Exam

Electrical Machines and Drives

Summer 2024

First name: Last name:

Matriculation number: Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 90 minutes.

Evaluation:

Task	1	2	3	4	$\sum$
Maximum score	11	12	8	11	42
Achieved score					

Task 1: Lifting magnet **[11 Points]**

A lifting magnet made of steel with the dimensions shown in Fig. 1 is to carry an iron load with a mass of 700 kg. The average field line length in the load is $l_{\mathrm{load}} = 250$ mm, the flux-carrying cross-section A_{load} is 0.0095 m^2. The winding of both coils have $N = 300$ turns. The magnet has a circular cross-section with a diameter of 70 mm. The roughness of the surfaces of the magnet and load results in an average air gap length of $\delta = 0.5$ mm. Neglect all leakage fluxes.

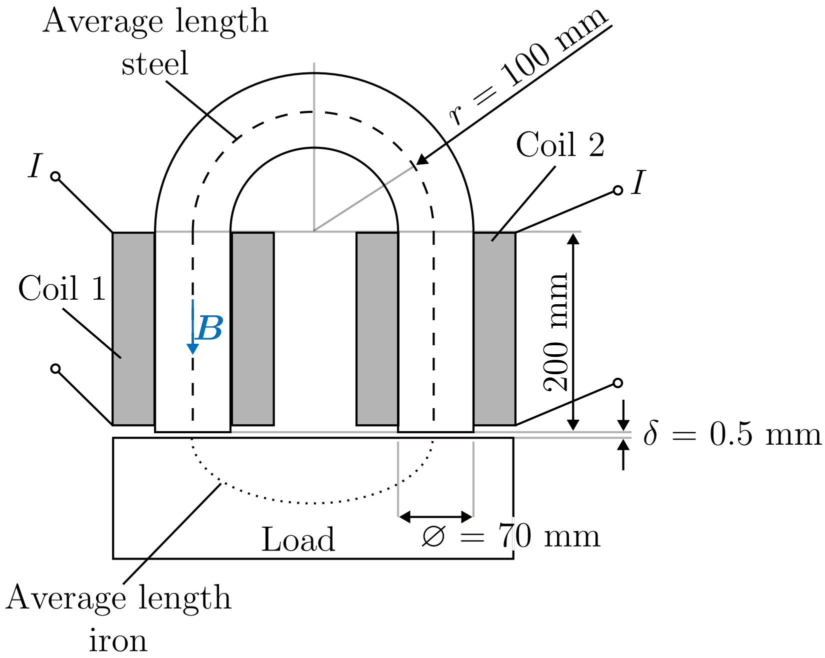


Figure 1: Sketch of the lifting magnet and load with their dimensions.

1.1 Formulate Ampère's circuital law in the general form and in a form adapted to the magnetic circuit shown here. Use the average field line length indication as the closed curve ∂S. [2 Points]

Answer:

The general form of Ampère's circuital law is defined as

$$\oint_{\partial S} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = \sum_k \theta_k$$

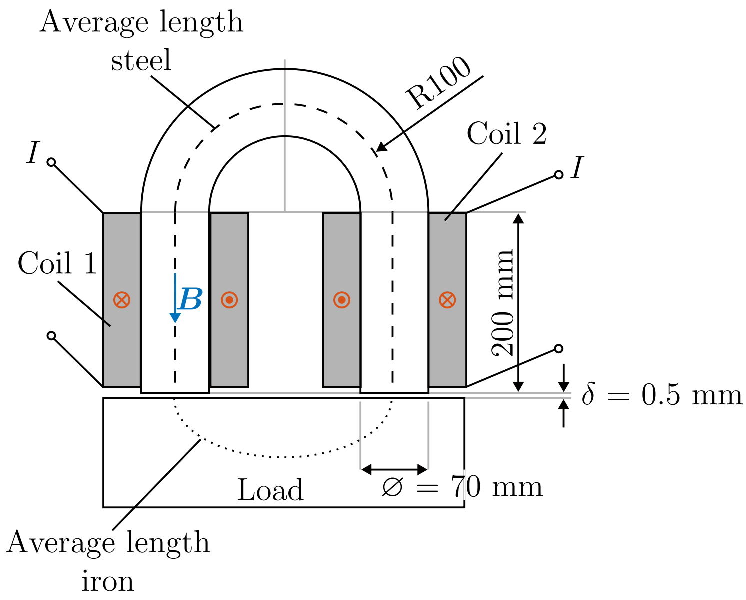
with the extension to the given task:

$$\sum_k \theta_k = l_{\mathrm{load}} H_{\mathrm{load}} + 2 l_\delta H_\delta + l_{\mathrm{m}} H_{\mathrm{m}}.$$

1.2 Add current direction symbols of the two coils to the above figure such that they fit to the already indicated magnetic flux orientation. [2 Points]

Answer:

The current direction is visualized in Sol.-Fig. 1 with red color.



Solution Figure 1: Necessary current direction marked in red.

1.3 How large is the weight force acting on the load assuming nominal gravity at sea level? Calculate the magnetic flux density in the air gap, so that the load floats straight. Use $F = \frac{B_\delta^2}{2\mu_0} 2A_\mathrm{m}$ to calculate the force with $\mu_0 = 4\pi \cdot 10^{-7}\ \frac{\mathrm{Vs}}{\mathrm{Am}}$ being the magnetic field constant. [2 Points]
Hint: if and only if you are not able to solve this subtask, use $B_\delta = 1.6$ T as a substitute result for the following questions.

Answer:
The air gap area is calculated with

$$A_\delta = \pi \frac{d^2}{4} = \pi \cdot \frac{(0.07\ \mathrm{m})^2}{4} = 0.0038\ \mathrm{m}^2$$

and the weight force is definded as follwos:

$$F = mg = 700\ \mathrm{kg} \cdot 9.81\ \mathrm{m/s}^2 = 6867\ \mathrm{N}.$$

Hence, and the given equation in the task, the flux density in the air gap is determined by:

$$B_\delta = \sqrt{\frac{6867\ \mathrm{kgm/s}^2 \cdot 2 \cdot 4\pi \cdot 10^{-7}\ \frac{\mathrm{Vs}}{\mathrm{Am}}}{2 \cdot 0.0038\ \mathrm{m}^2}} = 1.51\ \mathrm{T}.$$

1.4 How large is the necessary current I for both coils to achieve this lifting task? Use the magnetization curves in Fig. 2. [3 Points]

Answer:
The given magneto static situation is represented with

$$NI = \sum_k \theta_k = \sum_k l_k H_k,$$

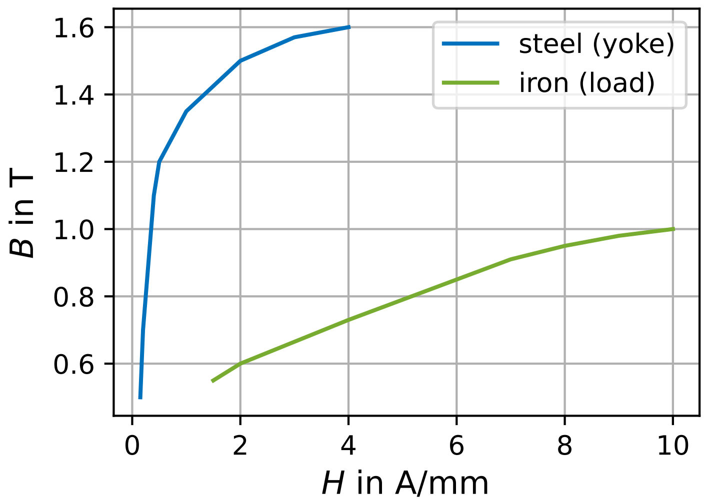


Figure 2: Magnetization curves for the yoke (steel) and the load (iron).

and, therefore, the single contributions must be determined. Since the leakage flux is neglected, the fluxes ϕ_δ in the air gap and ϕ_{load} in the iron are equal. This results into

$$\phi_\delta = \phi_{\mathrm{load}} = A_\delta B_\delta = A_{\mathrm{load}} B_{\mathrm{load}}$$

thus, the magnetic flux density in the load is determined as

$$B_{\mathrm{load}} = \frac{A_\delta}{A_{\mathrm{load}}} B_\delta = \frac{0.0038\ \mathrm{m}^2}{0.0095\ \mathrm{m}^2} \cdot 1.51\ \mathrm{T} = 0.60\ \mathrm{T},$$

and the corresponding magnetic field is $H_{\mathrm{load}} = 2000\ \frac{\mathrm{A}}{\mathrm{m}}$ derived from Fig. 2.
Also, the magnetic flux in the yoke ϕ_{m} is equal to the air gap flux, which results in

$$\phi_\delta = \phi_{\mathrm{m}} = A_\delta B_\delta = A_{\mathrm{m}} B_{\mathrm{m}},$$

hence, the flux density is calculated with

$$B_{\mathrm{m}} = \frac{A_\delta}{A_{\mathrm{m}}} B_\delta = \frac{0.0038\ \mathrm{m}^2}{0.0038\ \mathrm{m}^2} \cdot 1.51\ \mathrm{T} = 1.51\ \mathrm{T}$$

and the corresponding magnetic field is determined from Fig. 2 to $H_{\mathrm{m}} = 2000\ \frac{\mathrm{A}}{\mathrm{m}}$.
The magnetic field in the air gap is calculated with the magnetic constant and the relative permeability, which is $\mu_{\mathrm{r}} \approx 1$ for air. Therefore, only the magnetic constant μ_0 applies to:

$$H_\delta = \frac{B_\delta}{\mu_0} = \frac{1.51\ \mathrm{T}}{4\pi \cdot 10^{-7}\ \frac{\mathrm{Vs}}{\mathrm{Am}}} = 1201619\ \frac{\mathrm{A}}{\mathrm{m}}.$$

Hence, the magnetic voltage is calculated by

$$\begin{aligned}\theta &= 2l_\delta H_\delta + l_\mathrm{m} H_\mathrm{m} + l_\mathrm{load} H_\mathrm{load} \\ &= 2 \cdot \frac{0.5}{1000}\ \mathrm{m} \cdot 1201619\ \frac{\mathrm{A}}{\mathrm{m}} + (0.3142 + 2 \cdot 0.2)\ \mathrm{m} \cdot 2000\ \frac{\mathrm{A}}{\mathrm{m}} + 0.250\ \mathrm{m} \cdot 2000\ \frac{\mathrm{A}}{\mathrm{m}} = 3130\ \mathrm{A},\end{aligned}$$

which leads to the necessary current as follows:

$$I = \frac{\theta}{N} = \frac{3130\ \mathrm{A}}{600} = 5.22\ \mathrm{A}.$$

1.5 Determine the flux ϕ_coil through one coil. [1 Point]

Answer:
The flux through one coil is calculated with:

$$\phi_\mathrm{coil} = B_\mathrm{m} A_\mathrm{m} = 1.51\ \mathrm{T} \cdot 0.0038\ \mathrm{m}^2 = 5.74\ \mathrm{mVs}.$$

1.6 Calculate the flux linkage ψ_coil of one coil. [1 Point]

Answer:
The flux linkage of one coil is calculated by:

$$\psi_\mathrm{coil} = N\phi_\mathrm{coil} = 300 \cdot 5.74\ \mathrm{mVs} = 1.72\ \mathrm{Vs}.$$

Task 2: Separately excited DC machine **[12 Points]**

Table 1: DC machine parameters.

Symbol	Description	Values
$U_{\mathrm{a,n}}$	Nominal armature voltage	230 V
$I_{\mathrm{a,n}}$	Nominal armature current	22 A
$U_{\mathrm{f,n}}$	Nominal field voltage	230 V
$I_{\mathrm{f,n}}$	Nominal field current	0.5 A
P_{n}	Nominal power	4.5 kW
n_{n}	Nominal speed	1440 $^1/_{\mathrm{min}}$
n_0	No-load speed	1615 $^1/_{\mathrm{min}}$
p	Pole pair number	2
L_{a}	Armature inductance	6.92 H

2.1 Determine the torque T_{n} and the efficiency η_{n} at the nominal operating point. [2 Points]

Answer:

The nominal torque is

$$T_{\mathrm{n}} = \frac{P_{\mathrm{n}}}{\omega_{\mathrm{n}}} = \frac{P_{\mathrm{n}}}{n_{\mathrm{n}}}\frac{60}{2\pi}\frac{\mathrm{s}}{\mathrm{min}} = \frac{4.5\ \mathrm{kW}}{1440\ \frac{1}{\mathrm{min}}}\frac{60}{2\pi}\frac{\mathrm{s}}{\mathrm{min}} = 29.84\ \mathrm{Nm}.$$

The electrical input power is

$$P_{\mathrm{el,n}} = U_{\mathrm{a,n}}I_{\mathrm{a,n}} + U_{\mathrm{f,n}}I_{\mathrm{f,n}} = 5.06\ \mathrm{kW} + 115\ \mathrm{W} = 5.18\ \mathrm{kW}.$$

The resulting efficiency yields

$$\eta_{\mathrm{n}} = \frac{P_{\mathrm{n}}}{P_{\mathrm{el,n}}} = 86.87\ \%.$$

2.2 How large are the armature resistance R_{a} and the resulting armature losses $P_{\mathrm{l,a}}$ at the nominal operation neglecting iron and mechanical losses? [2 Points]

Hint: if and if only you are not able to solve this task, use $R_{\mathrm{a}} = 3.2\ \Omega$ and $\psi'_{\mathrm{f}} = 1.1$ Vs as a substitute result for the subsequent tasks.

Answer:

In the no-load case, the entire armature voltage is identical to the induced voltage given a constant field excitation

$$U_{\mathrm{i,0}} = U_{\mathrm{a,n}} = \omega_0\psi'_{\mathrm{f}}.$$

Hence, the effective field flux linkage is

$$\psi'_{\mathrm{f}} = \frac{U_{\mathrm{a,n}}}{\omega_0} = \frac{230\ \mathrm{V}}{1615\ \frac{1}{\mathrm{min}} \cdot \frac{2\pi}{60}\ \frac{\mathrm{min}}{\mathrm{s}}} = 1.36\ \mathrm{Vs}.$$

At nominal (loaded) operation, the armature voltage equation is

$$U_{\mathrm{a,n}} = R_{\mathrm{a}} I_{\mathrm{a,n}} + \omega_{\mathrm{n}} \psi_{\mathrm{f}}'$$

delivering the armature resistance

$$R_{\mathrm{a}} = \frac{U_{\mathrm{a,n}} - \omega_{\mathrm{n}} \psi_{\mathrm{f}}'}{I_{\mathrm{a,n}}} = \frac{230\ \mathrm{V} - 1440\ \frac{1}{\mathrm{min}} \cdot \frac{2\pi}{60}\ \frac{\mathrm{min}}{\mathrm{s}} \cdot 1.36\ \mathrm{Vs}}{22\ \mathrm{A}} = 1.13\ \Omega.$$

The nominal armature losses result in:

$$P_{\mathrm{l,a}} = R_{\mathrm{a}} I_{\mathrm{a,n}}^2 = 1.13\ \Omega \cdot 484\ \mathrm{A}^2 = 546.92\ \mathrm{W}.$$

2.3 For a new operating point at $n = 900\ ^1/_{\mathrm{min}}$, a torque of $T = 20$ Nm is to be achieved. For this purpose, an additional dropping resistor R_{d} is introduced into the armature circuit. Determine its required resistance value. [2 Points]
Hint: if and if only you are not able to solve this task, use $R_{\mathrm{d}} = 4.2\ \Omega$ as a substitute result for the subsequent tasks.

Answer:
Since the field winding is not affected by the dropping resistor or speed change, the excitation ψ_{f}' remains as previously calculated. Based on the torque equation, one can calculate the required armature current:

$$T = \psi_{\mathrm{f}}' I_{\mathrm{a}} \quad \Leftrightarrow \quad I_{\mathrm{a}} = \frac{T}{\psi_{\mathrm{f}}'} = \frac{20\ \mathrm{Nm}}{1.36\ \mathrm{Vs}} = 14.71\ \mathrm{A}.$$

The armature ohmic voltage drop (incl. dropping resistor) for the new operation point is

$$U_{\mathrm{R}} = U_{\mathrm{a,n}} - \omega \psi_{\mathrm{f}}' = 230\ \mathrm{V} - 900\ \frac{1}{\mathrm{min}} \cdot \frac{2\pi}{60}\ \frac{\mathrm{min}}{\mathrm{s}} 1.36\ \mathrm{Vs} = 101.82\ \mathrm{V} \overset{!}{=} I_{\mathrm{a}}(R_{\mathrm{a}} + R_{\mathrm{d}})$$

delivering the required dropping resistance of

$$R_{\mathrm{d}} = \frac{U_{\mathrm{R}}}{I_{\mathrm{a}}} - R_{\mathrm{a}} = \frac{101.82\ \mathrm{V}}{14.71\ \mathrm{A}} - 1.13\ \Omega = 5.79\ \Omega.$$

2.4 Determine the efficiency at this new operating point. What are the alternatives to introducing a dropping resistor to achieve the required torque at the new operating point and why should these alternatives be considered? [2 Points]

Answer:
The new total input power is

$$P_{\mathrm{el}} = U_{\mathrm{a,n}} I_{\mathrm{a}} + U_{\mathrm{f,n}} I_{\mathrm{f,n}} = 3.38\ \mathrm{kW} + 115\ \mathrm{W} = 3.50\ \mathrm{kW}$$

and the new mechanical output power is

$$P = Tn\frac{2\pi}{60}\frac{\mathrm{min}}{\mathrm{s}} = 20\ \mathrm{Nm}\cdot 900\ \frac{1}{\mathrm{min}}\cdot\frac{2\pi}{60}\frac{\mathrm{min}}{\mathrm{s}} = 1.88\ \mathrm{kW}$$

resulting in the new efficiency

$$\eta = \frac{P}{P_{\mathrm{el}}} = \frac{1.88\ \mathrm{kW}}{3.50\ \mathrm{kW}} = 53.71\ \%.$$

Possible alternatives are:

- Decreasing the armature voltage supply U_{a} (e.g., via a buck converter),
- Increasing the field excitation ψ'_{f} (e.g., via a boost converter in field circuit).

From above's calculation a significant decrease of the machine efficiency due to the dropping resistor can be observed, which motivates the alternatives (since they typically add only little addition losses).

2.5 Now the transient response of a machine supply fault should be investigated: calculate $i_{\mathrm{a}}(t)$ for $t \geq 0$ assuming $i_{\mathrm{a}}(t=0) = I_{\mathrm{a,n}}$ and $u_{\mathrm{a}}(t) = 0$ for $t \geq 0$ (short circuit of armature voltage supply). Assume further that the machine speed $n(t) = n_{\mathrm{n}}$ for $t \geq 0$ remains constant and that the field winding is unaffected by the armature supply fault, i.e., delivering nominal excitation. [2 Points]

Answer:
Starting point for solving the task is the armature current ODE:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{a}}(t) &= \frac{1}{L_{\mathrm{a}}}\left(u_{\mathrm{a}}(t) - R_{\mathrm{a}}i_{\mathrm{a}}(t) - u_{\mathrm{i}}(t)\right)\\ &= \frac{1}{L_{\mathrm{a}}}\left(-R'i_{\mathrm{a}}(t) - u_{\mathrm{i}}\right)\end{aligned}$$

with $R' = R_{\mathrm{a}} + R_{\mathrm{d}}$. Based on the subtask's assumptions the armature voltage at its terminals is zero $u_{\mathrm{a}}(t) = 0$ and the induced voltage is a constant $u_{\mathrm{i}}(t) = u_{\mathrm{i}}$. This results in a simple scalar, nonhomogeneous ODE. The homogeneous solution candidate is $x_{\mathrm{h}}(t) = e^{\lambda t}$, which delivers

$$\begin{aligned}\lambda e^{\lambda t} &= -\frac{R'}{L_{\mathrm{a}}}e^{\lambda t}\\ \Leftrightarrow \qquad \lambda &= -\frac{R'}{L_{\mathrm{a}}} = \frac{1}{\tau_{\mathrm{a}}}\end{aligned}$$

when inserted into the homogeneous part of the ODE leading to the time constant τ_{a}. To solve the entire inhomogenous ODE we apply the variation of parameter approach using

$$i_{\mathrm{a}}(t) = C(t)e^{\lambda t}.$$

Inserting into the ODE yields

$$\begin{aligned}
 & \dot{C}(t)e^{\lambda t} + C(t)\lambda e^{\lambda t} = C(t)\lambda e^{\lambda t} - \frac{1}{L_\mathrm{a}}u_\mathrm{i} \\
\Leftrightarrow \quad & \dot{C}(t)e^{\lambda t} = -\frac{1}{L_\mathrm{a}}u_\mathrm{i} \\
\Leftrightarrow \quad & \dot{C}(t) = -\frac{1}{L_\mathrm{a}}u_\mathrm{i}e^{-\lambda t}
\end{aligned}$$

delivering the integration result

$$C(t) = \frac{u_\mathrm{i}}{L_\mathrm{a}\lambda}e^{-\lambda t} + C_0 = -\frac{u_\mathrm{i}}{R'}e^{-\lambda t} + C_0.$$

The overall solution is then

$$i_\mathrm{a}(t) = C_0 e^{\lambda t} - \frac{u_\mathrm{i}}{R'}.$$

The integration constant C_0 can be found from the initial condition

$$i_\mathrm{a}(t=0) = I_\mathrm{a,n} = C_0 - \frac{u_\mathrm{i}}{R'} \quad \Leftrightarrow \quad C_0 = I_\mathrm{a,n} + \frac{u_\mathrm{i}}{R'}.$$

Hence, we finally receive

$$\begin{aligned}
i_\mathrm{a}(t) &= I_\mathrm{a,n}e^{\lambda t} + \frac{u_\mathrm{i}}{R'}(e^{\lambda t} - 1) = I_\mathrm{a,n}e^{-\frac{R'}{L_\mathrm{a}}t} + \frac{u_\mathrm{i}}{R'}(e^{-\frac{R'}{L_\mathrm{a}}t} - 1) \\
&= 22\ \mathrm{A} \cdot e^{-t/1\ \mathrm{s}} + \frac{205.08\ \mathrm{V}}{6.92\ \Omega}(e^{-t/1\ \mathrm{s}} - 1) \\
&= 22\ \mathrm{A} \cdot e^{-t/1\ \mathrm{s}} + 29.64\ \mathrm{A}(e^{-t/1\ \mathrm{s}} - 1).
\end{aligned}$$

2.6 Sketch the current response $i_\mathrm{a}(t)$ as a function of time t. Also calculate the steady-state value for $t \to \infty$. [2 Points]

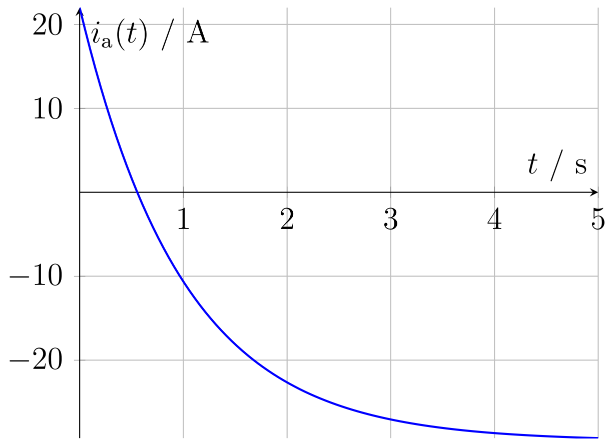
Hint: if and if only you have not solved the previous subtask, you can draw the current trajectory qualitatively to highlight the general system response characteristic for the given differential equation model. Also note, that the question addressing the steady-state armature current can be answered independently of the previous subtask.

Answer:

The steady-state final value can be directly obtained from the ODE (without solving it): setting ${}^{\mathrm{d}}/_{\mathrm{d}t}(i_\mathrm{a}(t)) = 0$ leading to

$$0 = \frac{1}{L_\mathrm{a}}\left(-R' i_\mathrm{a}(t\to\infty) - u_\mathrm{i}\right) \quad \Leftrightarrow \quad i_\mathrm{a}(t\to\infty) = -\frac{u_\mathrm{i}}{R'} = -29.64\ \mathrm{A}.$$

In Sol.-Fig. 2 the current response is visualized. Due to the constant induced voltage, the short-circuited armature winding current gets negative, that is, the mechanical load, which keeps the speed



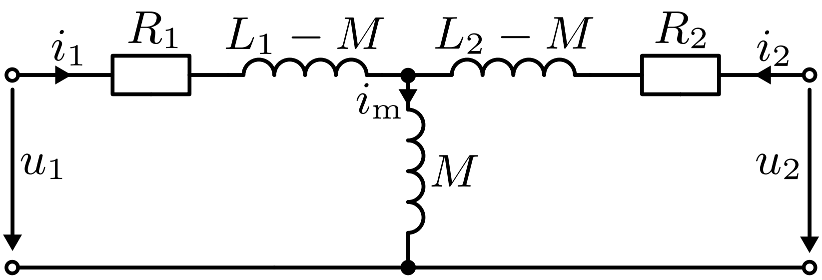
Solution Figure 2: Plot of the current response $i_{\mathrm{a}}(t)$.

constant (as assumed in the subtask description), provides mechanical power which is dissipated in heat within the armature circuit.

Task 3: Transformer [8 Points]

3.1 Draw and label the T-type equivalent circuit diagram of a single phase transformer. [1 Point]

Answer:



Solution Figure 3: T-type equivalent circuit diagram of a single phase transformer

3.2 The primary side has $N_1 = 23$ and the secondary side has $N_2 = 10$ turns. For $\underline{U}_1 = 230$ V, which secondary voltage U_2 would you expect for an unloaded, idealized transformer? [1 Point]

Answer:
For the described idealized case the voltage transformation ratio is

$$U_2 = \frac{U_1}{\ddot{\mathrm{u}}} = U_1 \frac{N_2}{N_1} = 230\ \mathrm{V} \cdot \frac{10}{23} = 100\ \mathrm{V}.$$

3.3 Which actual voltage U_2 is to be expected if the transformer is loaded with $\underline{I}_2 = -10$ A considering the following parameters: $R_1 \approx 0\ \Omega$, $R_2 = 0.1\ \Omega$, $M = 0.42$ H, $L_1 = 1$ H, $L_2 = 0.17$ H? [2 Points]

Answer:
The steady-state complex current and voltage phasors of the transformer are:

$$\begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \begin{bmatrix} R_1 + \mathrm{j}\omega_\mathrm{el} L_1 & \mathrm{j}\omega_\mathrm{el} M \\ \mathrm{j}\omega_\mathrm{el} M & R_2 + \mathrm{j}\omega_\mathrm{el} L_2 \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}.$$

Considering $R_1 \approx 0$ and rearranging the first line yields

$$\underline{I}_1 = \frac{\underline{U}_1 - \mathrm{j}\omega_\mathrm{el} M \underline{I}_2}{\mathrm{j}\omega_\mathrm{el} L_1}.$$

Inserting that into the second line

$$\underline{U}_2 = \mathrm{j}\omega_\mathrm{el} M \underline{I}_1 + (R_2 + \mathrm{j}\omega_\mathrm{el} L_2)\underline{I}_2$$

delivers

$$\underline{U}_2 = \frac{M}{L_1}\underline{U}_1 + \left[R_2 + \mathrm{j}\omega_\mathrm{el}\left(L_2 - \frac{M^2}{L_1}\right)\right]\underline{I}_2.$$

Inserting the given parameter values yields

$$\begin{aligned}\underline{U}_2 &= \frac{0.42\ \mathrm{H}}{1\ \mathrm{H}} \cdot 230\ \mathrm{V} + \left[0.1\ \Omega + \mathrm{j}314.16\ \frac{1}{\mathrm{s}}\left(0.17\ \mathrm{H} - \frac{(0.42\ \mathrm{H})^2}{1\ \mathrm{H}}\right)\right] \cdot (-10\ \mathrm{A}) \\ &= 95.6\ \mathrm{V} - \mathrm{j}20.11\ \mathrm{V}\end{aligned}$$

finally leading to

$$U_2 = 97.69\ \mathrm{V}.$$

3.4 Which primary prospective short-circuit current $I_{1,\mathrm{psc}}$ occurs if the secondary side is short-circuited? [2 Points]

Answer:
The short circuit relates to $U_2 = 0$ leading to

$$0 = \mathrm{j}\omega_{\mathrm{el}} M \underline{I}_1 + (R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2)\underline{I}_2 \quad \Leftrightarrow \quad \underline{I}_2 = \frac{-\mathrm{j}\omega_{\mathrm{el}} M}{R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2}\underline{I}_1.$$

Inserting into the primary voltage equations yields

$$\underline{I}_1 = \frac{1}{\mathrm{j}\omega_{\mathrm{el}} L_1}\underline{U}_1 + \frac{M}{L_1}\frac{\mathrm{j}\omega_{\mathrm{el}} M}{R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2}\underline{I}_1.$$

Solving for $\underline{I}_1$

$$\underline{I}_1 = \left(1 - \frac{M}{L_1}\frac{\mathrm{j}\omega_{\mathrm{el}} M}{R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2}\right)^{-1} \frac{1}{\mathrm{j}\omega_{\mathrm{el}} L_1}\underline{U}_1$$

and some rewriting

$$\begin{aligned}\underline{I}_1 &= \left(\frac{L_1 R_2 + \mathrm{j}\omega_{\mathrm{el}}(L_1 L_2 - M^2)}{L_1(R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2)}\right)^{-1} \frac{1}{\mathrm{j}\omega_{\mathrm{el}} L_1}\underline{U}_1 \\ &= \left(\frac{(R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2)}{L_1 R_2 + \mathrm{j}\omega_{\mathrm{el}}(L_1 L_2 - M^2)}\right)\frac{1}{\mathrm{j}\omega_{\mathrm{el}}}\underline{U}_1 \\ &= \left(\frac{(R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2)}{L_1 R_2 \mathrm{j}\omega_{\mathrm{el}} - \omega_{\mathrm{el}}^2(L_1 L_2 - M^2)}\right)\underline{U}_1 \\ &= -\left(\frac{(R_2 + \mathrm{j}\omega_{\mathrm{el}} L_2)(L_1 R_2 \mathrm{j}\omega_{\mathrm{el}} + \omega_{\mathrm{el}}^2(L_1 L_2 - M^2))}{(L_1 R_2 \omega_{\mathrm{el}})^2 + \omega_{\mathrm{el}}^4(L_1 L_2 - M^2)^2}\right)\underline{U}_1\end{aligned}$$

finally leads to

$$\begin{aligned}\underline{I}_{1,\mathrm{psc}} &= (4.4 \cdot 10^{-3}\ \frac{1}{\Omega} + \mathrm{j} \cdot 84.3 \cdot 10^{-3}\ \frac{1}{\Omega}) \cdot 230\ \mathrm{V} \\ &= 1.0\ \mathrm{A} + \mathrm{j} \cdot 19.40\ \mathrm{A},\end{aligned}$$

that is, is $I_{1,\mathrm{psc}} = 19.42\ \mathrm{A}$.

3.5 What idealized secondary output voltage results in the unloaded case when the transformer is reconfigured as an autotransformer connecting the terminals 1.1 and 2.2? [1 Point]

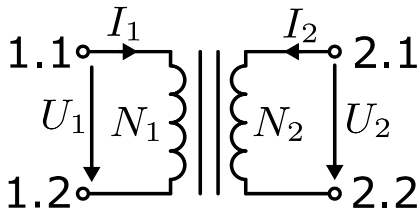
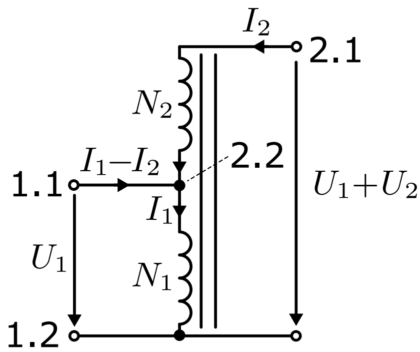


Figure 3: Transformer connection nomenclature

Answer:

The described case refers to a step-up autotransformer, cf. Sol.-Fig. 4. Hence, the new secondary output voltage is $U_{\mathrm{at,out}} = U_1 + U_2 = 330$ V.



Solution Figure 4: Step-up autotransformer

3.6 Calculate the change in primary current in the event of a short circuit on the secondary side of the autotransformer compared to the original galvanically isolated transformer. [1 Point]

Answer:

Due to the reconfiguration the effective short-circuit impedance is decreased and the resulting primary short-circuit current is increased:

$$I_{1,\mathrm{at,psc}} = \left(1 + \frac{N_1}{N_2}\right) I_{1,\mathrm{psc}} = (1 + 2.3) \cdot 19.42 \text{ A} = 64.09 \text{ A}.$$

The primary short-circuit current is 3.3 times higher than for the original transformer configuration.

Task 4: Synchronous generator [11 Points]

An externally excited synchronous generator is directly fed from a wind turbine without utilizing a gear. For the three-phase generator the parameters from Tab. 2 are known.

Ein elektrisch erregter Synchrongenerator wird von einer Windturbine direkt angetrieben (getriebeloser Windgenerator). Für den Drehstromgenerator sind die Parameter aus Tab. 2 bekannt.

Table 2: Parameters of the synchronous generator.

Symbol	Value	Symbol	Value
$P_{\mathrm{n,el}}$	-1.5 MW	l_{z}	700 mm
d_{s}	5 m	n_{n}	15 $\frac{1}{\mathrm{min}}$
N_{w}	3 windings per coil	q	2
$2p$	90	$\frac{y}{\rho_{\mathrm{p}}}$	$\frac{5}{6}$

4.1 Calculate the nominal torque T_{n} at the rated conditions of the generator. At this operating point, the efficiency is given with $\eta_{\mathrm{n}} = 95$ %. [2 Points]

Answer:
In the task, only the electrical power is given, therefore, calculating the mechanical power by

$$P_{\mathrm{n,mech}} = \frac{P_{\mathrm{n,el}}}{\eta_{\mathrm{n}}} = \frac{-1.5\ \mathrm{MW}}{0.95} = -1.578947\ \mathrm{MW}$$

and the torque at the nominal operating is determined as:

$$T_{\mathrm{n}} = \frac{P_{\mathrm{n,mech}}}{\omega_{\mathrm{n}}} = \frac{-1.578947\ \mathrm{MW}}{2\pi \cdot 0.25\ \frac{1}{\mathrm{s}}} = -1005.189\ \mathrm{kNm}.$$

4.2 How large is the number of winding turns per phase $N_{\mathrm{w,phase}}$, when the total 90 coils are parallelized in 5 groups? [1 Point]

Answer:
The number of winding turns per phase is:

$$N_{\mathrm{w,phase}} = \frac{2pqN_{\mathrm{w}}}{a} = \frac{90 \cdot 2 \cdot 3}{5} = 108.$$

4.3 Calculate the winding factor $\zeta_{\mathrm{w},k}$ for the fundamental wave ($k = 1$). [2 Points]

Answer:
The winding factor is defined by

$$\zeta_{\mathrm{w},k} = \zeta_{\mathrm{d},k}\zeta_{\mathrm{p},k},$$

where $\zeta_{\mathrm{d},k}$ represents the distribution and $\zeta_{\mathrm{p},k}$ the pitch factor. With $m = 3$ phases, the distribution factor calculates as

$$\zeta_{\mathrm{d},1} = \frac{\sin\left(\frac{\pi}{2m}\right)}{q\sin\left(\frac{\pi}{2mq}\right)} = \frac{\sin\left(\frac{\pi}{2\cdot 3}\right)}{2\cdot\sin\left(\frac{\pi}{2\cdot 3\cdot 2}\right)} = 0.966,$$

and, the pitch factor is determined with

$$\zeta_{\mathrm{p},1} = \sin\left(\frac{\pi}{2}\frac{y}{\rho_\mathrm{p}}\right) = \sin\left(\frac{\pi}{2}\cdot\frac{5}{6}\right) = 0.966,$$

which results in the winding factor by

$$\zeta_{\mathrm{w},1} = \zeta_{\mathrm{d},1}\zeta_{\mathrm{p},1} = 0.966\cdot 0.966 = 0.933.$$

4.4 The flux density of the fundamental wave of one phase is given with $\hat{B}_\delta^{(1)} = 1$ T. Determine at nominal speed n_n the flux per pole ϕ_δ and the induced voltage per phase $U_{\mathrm{i,phase}}$. [2 Points]

Answer:

The pole pitch as a distance in meter is calculated by

$$\tau_\mathrm{p} = \frac{d_\mathrm{si}\pi}{2p} = \frac{5\ \mathrm{m}\cdot\pi}{90} = 0.1745\ \mathrm{m},$$

and the effective cross-sectional area of the machine per pole is determined with

$$A_\delta = \tau_\mathrm{p} l_\mathrm{z} = 0.1745\ \mathrm{m}\cdot 0.7\ \mathrm{m} = 0.122\ \mathrm{m}^2.$$

Hence, the air gap flux is given with

$$\phi_\delta = A_\delta \hat{B}_\delta^{(1)} = 0.122\ \mathrm{m}^2\cdot 1\ \mathrm{T} = 0.122\ \mathrm{Vs},$$

the electrical angular frequency is determined by

$$\omega_\mathrm{el} = 2\pi f_\mathrm{mech} p = 2\pi\cdot 0.25\ \frac{1}{\mathrm{s}}\cdot 45 = 70.69\ \frac{1}{\mathrm{s}}$$

resulting in the induced voltage of

$$U_{\mathrm{i,phase}} = \omega_\mathrm{el} N_{\mathrm{w,phase}} \zeta_{\mathrm{w},1}\phi_\delta = 70.69\ \frac{1}{\mathrm{s}}\cdot 108\cdot 0.933\cdot 0.122\ \mathrm{Vs} = 869\ \mathrm{V}.$$

4.5 Assume that the flux density in the air gap is a superposition of sine waves. Draw the trajectories of the flux densities $B_\delta^{(1)}(\vartheta)$ and $B_\delta^{(3)}(\vartheta)$ for one phase in the template (Fig. 4). [2 Points]

Answer:

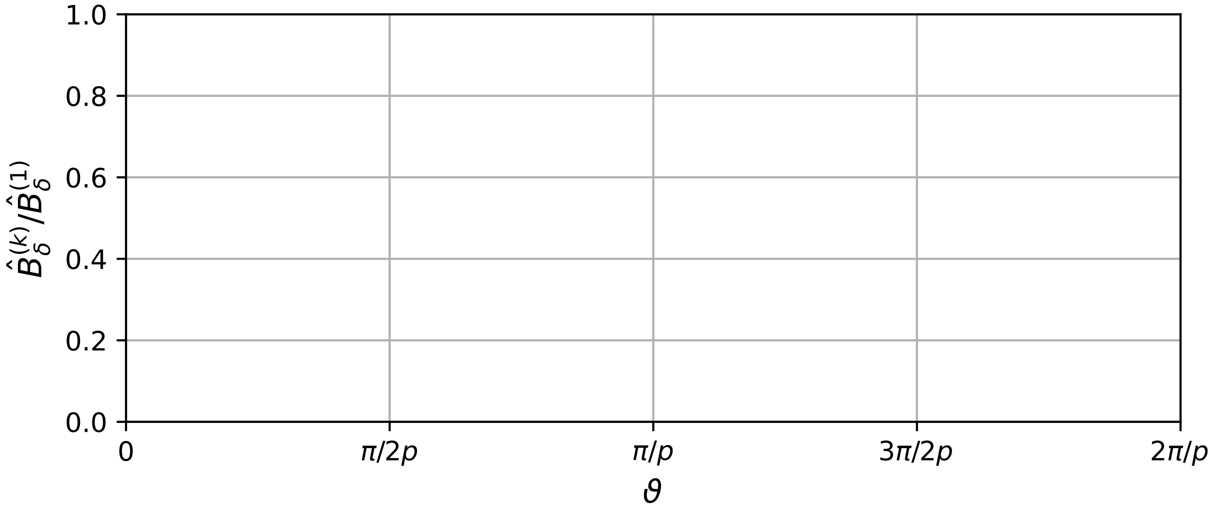


Figure 4: Template to draw the flux densities over the stator circumference ϑ.

The distribution factor for the third harmonic ($k = 3$) calculates as

$$\zeta_{\mathrm{d},3} = \frac{\sin\left(\frac{3\pi}{2m}\right)}{q\sin\left(\frac{3\pi}{2mq}\right)} = \frac{\sin\left(\frac{3\cdot\pi}{2\cdot 3}\right)}{2\cdot\sin\left(\frac{3\cdot\pi}{2\cdot 3\cdot 2}\right)} = 0.707,$$

and, the pitch factor is determined with

$$\zeta_{\mathrm{p},3} = \sin\left(3\cdot\frac{\pi}{2}\frac{y}{\rho_\mathrm{p}}\right) = \sin\left(3\cdot\frac{\pi}{2}\cdot\frac{5}{6}\right) = -0.707,$$

which results in the winding factor by

$$\zeta_{\mathrm{w},3} = \zeta_{\mathrm{d},3}\zeta_{\mathrm{p},3} = 0.707\cdot -0.707 = -0.500.$$

The winding factor must set into the relationship of the fundamental wave by

$$\frac{B_\delta^{(3)}(\vartheta)}{B_\delta^{(1)}(\vartheta)} = \frac{-0.5}{0.933} = -0.54,$$

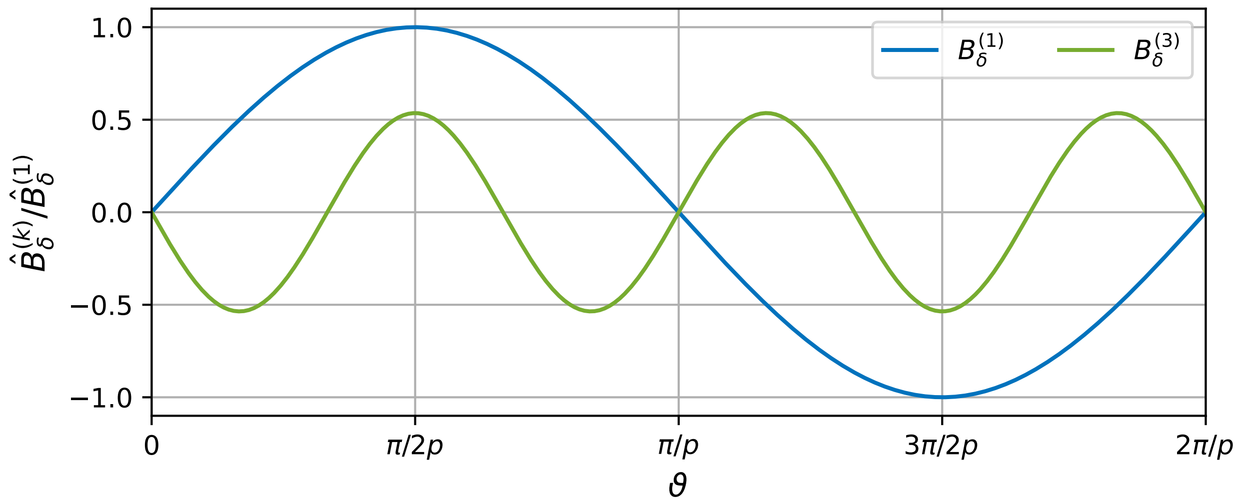
which is the amplitude for the third harmonic. Both trajectories are shown in Sol.-Fig. 5.

4.6 Is the third harmonic ($k = 3$) problematic for this type of machine? [1 Point]

Answer:
Due to the three-phase winding system of this machine, the third harmonic is canceled out and, therefore, no problem occurs.

4.7 Is it possible to connect the wind turbine directly to the grid? [1 Point]

Answer:



Solution Figure 5: Solution trajectories for $B_\delta^{(1)}(\vartheta)$ and $B_\delta^{(3)}(\vartheta)$.

No, due the the changing wind stream and the varying rotational speed of the generator an inverter is necessary to transform this changing generator frequency to a constant grid frequency.