

Exam

Electrical Machines and Drives

Summer 2025

First name:

Last name:

Matriculation number:

Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 90 minutes.

Evaluation:

Task	1	2	3	4	Σ
Maximum score	10	9	9	14	42
Achieved score					

Task 1: Electromagnetic behavior of an EI-150 core

[10 Points]

A standardized EI-150 core made of electrical steel sheet with the geometry from Fig. 1 is to be analyzed. Its air gap length is $\delta = 0.8$ mm and all parts have a depth of 50 mm. A coil with $N = 500$ turns around the middle yoke of the core leads to a magnetic flux density of $B_{\text{Fe}} = 1.2$ T in this yoke (at rated current). The magnetization curve of electrical steel is also depicted in Fig. 1.

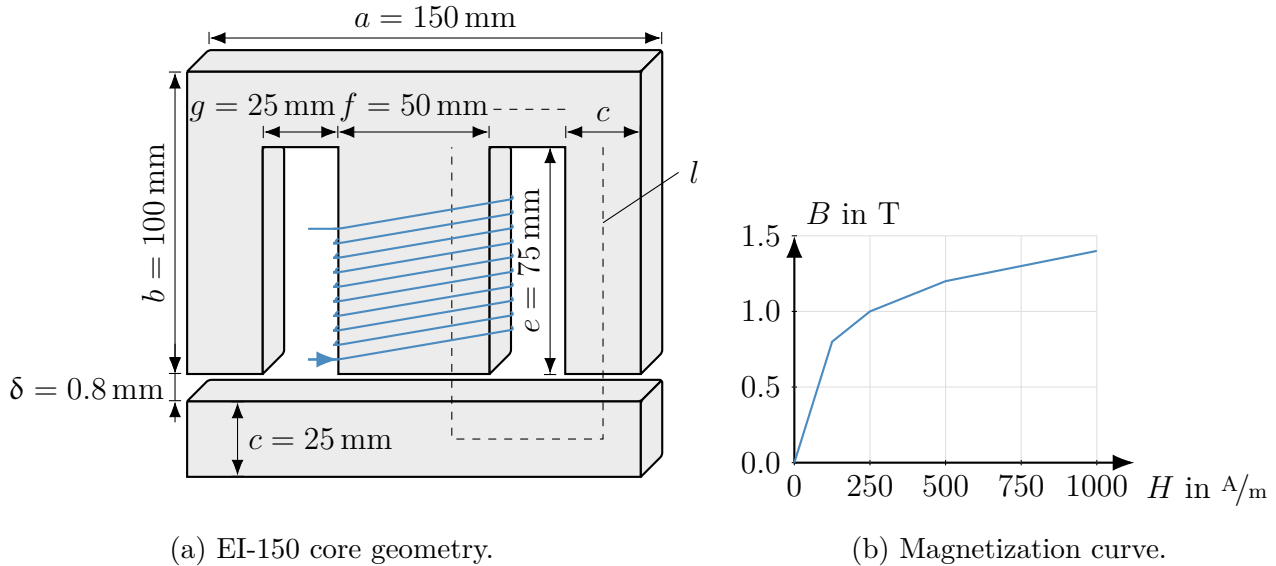


Fig. 1: EI-150 core and electrical steel magnetization curve.

1.1 Calculate the magnetic flux ϕ_{Fe} in the middle yoke.

[2 Points]

Answer:

The area of the middle yoke according to the figure and text description is given by

$$A_{\text{Fe}} = (50 \cdot 50) \text{ mm}^2 = 2500 \text{ mm}^2,$$

and, thus the flux results in:

$$\phi_{\text{Fe}} = B_{\text{Fe}} A_{\text{Fe}} = 1.2 \text{ T} \cdot 2500 \text{ mm}^2 = 3.0 \text{ mWb}.$$

1.2 How large is the magnetic flux density B_{δ} in the middle yoke's air gap, when all leakage fluxes are neglected and there is no expansion of the magnetic field lines in the air gap?

[2 Points]

Hint: if and only if you are not able to solve this subtask, use $B_{\delta} = 1.0$ T as a substitute result for the following questions.

Answer:

Without leakage fluxes, the magnetic flux in the core is identical to the magnetic flux in the air gap. Therefore,

$$\phi_{\text{Fe}} = \phi_{\delta} = B_{\text{Fe}} A_{\text{Fe}} = B_{\delta} A_{\delta},$$

and without the expansion of the magnetic field lines in the air gap both areas are identical, which results in:

$$B_{\delta} = B_{\text{Fe}} \frac{A_{\text{Fe}}}{A_{\delta}} = 1.2 \text{ T} \cdot 1 = 1.2 \text{ T}.$$

1.3 Determine the electrical steel sheet permeability μ_{Fe} as well as the magnetic field strength in both the air gap H_{δ} and in the electrical steel sheet H_{Fe} . Use $\mu_0 = 4\pi 10^{-7} \frac{\text{Vs}}{\text{Am}}$ for the magnetic field constant. [2 Points]

Answer:

The magnetic field strength in the air gap is given with

$$H_{\delta} = \frac{B_{\delta}}{\mu_0} = \frac{1.2 \text{ T}}{4\pi 10^{-7} \frac{\text{Vs}}{\text{m}}} = 954930 \frac{\text{A}}{\text{m}}.$$

The magnetic field strength H_{Fe} in the electrical steel sheet is determined with the magnetization curve in Fig. 1. Thus,

$$\mu_{\text{Fe}} = \frac{B_{\text{Fe}}}{H_{\text{Fe}}} = \frac{1.2 \text{ T}}{500 \frac{\text{A}}{\text{m}}} = 0.0024 \frac{\text{Vs}}{\text{Am}}$$

results.

1.4 Calculate the magnetomotive force θ along an idealized, closed field line through the middle and one outer yoke (cf. the field line contour l in Fig. 1a). [2 Points]

Answer:

The split magnetic circuit is symmetrical, therefore, it can be treated like an unsplit magnetic circuit and only one half needs to be taken into account. The magnetomotive force is given by

$$\theta = H_{\text{Fe}} l_{\text{Fe}} + H_{\delta} l_{\delta} = H_{\text{Fe}} (l_{\text{I}} + l_{\text{E}}) + H_{\delta} l_{\delta},$$

with

$$l_{\text{E}} = 2e + g + 2c = 2 \cdot 75 \text{ mm} + 25 \text{ mm} + 2 \cdot 25 \text{ mm} = 225 \text{ mm},$$

and

$$l_{\text{I}} = g + 2c = 25 \text{ mm} + 2 \cdot 25 \text{ mm} = 75 \text{ mm},$$

and $l_{\delta} = 1.6 \text{ mm}$. This results in:

$$\theta = 500 \frac{\text{A}}{\text{m}} \cdot 0.3 \text{ m} + 954930 \frac{\text{A}}{\text{m}} \cdot 0.0016 \text{ m} = 1677.9 \text{ A}.$$

1.5 After the I core has been connected to the E core, only a parasitic air gap of $\delta = 0.1 \text{ mm}$ remains. What is the necessary current I'_{n} to produce the same magnetic field density in the middle yoke compared to the previous configuration? [2 Points]

Answer:

With the magnetomotive force from the previous task

$$\theta = H_{\text{Fe}} l_{\text{Fe}} + H_{\delta} l_{\delta}$$

and

$$\theta = NI$$

the necessary current (for the initial air gap length) results in:

$$I_{\text{n}} = \frac{\theta}{N} = \frac{1677.9 \text{ A}}{500} = 3.4 \text{ A}.$$

With the reduced air gap the magnetomotive force changed to

$$\theta' = 500 \frac{\text{A}}{\text{m}} \cdot 0.3 \text{ m} + 954930 \frac{\text{A}}{\text{m}} \cdot 2 \cdot 0.0001 \text{ m} = 341 \text{ A},$$

which results in a necessary current of

$$I'_{\text{n}} = \frac{\theta'}{N} = \frac{341 \text{ A}}{500} = 0.68 \text{ A},$$

thus, less current is needed.

Task 2: Permanent magnet DC machine

[9 Points]

Permanent magnet DC (PMDC) machines are often used in low power applications, such as machine tools or micromobility. In those applications, low voltage ratings are common to ensure safety, in particular regarding touch protection. In the following, you will analyze the operation behavior of a PMDC machine with the parameters given in Tab. 1 for an electric scooter application. A gearbox for torque and speed adaptation can be neglected.

Tab. 1: PMDC machine parameters.

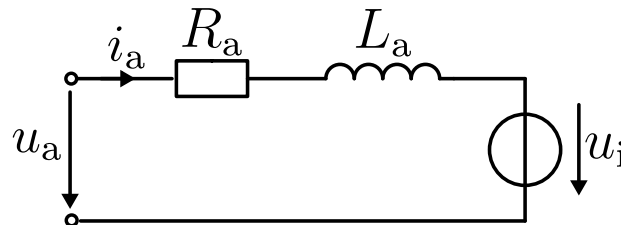
Symbol	Description	Values
U_n	Nominal voltage	48 V
I_n	Nominal current	5.75 A
T_n	Nominal torque	1.67 Nm
$P_{me,n}$	Nominal mech. power	200 W
R_a	Armature resistance	2.4 Ω
L_a	Armature inductance	3.7 H

2.1 Draw the equivalent circuit diagram of a PMDC machine.

[1 Point]

Answer:

Since the PMDC machine does not have a distinct field winding, the equivalent circuit diagram boils down to the armature circuit as shown in Sol.-Fig. 1.



Solution Fig. 1: Equivalent circuit diagram of the PMDC machine.

2.2 Determine the effective PM flux linkage ψ'_f for the nominal steady-state operating point. [1 Point]

Hint: if and if only you are not able to solve this task, use $\psi'_f = 0.4$ Vs as a substitute result for the subsequent tasks.

Answer:

The effective PM flux linkage can be derived from the machine's nominal torque and current:

$$\psi'_{f,n} = \frac{T_n}{I_n} = 0.29 \text{ Vs.}$$

2.3 What is the machine's nominal speed n_n and efficiency η_n ?

[2 Points]

Answer:

First, the induced voltage is calculated via

$$U_{i,n} = U_n - R_a I_n = 34.2 \text{ V}.$$

The nominal angular velocity is

$$\omega_n = \frac{U_{i,n}}{\psi'_{f,n}} = 117.9 \frac{1}{s}$$

resulting in

$$n_n = \omega_n \frac{60}{2\pi} \frac{s}{\min} = 1126.2 \frac{1}{\min}.$$

For determining the nominal efficiency, we first calculate the nominal electrical power

$$P_{el,n} = U_n I_n = 276 \text{ W}$$

and then use the already known nominal mechanical power to receive

$$\eta_n = \frac{P_{me,n}}{P_{el,n}} = 72.46 \text{ \%}.$$

2.4 Assume that the scooter's load torque T_L is quadratically depending on the speed since the air drag is dominant. This is represented via the equation below utilizing the friction coefficient b with:

$$T_L(\omega) = b\omega^2 \quad \text{with} \quad b = 0.001 \text{ Nms}^2.$$

Calculate the resulting operating point assuming the scooter drive is powered with the fixed, nominal voltage. [3 Points]

Answer:

The yet unknown operation point is determined by the intersection of the load torque and the machine's torque, which is given by

$$T(\omega) = I_a \psi'_f = \frac{U_n - \omega \psi'_f}{R_a} \psi'_f \stackrel{!}{=} b\omega^2 = T_L(\omega).$$

Resorting delivers a quadratic equation in ω :

$$b\omega^2 + \frac{\psi'^2_f}{R_a} \omega - \frac{\psi'_f U_n}{R_a} = 0.$$

The possible solutions of this equation are given by

$$\omega_{1,2} = -\frac{\psi'^2_f}{2bR_a} \pm \sqrt{\left(\frac{\psi'^2_f}{2bR_a}\right)^2 + \frac{\psi'_f U_n}{bR_a}}.$$

The positive solution is the relevant one, which results in

$$\omega = 60.63 \frac{1}{s}, \quad n = 578.94 \frac{1}{\min}.$$

The corresponding load torque is given by

$$T_L = b\omega^2 = 3.68 \text{ Nm}.$$

The machine's armature current is calculated by

$$I_a = \frac{U_n - \omega\psi'_f}{R_a} = 12.67 \text{ A}.$$

2.5 What is the theoretical maximum speed of the PMDC machine if unloaded? Discuss why this maximum speed is an inherent limitation of the PMDC machine. [2 Points]

Answer:

During no-load operation, the armature current is zero, which results in the induced voltage being equal to the nominal voltage:

$$U_i = U_n = \omega\psi'_f.$$

Rearranging this equation delivers the theoretical maximum speed:

$$\omega_{\max} = \frac{U_n}{\psi'_f} = 165.52 \frac{1}{s}, \quad n_{\max} = 1580.57 \frac{1}{\min}.$$

The maximum speed is an inherent limitation of the PMDC machine since the induced voltage is proportional to the speed and the flux linkage. If the speed exceeds this limit, the induced voltage would exceed the nominal voltage, which is not possible in practice. Also, a field weakening is not possible due to the permanent magnet excitation, i.e., the flux linkage is constant and cannot be reduced to increase the speed.

Task 3: Winding factors

[9 Points]

The cross-section of a permanent magnet synchronous machine (PMSM) is given in Fig. 2 with the parameters from Tab. 2. All windings per phase are connected in parallel ($a = 2$).

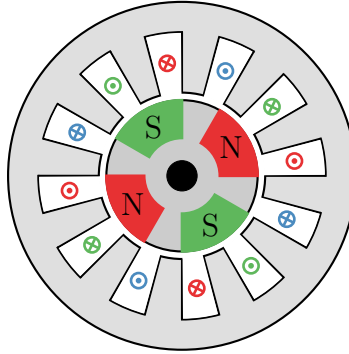


Fig. 2: Sketch of the utilized PMSM.

Tab. 2: Parameters of the PMSM.

Symbol	Value	Symbol	Value
l_z	0.45 m	d_{si}	0.3 m
n_n	2500 $1/\text{min}$	N_c	5 windings per coil

3.1 What is the number of slots Q , the number of phases m and the number of notches q ? In addition, calculate the pole pitch ρ_p . [2 Points]

Answer:

The number of slots can be counted and is $Q = 12$. The number of phases can be seen at the different colors and distribution of the stator winding, thus $m = 3$. The number of notches results in:

$$q = \frac{Q}{2pm} = \frac{12}{2 \cdot 2 \cdot 3} = 1.$$

In addition, the pole pitch is calculated by:

$$\rho_p = \frac{\pi}{p} = \frac{\pi}{2}.$$

3.2 Calculate the winding factor $\xi_{w,k}$ for the fundamental wave ($k = 1$). [2 Points]

Answer:

The winding factor is defined as

$$\xi_{w,k} = \xi_{d,k} \xi_{p,k},$$

with $k = 1$ for the fundamental wave. The distribution factor is calculated by

$$\xi_{d,1} = \frac{\sin\left(\frac{\pi}{2m}\right)}{q \sin\left(\frac{\pi}{2mq}\right)} = \frac{\sin\left(\frac{\pi}{2 \cdot 3}\right)}{1 \cdot \sin\left(\frac{\pi}{2 \cdot 3 \cdot 1}\right)} = 1$$

and, the pitch factor is

$$\xi_{p,1} = \sin\left(\frac{\pi}{2} \frac{y}{\rho_p}\right) = \sin\left(\frac{\pi}{2} \cdot \frac{3}{\pi/2}\right) = 0.14,$$

resulting in:

$$\xi_{w,1} = 1 \cdot 0.14 = 0.14.$$

3.3 The air gap flux density is given with $\hat{B}_\delta^{(1)} = 1.2$ T. Calculate the flux per pole ϕ_δ and the flux linkage ψ_{phase} . [4 Points]

Hint: if and only if you are not able to solve this task, use $\psi_{\text{phase}} = 0.5$ Vs as a substitute result for the following questions.

Answer:

The pole pitch as a distance in meter is calculated by

$$\tau_p = \frac{d_{s,i}\pi}{2p} = \frac{0.3 \text{ m} \cdot \pi}{2 \cdot 2} = 0.236 \text{ m},$$

and the effective cross-sectional area of the machine per pole is determined with

$$A_\delta = \tau_p l_z = 0.236 \text{ m} \cdot 0.45 \text{ m} = 0.106 \text{ m}^2.$$

Hence, air gap flux is given with

$$\phi_\delta = A_\delta \hat{B}_\delta^{(1)} = 0.106 \text{ m}^2 \cdot 1.2 \text{ T} = 0.127 \text{ Vs},$$

and the number of windings per phase is defined by

$$N_{\text{phase}} = \frac{2pqN_c}{a} = \frac{2 \cdot 2 \cdot 1 \cdot 5}{2} = 10,$$

resulting in the flux linkage

$$\psi_{\text{phase}} = \phi_\delta N_{\text{phase}} \xi_{w,1} = 0.127 \text{ Vs} \cdot 10 \cdot 0.14 = 0.18 \text{ Vs}.$$

3.4 Calculate fundamental component of the induced voltage $U_{i,\text{phase}}$ for one phase. [1 Point]

Answer:

The electrical angular frequency is defined by

$$\omega_{\text{el}} = 2\pi f_{\text{mech}} p = 2\pi \cdot \frac{2500}{60} \frac{1}{\text{s}} \cdot 2 = 523.6 \frac{1}{\text{s}},$$

and therefore, the induced voltage is:

$$U_{\text{i,phase}} = \omega_{\text{el}} \psi_{\text{phase}} = 523.6 \frac{1}{\text{s}} \cdot 0.18 \text{ Vs} = 94.2 \text{ V}.$$

Task 4: Synchronous machine

[14 Points]

A four-pole synchronous machine operates as a generator in a power plant and supplies 17 MW of active power to the power grid. The grid frequency is 50 Hz. The stator ohmic winding resistance of the synchronous machine is negligible and the excitation current is set to a constant value of 100 A. Fig. 3 shows the phasor diagram of the synchronous machine at a given operating point.

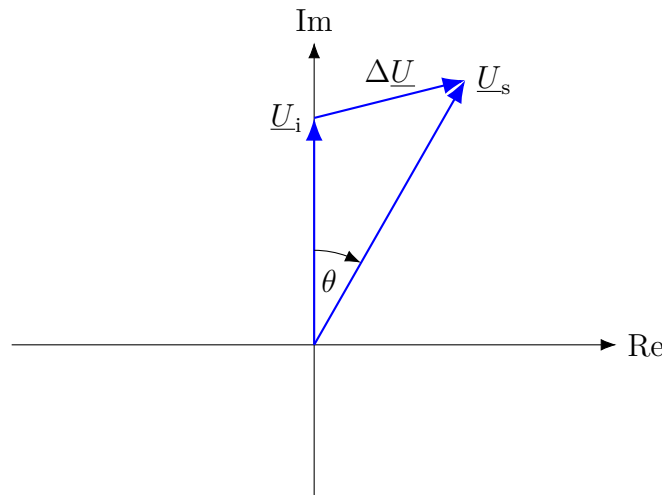


Fig. 3: Phasor diagram of a synchronous machine with scaling 1 kV = 1 cm and 1 kA = 1 cm.

4.1 Determine the operating mode of the synchronous machine based on the phasor diagram. [1 Point]

Answer:

Since the load angle θ is negative, the synchronous machine operates as a generator (i.e., feeds active power into the grid). The induced voltage U_i amplitude is larger than the stator voltage amplitude U_s , which is characteristic for an under excited generator which draws reactive power from the grid.

4.2 Determine the short-circuit current I_{sc} and add its phasor \underline{I}_{sc} to Fig. 3. [2 Points]

Hint: if and if only you are not able to solve this task, assume $I_{sc} = 3$ kA as a substitute result.

Answer:

The active power is calculated by

$$P = 3U_s I_s \cos(\varphi)$$

with φ being the phase angle between the stator voltage and stator current phasors. Hence, $I_s \cos(\varphi)$ determines the active current component which is in line with the stator voltage phasor. The stator current phasor is (neglecting the ohmic stator resistance):

$$\underline{I}_s = \frac{\underline{U}_s}{jX_s} - \frac{\underline{U}_i}{jX_s} = -j\frac{\underline{U}_s}{X_s} + j\frac{\underline{U}_i}{X_s}.$$

The first component of \underline{I}_s is orthogonal to the stator voltage phasor and, therefore, does not contribute

to the active power. The second component is relevant for the active power and can be rewritten as

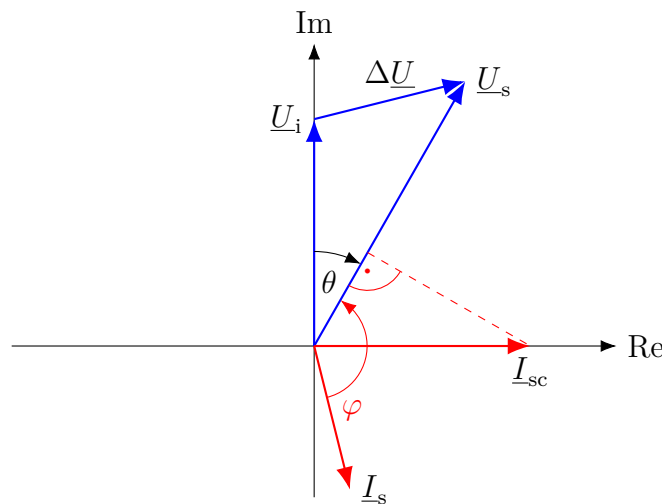
$$j \frac{U_i}{X_s} = -\underline{I}_{sc},$$

i.e., is the negative short-circuit current phasor. Hence, the short-circuit current is lagging by $\pi/2$ to the inner voltage phasor and needs to be aligned with the real axis given the inner voltage phasor orientation with the imaginary axis. Applying some trigonometric identities to Sol.-Fig. 2 it can be found that the fraction of the short-circuit current aligned with the stator voltage phasor is given by

$$I_{sc} \sin(\theta) \stackrel{!}{=} I_s \cos(\varphi).$$

Hence, we can find the short-circuit current phasor length from the active power and the stator voltage:

$$I_{sc} = \frac{P}{3U_s \sin(\theta)} = \frac{17 \text{ MW}}{3 \cdot 4 \text{ kV} \cdot 0.496} = 2.86 \text{ kA}.$$



Solution Fig. 2: Phasor diagram of a synchronous machine with solution phasors

4.3 Determine the synchronous reactance X_s .

[1 Point]

Hint: if and if only you are not able to solve this task, assume $X_s = 1 \Omega$ as a substitute result.

Answer:

The synchronous reactance is the fraction of the inner voltage and the short-circuit current:

$$X_s = \frac{U_i}{I_{sc}} = \frac{3 \text{ kV}}{2.86 \text{ kA}} = 1.05 \Omega.$$

4.4 Determine the stator current I_s , the phase shift angle φ between stator current and voltage, and add the phasor \underline{I}_s to Fig. 3.

[3 Points]

Hint: if and if only you are not able to solve this task, assume $I_s = 3 \text{ kA}$ and $\varphi = 150^\circ$ as substitute

results.

Answer:

The stator current can be derived from the voltage difference between the inner voltage and the stator voltage:

$$I_s = \frac{\Delta U}{X_s} = \frac{2.06 \text{ kV}}{1.05 \Omega} = 1.96 \text{ kA}.$$

The phase shift angle φ is given by

$$\varphi = \arccos\left(\frac{P}{3U_s I_s}\right) = \arccos\left(\frac{-17 \text{ MW}}{3 \cdot 4 \text{ kV} \cdot 1.96 \text{ kA}}\right) = \arccos(-0.723) = 136.29^\circ.$$

With this information the stator current phasor can be added to Sol.-Fig. 2. The stator current phasor is advancing the stator voltage phasor by the angle φ .

4.5 Determine the power factor $\cos(\varphi)$, the reactive power Q and the apparent power S . [3 Points]

Answer:

Based on the previous result regarding φ the power factor is given by

$$\cos(\varphi) = \cos(136.29^\circ) = -0.723.$$

The reactive power is given by

$$Q = 3U_s I_s \sin(\varphi) = 3 \cdot 4 \text{ kV} \cdot 1.96 \text{ kA} \cdot \sin(136.29^\circ) = 16.25 \text{ MVA}.$$

Consequently, the apparent power results in

$$S = \sqrt{P^2 + Q^2} = \sqrt{(17 \text{ MW})^2 + (16.25 \text{ MVA})^2} = 23.52 \text{ MVA}.$$

4.6 What is the generator's torque T at the given operating point and what is its theoretical maximum stable torque for the given excitation? [2 Points]

Answer:

The generator's torque at the given operating point can be calculated using

$$T = \frac{P_{\text{me}}}{\omega_r} = \frac{P}{\frac{1}{p} f_s \cdot 2\pi} = \frac{17 \text{ MW}}{\frac{1}{2} 50 \text{ Hz} \cdot 2\pi} = 108.23 \text{ kNm}.$$

Here, p is the pole pair number of the generator and f_s is the grid frequency. Since (ohmic) losses are neglected, above the active grid power is equal to the mechanical power P_{me} . The theoretical maximum stable torque is produced at the tipping point ($\theta = 90^\circ$) leading to

$$T_{\text{max}} = \frac{T}{\sin(\theta)} = \frac{108.23 \text{ kNm}}{\sin(29.75^\circ)} = 218.21 \text{ kNm}.$$

4.7 Now, the generator should only supply the active power specified above to the grid, but no reactive power. To what value must the excitation current I_f be changed to achieve this? [2 Points]

Answer:

The described operation point is characterized by $\cos(\varphi) = 1$, i.e.,

$$|P| = |S| = 3U_s I_s.$$

Hence, the new stator current is given by

$$I_s = \frac{|P|}{3U_s} = \frac{17 \text{ MW}}{3 \cdot 4 \text{ kV}} = 1.42 \text{ kA}.$$

This leads to the new delta voltage

$$\Delta U = I_s X_s = 1.42 \text{ kA} \cdot 1.05 \Omega = 1.49 \text{ kV}.$$

The new inner voltage is given by

$$U_i = \sqrt{U_s^2 + (\Delta U)^2} = \sqrt{(4 \text{ kV})^2 + (1.49 \text{ kV})^2} = 4.27 \text{ kV}.$$

As the inner voltage is directly proportional to the excitation current, the new excitation current can be calculated via the ratio of the new inner voltage and the old inner voltage:

$$I_f = \frac{U_i}{U_{i,\text{old}}} I_{f,\text{old}} = \frac{4.27 \text{ kV}}{3.0 \text{ kV}} \cdot 100 \text{ A} = 142.33 \text{ A}.$$

Hence, the excitation current must be increased compared to the previous operating point.