameters.

Symbol	Description	Value
R_s	Stator resistance	30 mΩ
$L'_\mathrm{s,d}$	d-axis inductance	0.25 mH
$L'_\mathrm{s,q}$	q-axis inductance	0.40 mH
ψ_pm	PM flux linkage	0.12 Vs

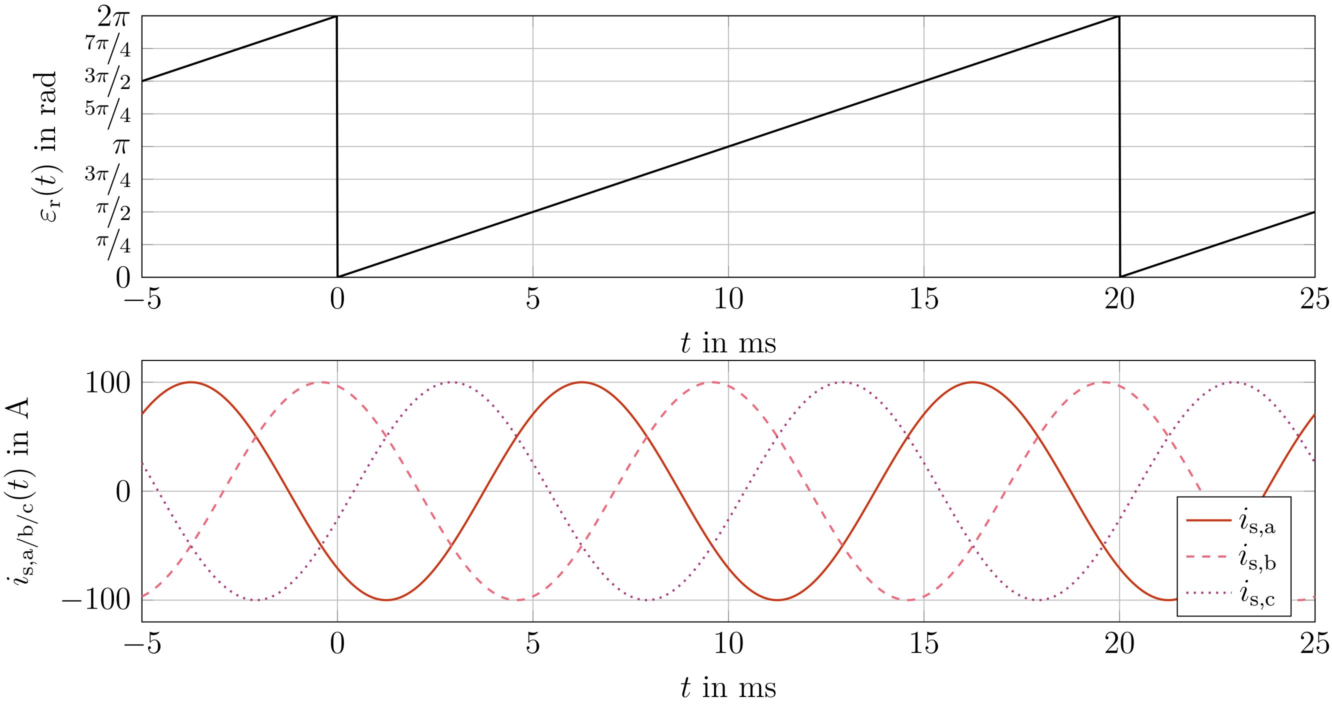


Fig. 4: Measured mechanical rotor angle $\varepsilon_\mathrm{r}(t)$ and three-phase stator currents $i_\mathrm{s,a}(t)$, $i_\mathrm{s,b}(t)$, $i_\mathrm{s,c}(t)$.

4.1 Determine the pole pair number p as well as the mechanical and electrical rotational frequencies f_r and $f_\mathrm{r,el}$ from Fig. 4. [3 Points]

Hint: If and if only you are not able to solve this task, use the following alternative values for the following tasks: $p = 3$, $f_\mathrm{r} = 25$ Hz, $f_\mathrm{r,el} = 75$ Hz.

Answer:

From the upper plot, one mechanical period occurs within 20 ms, hence

$$T_\mathrm{r} = 20\ \mathrm{ms} \ \Rightarrow\ f_\mathrm{r} = \frac{1}{T_\mathrm{r}} = 50\ \mathrm{Hz}.$$

From the lower plot, two electrical current periods occur within 20 ms, hence

$$T_\mathrm{r,el} = \frac{20\ \mathrm{ms}}{2} = 10\ \mathrm{ms} \ \Rightarrow\ f_\mathrm{r,el} = \frac{1}{T_\mathrm{r,el}} = 100\ \mathrm{Hz}.$$

Therefore, the pole pair number is

$$p = \frac{f_\mathrm{r,el}}{f_\mathrm{r}} = \frac{100}{50} = 2.$$

4.2 Determine the electrical load angle θ from Fig. 4, i.e., the angle between the rotor flux linkage and the stator current space vector. [2 Points]
Hint: Due to the visual uncertainty, only an approximate estimate is expected. If and if only you are not able to solve this task, use the following alternative value for the following tasks: $\theta = 225°$.

Answer:
From Fig. 4 it can be seen that the current period is leading the mechanical period by approximately 3.75 ms, which corresponds to an a mechanical angle of

$$\varepsilon_\mathrm{lead} = \frac{3.75\ \mathrm{ms}}{20\ \mathrm{ms}} \cdot 360° = 67.5°.$$

The electrical angle is p times the mechanical angle, hence

$$\theta = p \cdot \varepsilon_\mathrm{lead} = 2 \cdot 67.5° = 135°.$$

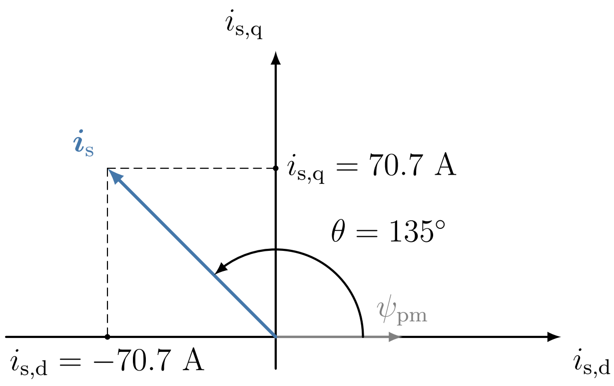
4.3 Determine the steady-state dq-current components $i_\mathrm{s,d}$ and $i_\mathrm{s,q}$. [1 Point]
Hint: If and if only you are not able to solve this task, use the following alternative values for the following tasks: $i_\mathrm{s,d} = -70.71$ A, $i_\mathrm{s,q} = -70.71$ A.

Answer:
With rotor-flux orientation, the d-axis is aligned with ψ_pm, hence the dq-currents can be computed from the stator current amplitude $\hat{i}_\mathrm{s}$ and the load angle θ as

$$i_\mathrm{s,d} = \hat{i}_\mathrm{s} \cos(\theta), \qquad i_\mathrm{s,q} = \hat{i}_\mathrm{s} \sin(\theta).$$

With $\hat{i}_\mathrm{s} = 100$ A and $\theta = 135°$, one obtains

$$i_\mathrm{s,d} = 100\ \mathrm{A} \cos(135°) = -70.71\ \mathrm{A}, \qquad i_\mathrm{s,q} = 100\ \mathrm{A} \sin(135°) = 70.71\ \mathrm{A}.$$



Solution Fig. 3: Visual vector representation of the situation from Fig. 4

4.4 Calculate the electromagnetic torque T and the mechanical output power P_{me}. [2 Points]
Hint: If and if only you are not able to solve this task, use the following alternative values for the following tasks: $P_{me} = -8.64$ kW.

Answer:
The torque is

$$T = \frac{3}{2} p\, i_{s,q} \left(\psi_{pm} + \left(L'_{s,d} - L'_{s,q}\right) i_{s,d}\right).$$

Inserting the given parameters and the computed dq-currents yields

$$T = \frac{3}{2} \cdot 2 \cdot 70.71 \text{ A} \left(0.12 \text{ Vs} + (0.25 \text{ mH} - 0.40 \text{ mH}) \cdot -70.71 \text{ A}\right) = 27.71 \text{ Nm}.$$

The mechanical power is $P_{me} = T\omega_r$, where $\omega_r = 2\pi f_r$ is the mechanical angular velocity. With $f_r = 50$ Hz, one obtains

$$P_{me} = 27.71 \text{ Nm} \cdot 2\pi \cdot 50 \text{ Hz} = 8.70 \text{ kW}.$$

4.5 Calculate the electrical input power P_{el} and the efficiency η. [3 Points]

Answer:
Flux linkages:

$$\psi_{s,d} = L'_{s,d} i_{s,d} + \psi_{pm} = 0.25 \text{ mH} \cdot -70.71 \text{ A} + 0.12 \text{ Vs} = -0.01768 \text{ Vs} + 0.12 \text{ Vs} = 0.10232 \text{ Vs},$$

$$\psi_{s,q} = L'_{s,q} i_{s,q} = 0.40 \text{ mH} \cdot 70.71 \text{ A} = 0.02828 \text{ Vs}.$$

Voltages in steady state:

$$u_{s,d} = R_s i_{s,d} - \omega_{r,el}\psi_{s,q} = 30 \text{ m}\Omega \cdot -70.71 \text{ A} - 628.32 \tfrac{1}{\text{s}} \cdot 0.02828 \text{ Vs} = -2.12 \text{ V} - 17.77 \text{ V} = -19.89 \text{ V},$$

$$u_{s,q} = R_s i_{s,q} + \omega_{r,el}\psi_{s,d} = 30 \text{ m}\Omega \cdot 70.71 \text{ A} + 628.32 \tfrac{1}{\text{s}} \cdot 0.10232 \text{ Vs} = 2.12 \text{ V} + 64.29 \text{ V} = 66.41 \text{ V}.$$

Electrical input power:

$$P_{\mathrm{el}} = \frac{3}{2}\left(u_{\mathrm{s,d}} i_{\mathrm{s,d}} + u_{\mathrm{s,q}} i_{\mathrm{s,q}}\right) = \frac{3}{2}\left(-19.89\ \mathrm{V} \cdot -70.71\ \mathrm{A} + 66.41\ \mathrm{V} \cdot 70.71\ \mathrm{A}\right) = 9.15\ \mathrm{kW}.$$

Efficiency:

$$\eta = \frac{P_{\mathrm{me}}}{P_{\mathrm{el}}} = \frac{8.70\ \mathrm{kW}}{9.15\ \mathrm{kW}} = 0.951.$$

Consistency check: copper loss $P_{\mathrm{Cu}} = P_{\mathrm{el}} - P_{\mathrm{me}} = 0.45\ \mathrm{kW} = \frac{3}{2} R_{\mathrm{s}}(i_{\mathrm{s,d}}^2 + i_{\mathrm{s,q}}^2)$.

4.6 The PMSM is overheated above the Curie temperature, causing permanent demagnetization of the magnets. Qualitatively discuss the effect on torque and mechanical power at the same operating point. Is electromagnetic energy conversion still possible? [2 Points]

Hint: This question is largely independent of the previous tasks and can be answered based on the general understanding of PMSMs and the torque equation.

Answer:

A total demagnetization reduces the permanent magnet flux linkage to zero $\psi_{\mathrm{pm}} = 0$. Hence, the torque equation simplifies to only the reluctance torque contribution:

$$T = \frac{3}{2} p\, i_{\mathrm{s,q}} \left(\left(L'_{\mathrm{s,d}} - L'_{\mathrm{s,q}}\right) i_{\mathrm{s,d}}\right).$$

Due to the motor's saliency some torque generation and energy conversion is still possible, however, for the above motors parameters, the PM-based torque is dominating for the given operating point, hence a significant reduction of torque and mechanical power is expected.