Exam

Electrical Machines and Drives

Summer 2026

First name: Last name:

Matriculation number: Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 120 minutes.

Evaluation:

Task	1	2	3	4	$\sum$
Maximum score	10	11	12	9	42
Achieved score					

Task 1: Inductive distance measurement **[10 Points]**

An inductive distance measurement system is given in Fig. 1, where the iron core is at a fixed position and the target can be moved. The iron core is excited by the coil, which generates a magnetic field in the core and the air gap. The parameters of the system are given in Tab. 1.

Tab. 1: Parameters of the inductive measurement system.

Symbol	Description	Value
l_x	Air gap length	0.01 m
A_x	Air gap cross section	0.0001 m^2
N	Winding turns	500
μ_0	Magnetic field constant	$4\pi 10^{-7}$ $\frac{\mathrm{Vs}}{\mathrm{Am}}$
J	Current density	3 $\frac{\mathrm{A}}{\mathrm{mm}^2}$

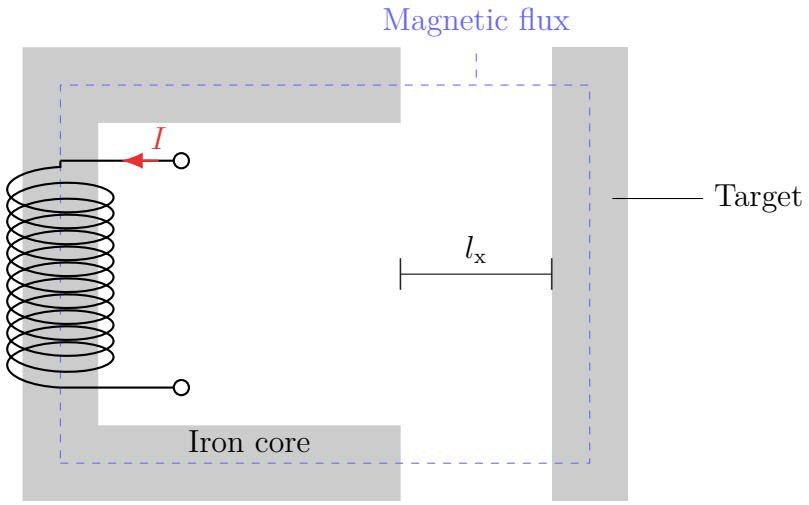


Fig. 1: Inductive distance measurement system.

1.1 Calculate the inductance value L_x for the given air gap length l_x. For simplicity, neglect the reluctances of the iron core and target. [1 Point]

Hint: if and only if you cannot solve this subtask, use $L_\mathrm{x} = 3.14$ mH for the following tasks.

Answer:

The inductance of a coil is given by:

$$L_\mathrm{x} = \frac{N^2}{R_\mathrm{m}} = \frac{N^2}{\frac{2l_\mathrm{x}}{\mu_0 A_\mathrm{x}}} = \frac{N^2 \mu_0 A_\mathrm{x}}{2l_\mathrm{x}} = 1.57 \text{ mH}.$$

1.2 The coil is excited with a sinusoidal voltage (represented as a complex phasor) of $\underline{U} = 10$ V at a frequency of 1 kHz. Neglect the ohmic resistance of the coil and calculate the current $\underline{I}$ through it using complex numbers. [1 Point]

Hint: if and only if you cannot solve this subtask, use $\underline{I} = -2.1\text{j}$ A for the following tasks.

Answer:
The complex resistance is given with

$$\underline{Z}_\text{L} = \text{j}2\pi f L_\text{x} = 9.9\text{j}\ \Omega,$$

resulting in a complex current of:

$$\underline{I} = \frac{\underline{U}}{\underline{Z}_\text{L}} = -1.013\text{j}\ \text{A}.$$

1.3 Calculate the complex magnetic flux linkage $\underline{\Psi}$ of the coil based on the previous subtask's simplifications. [1 Point]

Answer:
The complex flux linkage is given as:

$$\underline{\Psi} = L_\text{x}\underline{I} = -1.6\text{j}\ \text{mVs}.$$

1.4 Use the magnitude of the calculated current $\underline{I}$ from the previous subtask and determine the ohmic winding resistance based on that. Then, calculate the impedance of the coil and the resulting new current I'. The average winding length is given with $l_\text{w} = 0.2$ m and an electrical conductivity of $\kappa_\text{Cu} = 50 \cdot 10^6\ \frac{\text{S}}{\text{m}}$. The permissible current density J for the application is given in Tab. 1. [2 Points]
Hint: if and only if you cannot solve this subtask, use $R = 1.4\ \Omega$ for the following tasks.

Answer:
Given the current density, the required cross-sectional area is calculated as follows:

$$q = \frac{|\underline{I}|}{J} = 0.338\ \text{mm}^2,$$

and the resistance as

$$R = \frac{1}{\kappa_\text{Cu}} \frac{N l_\text{w}}{q} = 5.9\ \Omega,$$

which leads to an absolute impedance of:

$$Z = \sqrt{R^2 + Z_\text{L}^2} = 11.51\ \Omega.$$

Therefore, the current is calculated by:

$$I' = \frac{|\underline{U}|}{Z} = 0.87\ \text{A}.$$

1.5 Calculate the ohmic power loss of the coil with the new current I'. [1 Point]

Answer:
The power loss is given with:

$$P = RI'^2 = 4.5 \text{ W}.$$

1.6 What is the minimum distance to the target? Due to the measurement uncertainty, the coil current should not fall below the minimum measurable current $I_{\min} = 10$ mA. The frequency f and the applied voltage $\underline{U}$ to the coil remain unchanged. [2 Points]

Answer:
With the minimum current the maximum impedance can be calculated with

$$Z_{\max} = \frac{|\underline{U}|}{I_{\min}},$$

which results in the reactance resistance as:

$$X_{\mathrm{L,max}} = \sqrt{Z_{\max}^2 - R^2}.$$

The maximum inductance is given by:

$$L_{\max} = \frac{X_{\mathrm{L,max}}}{2\pi f},$$

which results in a minimum distance of:

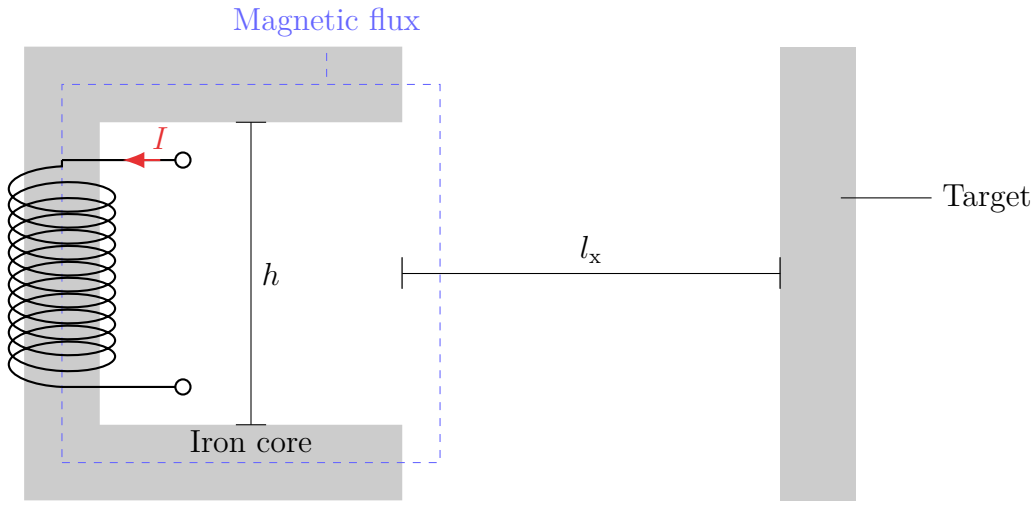
$$l_{\min} = \frac{N^2 \mu_0 A_{\mathrm{x}}}{2L_{\max}} = 98.7 \text{ µm}.$$

1.7 Is the maximum distance to the target limited? If yes, by what? [2 Points]

Answer:
A larger air gap leads to smaller inductance values, which results in larger currents. Therefore, the maximum distance is limited by the maximum current that can be provided by

- the power supply and
- that can be measured by the measurement system.

If the power supply and the measurement system are powerful enough, the maximum distance is limited by the dimensions of the iron core. If the distance from the iron core to the target is larger than the high h of the iron core, the magnetic field lines are no longer closed through the target, but through the iron core and its inner air gap. The corresponding sketch is visualized in Sol.-Fig. 1.



Solution Fig. 1: Inductive distance measurement system with $l_x \gg h$.

Task 2: Regenerative braking with a shunt DC drive **[11 Points]**

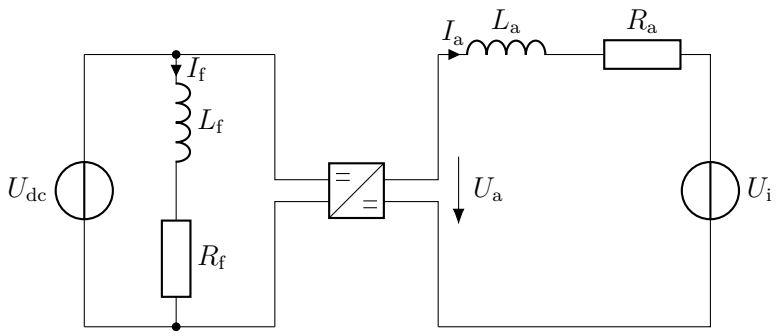
A shunt DC machine is connected to a 240 V DC link. The field winding is supplied directly from the DC link. A lossless four-quadrant DC/DC converter supplies the armature from the DC link. Positive armature power $P_\mathrm{a} = U_\mathrm{a} I_\mathrm{a}$ enters the machine. Use the parameters in Tab. 2. Neglect mechanical, iron, and converter losses. During braking, I_f and I_a are constant.

Tab. 2: Parameters of the DC drive.

Symbol	Description	Value
U_dc	DC-link voltage	240 V
R_a	Armature resistance	0.5 Ω
L_a	Armature inductance	15 mH
R_f	Field resistance	120 Ω
L'_f	Effective field inductance	0.6 H
J	Effective inertia	2.0 kgm^2

2.1 Draw the described drive layout incl. the equivalent circuit diagrams of the field winding and the armature (the DC/DC converter's internal structure shall not be sketched). [1 Point]

Answer:



Solution Fig. 2: DC shunt drive

2.2 Calculate the steady-state field current I_f and the effective flux ψ'_f. [1 Point]
Hint: if and only if you cannot solve this subtask, use $I_\mathrm{f} = 1$ A and $\psi'_\mathrm{f} = 0.6$ Vs in the next subtasks.

Answer:
The field branch is connected directly to the DC link. Hence,

$$I_\mathrm{f} = \frac{U_\mathrm{dc}}{R_\mathrm{f}} = 2 \text{ A}, \qquad \psi'_\mathrm{f} = L'_\mathrm{f} I_\mathrm{f} = 1.2 \text{ Vs}.$$

2.3 At $n = 1500$ $^1/_\mathrm{min}$, the converter sets $I_\mathrm{a} = -20$ A. What armature voltage U_a is required to maintain this current? What is the resulting torque T? [2 Points]

Hint: if and only if you cannot solve this subtask, use $U_\mathrm{a} = 150$ V and $T = -20$ Nm in the next subtasks.

Answer:

$$\omega = \frac{2\pi n}{60}\,\frac{\mathrm{min}}{\mathrm{s}} = 157.08\ \frac{1}{\mathrm{s}}, \qquad U_\mathrm{i} = \omega\psi_\mathrm{f}' = 188.50\ \mathrm{V},$$
$$U_\mathrm{a} = R_\mathrm{a} I_\mathrm{a} + U_\mathrm{i} = 178.50\ \mathrm{V}, \qquad T = \psi_\mathrm{f}' I_\mathrm{a} = -24\ \mathrm{Nm}.$$

2.4 Determine the resulting armature power P_a, mechanical power P_me, the total copper losses P_Cu as well as the resulting efficiency η for the given operating point. [2 Points]

Answer:
The relevant power terms are

$$P_\mathrm{a} = U_\mathrm{a} I_\mathrm{a} = -3.57\ \mathrm{kW}, \quad P_\mathrm{me} = T\omega = -3.77\ \mathrm{kW}, \quad P_\mathrm{Cu} = R_\mathrm{a} I_\mathrm{a}^2 + R_\mathrm{f} I_\mathrm{f}^2 = 200\ \mathrm{W} + 480\ \mathrm{W} = 680\ \mathrm{W},$$

resulting in an efficiency of

$$\eta = \frac{P_\mathrm{a} + P_\mathrm{Cu,f}}{P_\mathrm{me}} = \frac{-3.57\ \mathrm{kW} + 0.48\ \mathrm{kW}}{-3.77\ \mathrm{kW}} = \frac{-3.09\ \mathrm{kW}}{-3.77\ \mathrm{kW}} = 82.0\ \%.$$

Here, it must be considered that the drive is operating in generation mode, so that the mechanical power is negative. The copper losses of the field winding are added to the armature power, since they are supplied from the DC link.

2.5 During a stopping procedure, the speed falls from $n_1 = 1500\ {}^1/_\mathrm{min}$ to $n_2 = 0\ {}^1/_\mathrm{min}$ at constant $I_\mathrm{a} = -20$ A. Assuming zero load torque during the procedure, the machine's torque changes the speed according to

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = T.$$

What time Δt does it take to stop and what is the resulting kinetic energy change ΔE_kin? [2 Points]
Hint: if and only if you cannot solve this subtask, use $\Delta t = 10$ s and $\Delta E_\mathrm{kin} = -30$ kJ in the next subtask.

Answer:
Since $T = -24$ Nm is constant,

$$\Delta t = \frac{J(\omega_2 - \omega_1)}{T} = 13.09\ \mathrm{s}.$$

The kinetic-energy decrease is

$$\Delta E_\mathrm{kin} = \frac{J}{2}\left(\omega_2^2 - \omega_1^2\right) = -24.67\ \mathrm{kJ}.$$

2.6 How much electrical energy E_rec is recovered during the procedure? [1 Point]

Answer:
The loss work during the stopping procedure is

$$W_{\mathrm{Cu}} = P_{\mathrm{Cu}} \Delta t = 680\ \mathrm{W} \cdot 13.09\ \mathrm{s} = 8.89\ \mathrm{kJ}.$$

The recovered energy is then

$$E_{\mathrm{rec}} = |\Delta E_{\mathrm{kin}}| - W_{\mathrm{Cu}} = 24.67\ \mathrm{kJ} - 8.89\ \mathrm{kJ} = 15.78\ \mathrm{kJ}.$$

2.7 Calculate the maximum achievable speed n_{max} with the given drive setup considering the electrical constraints and neglecting mechanical limitations. What component could be added to the drive to extend its speed range? [2 Points]

Answer:
At no load, $I_{\mathrm{a}} = 0$ and $U_{\mathrm{a}} = U_{\mathrm{i}} = \omega \psi_{\mathrm{f}}'$. Hence,

$$\omega_{\mathrm{max}} = \frac{U_{\mathrm{dc}}}{\psi_{\mathrm{f}}'} = 200\ \tfrac{1}{\mathrm{s}}, \qquad n_{\mathrm{max}} = \omega_{\mathrm{max}} \frac{60}{2\pi} = 1909.9\ \tfrac{1}{\mathrm{min}}.$$

A controllable DC/DC converter could be added to the field circuit to reduce I_{f} and ψ_{f}' above base speed. The available torque then decreases.

Task 3: Three-phase stator winding design for harmonic suppression **[12 Points]**

A two-pole, three-phase stator has $Q = 18$ uniformly spaced slots and is supplied at $f = 50$ Hz. Assuming a single-layer winding: each slot contains one coil side and all nine coils have the same integer span y_s. The phase sequence is *a*-*b*-*c*. The design goal is to minimize the fifth space harmonic while maintaining a high fundamental winding factor. The relative reduction of the k-th harmonic r_k is given by

$$r_k = \frac{|\hat{B}^{(k)}|}{|\hat{B}^{(1)}|} = \frac{|\xi_{\mathrm{w},k}|}{k|\xi_{\mathrm{w},1}|}.$$

3.1 Determine the direction and synchronous mechanical speed of the fifth and seventh flux space harmonics, i.e., the mechanical speed at which the corresponding harmonic rotates synchronously with the fundamental excitation frequency. In addition, name at least two adverse effects of harmonics on the machine operation. [2 Points]

Answer:

$$n_1 = \frac{f}{p} \cdot 60 \ \tfrac{\mathrm{s}}{\mathrm{min}} = 3000 \ \tfrac{1}{\mathrm{min}}.$$

The fifth harmonic is negative sequence and the seventh is positive sequence:

$$n_5 = -\frac{n_1}{5} = -600 \ \tfrac{1}{\mathrm{min}}, \qquad n_7 = \frac{n_1}{7} = 428.6 \ \tfrac{1}{\mathrm{min}}.$$

Possible adverse effects are current harmonics, torque harmonics, additional losses, acoustic noise, and vibration.

3.2 Determine the pole-pair number p, the number of slots per phase and pole q as well as the pole pitch ρ_p [3 Points]
Hint: if and only if you are not able to solve this task, make reasonable assumptions for the addressed parameters in the following subtasks.

Answer:
Two poles give $p = 1$. For a three-phase winding with $m = 3$ we further have

$$q = \frac{Q}{2pm} = \frac{18}{2 \cdot 1 \cdot 3} = 3.$$

The pole pitch in number of slots is then

$$\rho_\mathrm{p} = \frac{Q}{2p} = \frac{18}{2 \cdot 1} = 9,$$

which corresponds to $\rho_\mathrm{p} = 180°$ in electrical degrees.

3.3 Check whether an integer coil span can cancel the fifth harmonic exactly in this stator. Explain the result. [2 Points]

Answer:

For $q = 3$, the distribution factor for the fifth harmonic is

$$\xi_{\mathrm{d},5} = \frac{\sin(5\pi/6)}{3\sin(5\pi/18)} = 0.2176 \neq 0.$$

Exact cancellation therefore requires a pitch factor of $\xi_{\mathrm{p},5} = 0$:

$$\sin\left(\frac{5\pi}{2}\frac{y}{9}\right) = 0 \quad \Rightarrow \quad y = \frac{18r}{5}, \qquad r \in \mathbb{Z}.$$

Within one pole pitch, the target values are 3.6 and 7.2 slots. Neither is an integer. Exact cancellation is impossible with a uniform integer-span winding in this stator.

3.4 Subject to $|\xi_{\mathrm{w},1}| \geq 0.8$, compare the practical single-layer spans $y_\mathrm{s} = 5, 7, 9$ for the $Q = 18$ stator design and select the span with the smallest r_5. [2 Points]

Answer:

y_s	$\lvert\xi_{\mathrm{w},1}\rvert$	$\lvert\xi_{\mathrm{w},5}\rvert$	r_5
5	0.7352	0.2044	5.56 %
7	0.9019	0.0378	0.84 %
9	0.9598	0.2176	4.53 %

The best realizable choice is

$$y_\mathrm{s} = 7.$$

3.5 Complete the single-layer winding layout in Fig. 2. Enter phase and current direction in every slot. Use the phase sequence *a*-*b*-*c* and $y_\mathrm{s} = 7$. The end-winding connections are not required to be sketched; a valid symmetric layout is sufficient. [3 Points]

Answer:

For phase a, the positive conductor side a^+ in slot 1 is given. With the selected coil span $y = 7$, the corresponding negative coil side is located seven slots further:

$$1 : a^+ \leftrightarrow 8 : a^-.$$

Since $q = 3$, phase a consists of three distributed coils. The corresponding coil phasors are displaced by one electrical slot angle. Starting from the first coil, the two remaining phase-a coils are therefore chosen as

$$16 : a^+ \leftrightarrow 9 : a^-, \qquad 17 : a^+ \leftrightarrow 6 : a^-.$$

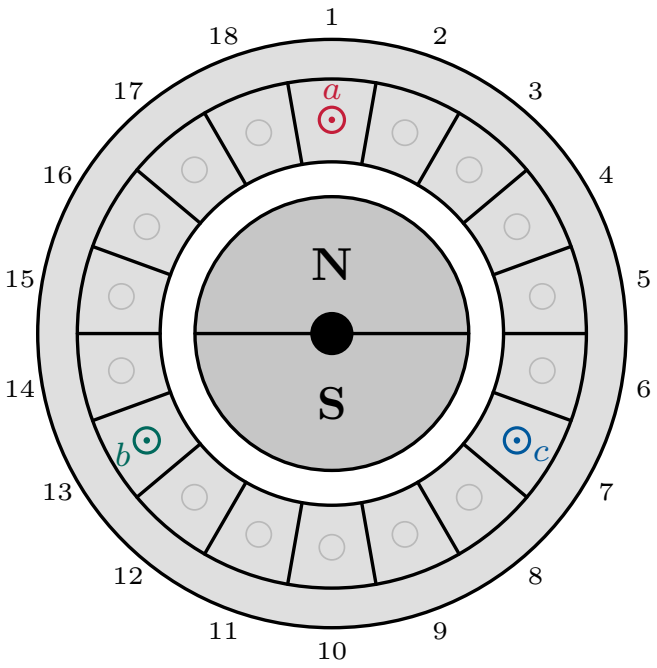


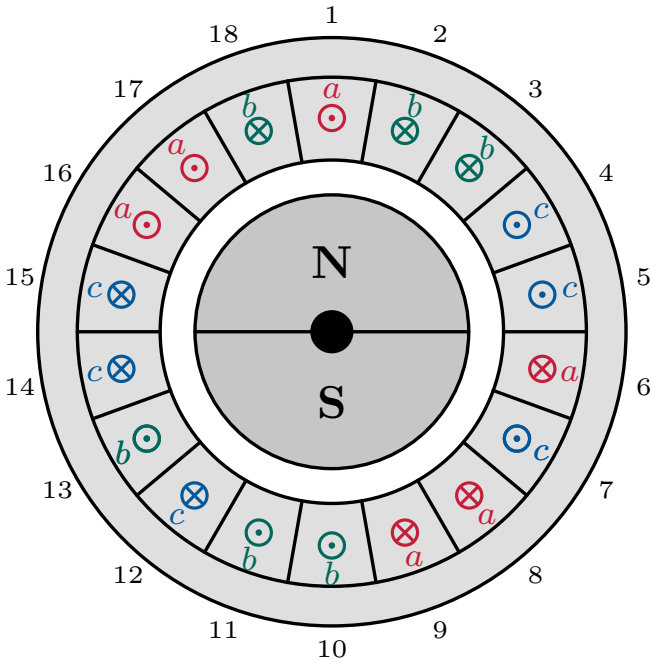
Fig. 2: Winding-layout template with given starting conductors per phase.

Thus, the complete phase-a winding is

$$1 : a^+,\ 6 : a^-,\ 8 : a^-,\ 9 : a^-,\ 16 : a^+,\ 17 : a^+.$$

The phases b and c are constructed analogously by applying the required 120° electrical phase displacement to the complete phase-a winding. This corresponds to a shift of six slots. Using the given reference conductors b^+ in slot 13 and c^+ in slot 7 yields the complete symmetric three-phase single-layer winding. Hence, the results are shown in the following table and in Sol.-Fig. 3.

phase	coil 1	coil 2	coil 3
a	$1^+ - 8^-$	$16^+ - 9^-$	$17^+ - 6^-$
b	$10^+ - 3^-$	$11^+ - 18^-$	$13^+ - 2^-$
c	$4^+ - 15^-$	$5^+ - 12^-$	$7^+ - 14^-$



Solution Fig. 3: Completed 18-slot single-layer winding layout.

Task 4: Induction machine **[9 Points]**

Given is a three-phase squirrel cage induction machine with the following parameters in Tab. 3.

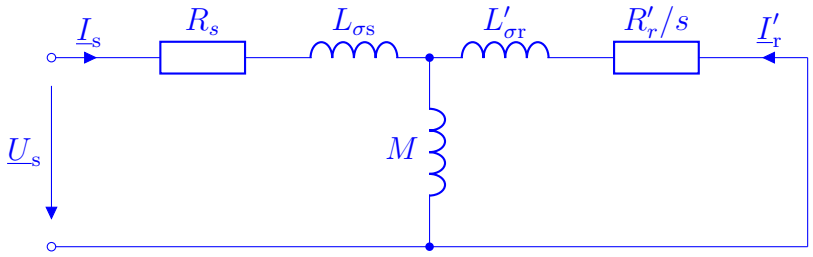
Tab. 3: Parameters of the induction machine.

Symbol	Description	Value
U_{LL}	Line-to-line voltage	400 V
$f_{\mathrm{s,n}}$	Nominal stator frequency	50 Hz
n_{n}	Nominal rotor speed	926 $\frac{1}{\mathrm{min}}$
T_{n}	Nominal torque	25 Nm

4.1 Draw the equivalent circuit diagram of the induction machine in steady state and label its components. [2 Points]

Answer:

The equivalent circuit diagram of the induction machine in steady state is shown in Sol.-Fig. 4.



Solution Fig. 4: Equivalent circuit diagram of the induction machine in steady state.

4.2 Calculate the number of pole pairs and the synchronous (no-load) speed of the machine. [1 Point]

Answer:

The stator frequency is given with $f_{\mathrm{s,n}} = 50$ Hz, which leads to a synchronous speed of:

$$n_{\mathrm{syn,el}} = f_{\mathrm{s,n}} \cdot 60 \ \frac{\mathrm{s}}{\mathrm{min}} = 3000 \ \frac{1}{\mathrm{min}}.$$

The nominal speed is given with $n_{\mathrm{n}} = 926 \ \frac{1}{\mathrm{min}}$, which is associated with a synchronous speed of $n_{\mathrm{syn,mech}} = 1000 \ \frac{1}{\mathrm{min}}$. Therefore, the pole pair number results in:

$$p = \frac{n_{\mathrm{syn,el}}}{n_{\mathrm{syn,mech}}} = 3.$$

4.3 Determine the slip frequency and the slip ratio at the nominal operating point. [1 Point]

Answer:

The slip frequency is calculated with

$$\omega_{\mathrm{slip,n}} = \omega_{\mathrm{s,n}} - \omega_{\mathrm{r,el}} = 23.2\ \tfrac{1}{\mathrm{s}},$$

which leads to the slip ratio as:

$$\omega_{\mathrm{slip,n}} = s\omega_{\mathrm{s,n}} \Rightarrow s_{\mathrm{n}} = \frac{\omega_{\mathrm{slip,n}}}{\omega_{\mathrm{s,n}}} = 0.074.$$

4.4 Fig. 3 shows the quadratic load curve for a pump application (green) and the torque curve of the induction motor (orange). It is assumed that the induction motor operates in steady state with the same values for U_{s} and ω_{s} as in the previous subtasks. Determine the torque and speed of the machine at the resulting operating point. What is the normalized slip at this point? [2 Points]

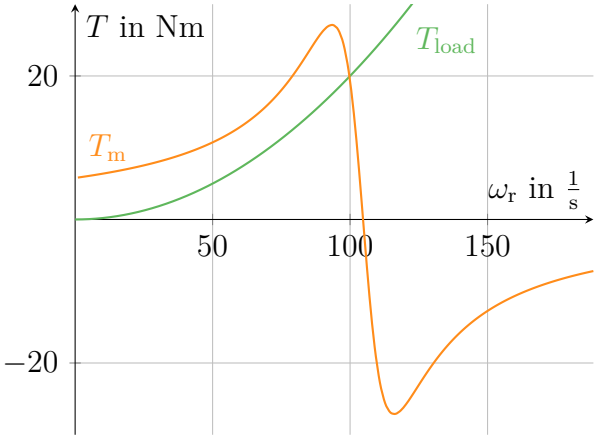


Fig. 3: Visualization of the quadratic load curve in green and the IM torque curve in orange.

Answer:

The operating point is determined at the intersection of the load curve and the torque curve. Thus,

$$T = 20\ \mathrm{Nm}, \quad \omega_{\mathrm{r}} = 100\ \tfrac{1}{\mathrm{s}},$$

and, the slip frequency and normalized slip are calculated as follows:

$$\omega_{\mathrm{slip}} = \omega_{\mathrm{s}} - \omega_{\mathrm{r,el}} = 14.2\ \tfrac{1}{\mathrm{s}}, \quad s = \frac{\omega_{\mathrm{slip}}}{\omega_{\mathrm{s}}} = 0.045.$$

4.5 Assuming that the pump drive is operated via a voltage source inverter operating at its maximum output voltage (i.e., U_{s} is fixed), what can be done to increase the drive's speed beyond the above operating point? What is the resulting effect on the torque? [2 Points]

Answer:

To accelerate the machine further, the stator frequency must be increased and due to the limited (DC-link) voltage, flux weakening must be applied. During flux weakening, the flux decreases with $\Psi_{\mathrm{s}} \sim 1/\omega_{\mathrm{s}}$, which leads to a decrease in torque with $T_{\max} \sim 1/\omega_{\mathrm{s}}^2$.

4.6 Sketch the torque-speed characteristic of the induction machine for the operating point at maximum speed and $s = s_{\max}$ for the given load curve in Fig. 4. [1 Point]
Hint: a qualitative sketch that captures the essential features is sufficient.

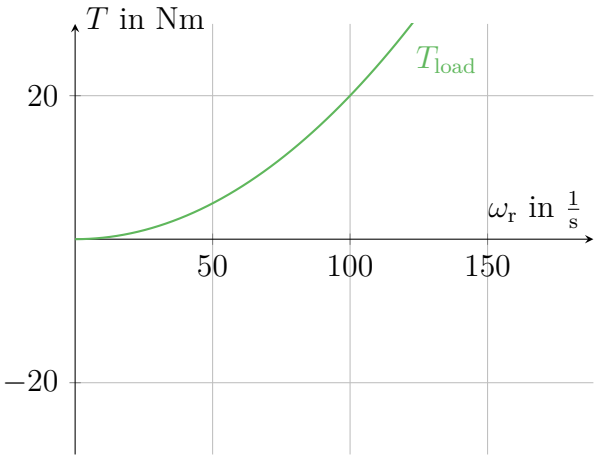
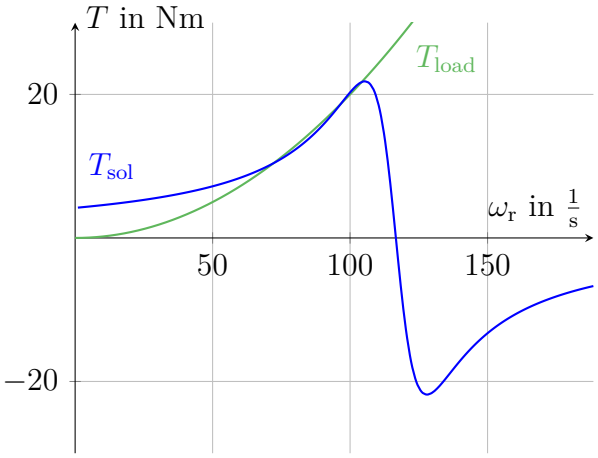


Fig. 4: Given quadratic load curve in green.

Answer:
At slip $s = s_{\max}$ the maximum torque is reached. Therefore, the maximum torque must have an intersection with the load curve as shown in Sol.-Fig. 5.



Solution Fig. 5: Maximum speed at $s = s_{\max}$ for the given operating point.