Mock-up Exam

Electrical Machines and Drives

Summer 2024

First name: Last name:

Matriculation number: Study program:

Instructions:

- You can only take part in the exam, if you are registered in the campus management system.
- Prepare your student ID and a photo ID card on your desk.
- Label each exam sheet with your name. Start a new exam sheet for each task.
- Answers must be given with a complete, comprehensible solution. Answers without any context will not be considered. Answers are accepted in German and English.
- Permitted tools are (exclusively): black / blue pens (indelible ink), triangle, a non-programmable calculator without graphic display and two DIN A4 cheat sheets.
- The exam time is 90 minutes.

Evaluation:

Task	1	2	3	4	$\sum$
Maximum score	8	12	9	13	42
Achieved score					

Task 1: Fundamentals [8 Points]

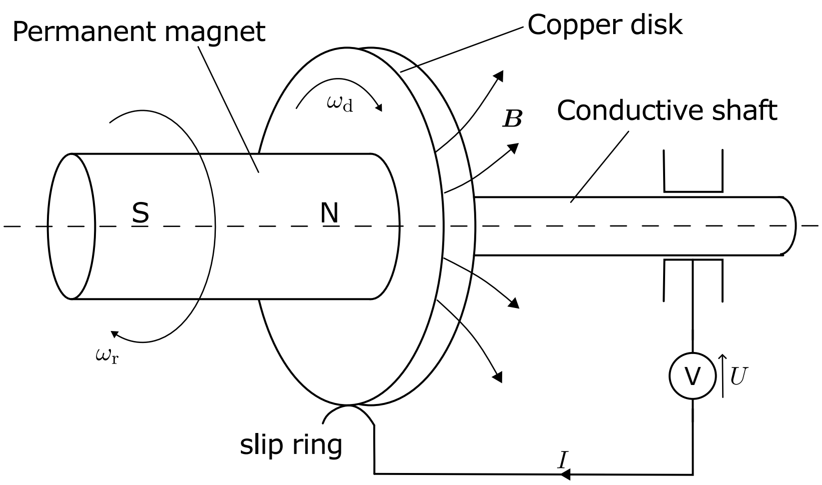


Figure 1: Faraday's disk (rotating copper disk in a homogenous magnetic field)

1.1 The disc from Fig. 1 has a diameter of $d = 60$ cm and is rotating with the circumferential speed $v_\mathrm{d} = 100\ \frac{\mathrm{m}}{\mathrm{s}}$. What is the rotational speed and angular velocity of the copper disk? [2 Points]

Answer:

The rotational speed is given by

$$n_\mathrm{d} = \frac{v_\mathrm{d}}{2\pi r} = \frac{100\ \frac{\mathrm{m}}{\mathrm{s}} \cdot 60\ \frac{\mathrm{s}}{\mathrm{min}}}{2\pi \cdot 0.3\ \mathrm{m}} = 3183\ \frac{1}{\mathrm{min}}.$$

The angular velocity is

$$\omega_\mathrm{d} = 2\pi n_\mathrm{d} \frac{1}{60}\ \frac{\mathrm{min}}{\mathrm{s}} = 333\ \frac{1}{\mathrm{s}}.$$

1.2 Assuming that the permanent magnet is not rotating ($\omega_\mathrm{r} = 0$) while delivering a homogenous and constant magnetic field with $B = 1.8$ T, what is the measured induced voltage U? [2 Points]

Answer:

Faraday's law of induction reads

$$u_\mathrm{i} = \oint_{\partial\mathcal{S}} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$$

where $\boldsymbol{E}$ is the electric field and $\partial\mathcal{S}$ is the boundary of the surface $\mathcal{S}$, which is the copper disk in this case. The electric field is a result of the rotating copper disk within the static magnetic field pointing from the center to the edge of the disk along the radial direction:

$$\boldsymbol{E} = \boldsymbol{v} \times \boldsymbol{B} = v(r)B\boldsymbol{e}_\mathrm{r}.$$

Here, $\boldsymbol{v}(r) = 2\pi r n_\mathrm{d}$ is the velocity field of the disk for a given radius element r and $\boldsymbol{e}_\mathrm{r}$ is the unit vector in the radial direction. The induced voltage is then

$$u_\mathrm{i} = \int_0^{d/2} v(r) B \mathrm{d}r = B \int_{r=0}^{d/2} 2\pi r n_\mathrm{d} \mathrm{d}r = 2\pi B n_\mathrm{d} \left[\frac{r^2}{2}\right]_0^{d/2} = \pi B n_\mathrm{d} \frac{d^2}{4} = \pi \cdot 1.8~\mathrm{T} \cdot 53.05~\frac{1}{\mathrm{s}} \cdot 0.09~\mathrm{m}$$
$$= 27~\mathrm{V}.$$

1.3 Assume that the volt meter is exchanged for a resistor with $R = 1~\Omega$. How big are the resulting current I and electrical power P? Is the disc operating as a motor or generator? [2 Points]

Answer:

The current is given by Ohm's law

$$I = -U/R = -27~\mathrm{V}/1~\Omega = -27~\mathrm{A}.$$

The current is to be counted negative as the voltage and current in the above figure are oriented in opposite directions (load convention). The electrical power is

$$P = IU = 27~\mathrm{A} \cdot -27~\mathrm{V} = -729~\mathrm{W}.$$

The disc is operating as a generator, since the mechanical energy (which is introduced to rotate the disc) is converted into electrical energy. Also, there is no electrical energy source to power the disc, so it cannot operate as a motor.

1.4 Discuss the three following cases regarding the presence of an induced voltage: [2 Points]

- The disc is at standstill, but the permanent magnet is rotating.
- The disc and the permanent magnet are rotating, but with different speeds.
- The disc and the permanent magnets are at standstill, but the electrical circuit is rotating.

Answer:

- In the first case, there is no induced voltage as the magnetic field of the magnet is symmetric around the rotation axis, that is, the flux through the disc remains constant despite the magnet's rotation.
- In the second case it is only important to note that the disc is rotating (like in the original setup), which is the important factor for the induced voltage. The relative speed between the disc and the magnetic is not relevant due to the discussed rotational symmetry of the magnetic field.
- In the third case, the electrical circuit rotates through the constant magnetic field of the magnets which is analogously to the original setup of a rotating disc. Consequently, in this final case also a voltage is induced.

Task 2: DC machine **[12 Points]**

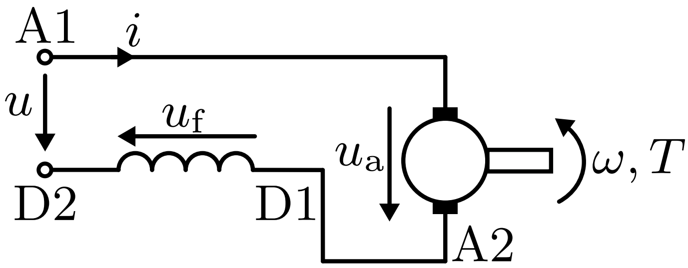
2.1 What are the three main connection types for DC machines? Draw the equivalent circuit diagrams and add the respective current and voltage equations in the steady state. [3 Points]

Answer:

The three types are:

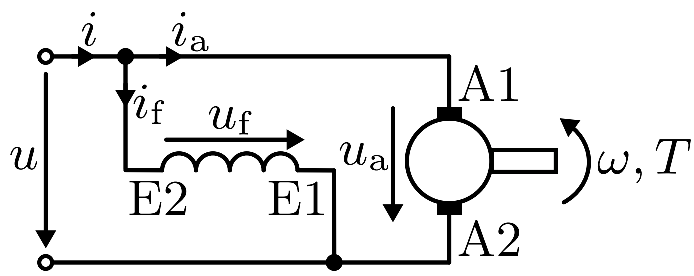
Series DC machine:

$$U = U_\mathrm{a} + U_\mathrm{f} = (R_\mathrm{a} + R_\mathrm{f})I + \omega L_\mathrm{f}' I, \qquad I = I_\mathrm{a} = I_\mathrm{f}.$$



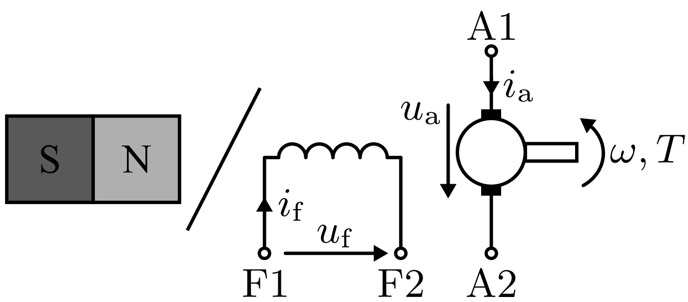
Shunt DC machine:

$$U = U_\mathrm{a} = U_\mathrm{f}, \quad I = I_\mathrm{a} + I_\mathrm{f}, \quad I_\mathrm{f} = \frac{U_\mathrm{f}}{R_\mathrm{f}}, \quad I_\mathrm{a} = \frac{1 - L_\mathrm{f}'/R_\mathrm{f}\omega}{R_\mathrm{a}} U$$



Separately-excited DC machine:

$$U_\mathrm{a} = R_\mathrm{a} I_\mathrm{a} + \omega L_\mathrm{f}' I_\mathrm{f}, \quad U_\mathrm{f} = R_\mathrm{f} I_\mathrm{f}$$



2.2 Now consider a DC machine with the parameters given in Tab. 1. To which of the above connection type can the parameter set belong? [1 Point]

Answer:

Since the field and armature voltage are equal while their currents are different, the machine could

Table 1: Characteristics of the given DC machine.

Symbol	Description	Values
$U_{\mathrm{a,n}}$	Nominal armature voltage	230 V
$I_{\mathrm{a,n}}$	Nominal armature current	20 A
$U_{\mathrm{f,n}}$	Nominal field voltage	230 V
$I_{\mathrm{f,n}}$	Nominal field current	1.1 A
R_{a}	Armature resistance	1.2 Ω
R_{f}	Field resistance	42.0 Ω
P_{n}	Nominal power	4.1 kW
n_{n}	Nominal speed	1500 $\frac{1}{\mathrm{min}}$

be a shunt DC machine. However, a seperately-excited DC machine could also be possible if two different power supplies are used for the field and armature.

2.3 Calculate the nominal torque T_{n}. [2 Points]

Answer:
The nominal mechanical power is given by

$$P_{\mathrm{n}} = T_{\mathrm{n}}\omega_{\mathrm{r,n}} = T_{\mathrm{n}} 2\pi n_{\mathrm{n}} \frac{1}{60}\frac{\mathrm{min}}{\mathrm{s}}.$$

Hence, we can calculate the nominal torque as

$$T_{\mathrm{n}} = \frac{P_{\mathrm{n}}}{n_{\mathrm{n}}} \cdot \frac{60}{2\pi}\frac{\mathrm{s}}{\mathrm{min}} = 26.1\ \mathrm{Nm}.$$

2.4 Determine the nominal efficiency η_{n} of the entire machine. [2 Points]

Answer:
The nominal electrical power is

$$P_{\mathrm{el,n}} = U_{\mathrm{a,n}}I_{\mathrm{a,n}} + U_{\mathrm{f,n}}I_{\mathrm{f,n}} = 230\ \mathrm{V}\cdot 20\ \mathrm{A} + 230\ \mathrm{V}\cdot 1.1\ \mathrm{A} = 4853\ \mathrm{W}.$$

The efficiency is then

$$\eta_{\mathrm{n}} = \frac{P_{\mathrm{n}}}{P_{\mathrm{el,n}}} = \frac{4100\ \mathrm{W}}{4853\ \mathrm{W}} = 0.85 = 85\ \%.$$

2.5 Calculate the armature starting current $I_{\mathrm{a,0}}$ and the resulting starting torque T_0. [2 Points]

Answer:
The armature starting current is given by

$$I_{\mathrm{a,0}} = \frac{U_{\mathrm{a,n}}}{R_{\mathrm{a}}} = \frac{230\ \mathrm{V}}{1.2\ \Omega} = 191.67\ \mathrm{A}.$$

For the torque we first calculate the effective field excitation

$$T_\mathrm{n} = \psi_\mathrm{f}' I_\mathrm{a,n} \quad \Leftrightarrow \quad \psi_\mathrm{f}' = \frac{T_\mathrm{n}}{I_\mathrm{a,n}} = \frac{26.1\ \mathrm{Nm}}{20\ \mathrm{A}} = 1.305\ \mathrm{Vs}$$

and then the starting torque

$$T_0 = \psi_\mathrm{f}' I_\mathrm{a,0} = 1.305\ \mathrm{Vs} \cdot 191.67\ \mathrm{A} = 250.13\ \mathrm{Nm}.$$

2.6 Discuss potential operation issues of the found starting torque and current values compared to the machine's nominal operation. Propose potential remedies to address these issues. [2 Points]

Answer:

The starting current is significantly higher than the nominal current, which can lead to thermal issues in the machine. The starting torque is also higher than the nominal torque, which can lead to mechanical issues. To address these issues, potential remedies could be:

- Vary the armature voltage by an adaptive power supply.
- Vary the field excitation by an adaptive power supply (if separately-excited type).
- Introduce dropping resistors in the armature circuit during starting.

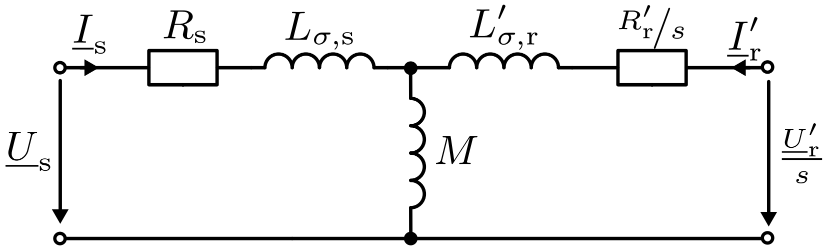
If these remedies are necessary highly depends on the specific application of the machine and in particular how fast the machine is accelerated as this will reduce the armature current due to the induced voltage.

Task 3: Induction machine **[9 Points]**

3.1 Draw and label the stationary equivalent circuit diagram of the general induction machine. What simplification can you make if the machine is at standstill ($\omega_\mathrm{r} = 0\ \frac{1}{\mathrm{s}}$)? [1 Point]

Answer:

The general induction machine equivalent circuit diagram is given by:



If $\omega_\mathrm{r} = 0\ \frac{1}{\mathrm{s}}$ the mechanical rotor frequency is zero and the slip angular frequency is equal to the electrical angular frequency of the stator excitation: $\omega_\mathrm{s} = \omega_\mathrm{slip}$. Consequently, the slip ratio is equal to one ($s = \omega_\mathrm{slip}/\omega_\mathrm{s} = 1$) and could be dropped from the equivalent circuit diagram.

3.2 From now on consider a **squire cage induction machine** with the parameters from Tab. 2. Calculate the no-load speed n_0. [2 Points]

Table 2: Characteristics of the given induction machine.

Symbol	Description	Values
U_n	Nominal voltage	400 V
$f_{\mathrm{s,n}}$	Nominal frequency	60 Hz
P_n	Nominal power	20 kW
n_n	Nominal speed	$1700\ \frac{1}{\mathrm{min}}$
p	Pole pair number	2
R_s	Stator resistance	0 Ω
R'_r	Rotor resistance	2 Ω
M	Mutual inductance	70 mH
$L_{\sigma,\mathrm{s}}$	Stator leakage inductance	2 mH
$L'_{\sigma,\mathrm{r}}$	Rotor leakage inductance	2 mH

Answer:

The no-load case is characterized by

$$s = \frac{\omega_\mathrm{slip}}{\omega_\mathrm{s}} = \frac{0}{\omega_\mathrm{s}} = 0, \qquad \omega_\mathrm{s} = \omega_\mathrm{r,el}.$$

Hence, we have

$$\omega_\mathrm{s} = 2\pi f_\mathrm{s} = 2\pi \cdot 60\ \mathrm{Hz} = 377\ \frac{1}{\mathrm{s}} = \omega_\mathrm{r,el}.$$

Converting this to the mechanical speed, we get

$$n_0 = \frac{\omega_{\mathrm{r,el}}}{2\pi p} \cdot 60\ \frac{\mathrm{s}}{\mathrm{min}} = 1800\ \frac{1}{\mathrm{min}}.$$

3.3 Calculate the nominal torque T_n and nominal slip s_n. [2 Points]

Answer:

The nominal mechanical power is given by

$$P_\mathrm{n} = T_\mathrm{n}\omega_{\mathrm{r,n}} = T_\mathrm{n} 2\pi n_\mathrm{n} \frac{1}{60}\ \frac{\mathrm{min}}{\mathrm{s}}.$$

Hence, we can calculate the nominal torque as

$$T_\mathrm{n} = \frac{P_\mathrm{n} 60}{2\pi n_\mathrm{n}} \frac{\mathrm{s}}{\mathrm{min}} = 112.35\ \mathrm{Nm}.$$

The slip angular frequency is

$$\omega_{\mathrm{slip,n}} = \omega_{\mathrm{s,n}} - \omega_{\mathrm{r,el,n}} = 2\pi(60\ \frac{1}{\mathrm{s}} - \frac{2 \cdot 1700}{60}\ \frac{1}{\mathrm{s}}) = 20.94\ \frac{1}{\mathrm{s}}$$

leading to a nominal slip of

$$s_\mathrm{n} = \frac{\omega_{\mathrm{slip,n}}}{\omega_{\mathrm{s,n}}} = \frac{20.94\ \frac{1}{\mathrm{s}}}{377\ \frac{1}{\mathrm{s}}} = 0.0556.$$

3.4 Determine the nominal electrical power $P_{\mathrm{el,n}}$ and the efficiency η_n. For this purpose neglect the impact of the stator leakage inductance. [2 Points]

Answer:

To determine the requested values, we first need to calculate the machine power losses. Since the stator ohmic losses can be neglected, we only have to consider the rotor ohmic losses. The nominal RMS rotor current is given by

$$I_{\mathrm{r,n}} = \frac{U_\mathrm{n}}{\sqrt{3}} \frac{1}{\sqrt{\frac{R'_\mathrm{r}}{s_\mathrm{n}}^2 + \left(\omega_\mathrm{s} L'_{\sigma,\mathrm{r}}\right)^2}} = 230.94\ \mathrm{V} \cdot \frac{1}{\sqrt{(35.71\ \Omega)^2 + (0.75\ \Omega)^2}} = 230.94\ \mathrm{V} \cdot \frac{1}{35.72\ \Omega} = 6.47\ \mathrm{A}.$$

The power losses of the rotor are then

$$P_{\mathrm{r,l,n}} = \frac{3}{2}(I_{\mathrm{r,n}})^2 \frac{R'_\mathrm{r}}{s_\mathrm{n}} = \frac{3}{2} \cdot 31.80\ \mathrm{A}^2 \cdot 35.72\ \Omega = 1492.91\ \mathrm{W}.$$

The total electrical power sums up to

$$P_{\mathrm{el,n}} = P_\mathrm{n} + P_{\mathrm{r,l,n}} = 20\ \mathrm{kW} + 1.49\ \mathrm{kW} = 21.49\ \mathrm{kW}.$$

The efficiency is then

$$\eta_\mathrm{n} = \frac{P_\mathrm{n}}{P_\mathrm{el,n}} = \frac{20\ \mathrm{kW}}{21.49\ \mathrm{kW}} = 0.931 = 93.1\ \%.$$

3.5 Determine the maximum possible torque T_max of the machine. Which speed n_max corresponds to that operating point? [2 Points]

Answer:

The maximum achievable torque is

$$T_\mathrm{max} = \frac{3}{2}p\frac{U_\mathrm{s}^2}{\omega_\mathrm{s}^2}\frac{M^2}{\sigma(L_{\sigma,\mathrm{s}}+M)^2(L'_{\sigma,\mathrm{r}}+M)} = \frac{3}{2}\cdot 2\cdot\frac{(230.94\ \mathrm{V})^2}{(376.99\ \frac{1}{\mathrm{s}})^2}\cdot\frac{(70\ \mathrm{mH})^2}{0.055\cdot(72\ \mathrm{mH})^3} = 268.72\ \mathrm{Nm}.$$

The corresponding slip frequency is

$$\omega_\mathrm{max} = \frac{R'_\mathrm{r}}{\sigma(L'_{\sigma,\mathrm{r}}+M)} = \frac{2\ \Omega}{0.055\cdot(72\ \mathrm{mH})} = 416.6\ \frac{1}{\mathrm{s}}$$

leading to a rotor angular speed of

$$\omega_\mathrm{r,max} = \frac{1}{p}(\omega_\mathrm{s} - \omega_\mathrm{max}) = -19.81\ \frac{1}{\mathrm{s}}$$

resulting in a mechanical speed of

$$n_\mathrm{max} = \frac{\omega_{r,\mathrm{max}}}{2\pi}\cdot 60\ \frac{\mathrm{s}}{\mathrm{min}} = -2.07\ \frac{1}{\mathrm{min}}.$$

Hence, the maximum torque is achieved at a negative speed which is necessary due to the relatively large slip frequency at the maximum torque operating point (which is mostly due to the relatively large rotor resistance).

Task 4: Synchronous machine **[13 Points]**

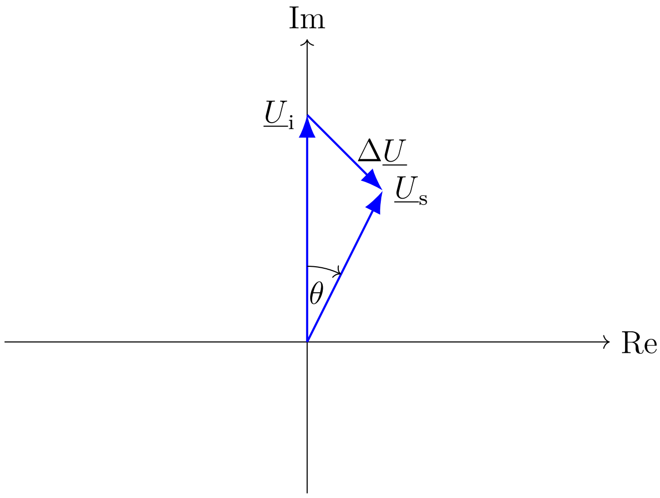


Figure 2: Phasor diagram of a synchronous machine with scaling 1 kV = 1 cm and 1 kA = 1 cm.

4.1 Determine the operating mode of the machine characterized by Fig. 2. [2 Points]

Answer:

The machine is operating as an overexcited generator, since the excitation voltage $\underline{U}_{\mathrm{i}}$ is leading the stator voltage $\underline{U}_{\mathrm{s}}$ by an angle θ and the induced voltage amplitude is larger than the stator voltage amplitude.

4.2 An experiment revealed the short-circuit current $I_{\mathrm{s,sc}} = 1.33$ kA for a nominal field excitation current $I_{\mathrm{f}} = 100$ A. Insert the short-circuit current into the above sketch and calculate the synchronous reactance X_{s}. [2 Points]

Answer:

The short-circuit current $\underline{I}_{\mathrm{s,sc}}$ is orthogonal to $\underline{U}_{\mathrm{i}}$ and lagging behind by 90°, that is, pointing towards the real axis with a length of 1.33 cm. The synchronous reactance is then

$$X_{\mathrm{s}} = \frac{\underline{U}_{\mathrm{i}}}{\underline{I}_{\mathrm{s,sc}}} = \frac{3\ \mathrm{kV}}{1.33\ \mathrm{kA}} = 2.26\ \Omega.$$

Here, the induced voltage $\underline{U}_{\mathrm{i}}$ has been obtained from optical measurement of the phasor diagram and also represents the open-circuit voltage. The completed phasor diagram is shown in Sol.-Fig. 1.

4.3 Determine the stator current $\underline{I}_{\mathrm{s}}$, the power factor $\cos(\varphi)$ and the corresponding angle φ. Add those into the above diagram. The nominal active power is $P = -4$ MW while the ohmic stator resistance can be neglected. [3 Points]

Answer:

The stator current is given by

$$I_\mathrm{s} = \frac{\Delta U}{X_\mathrm{s}} = \frac{1.41\ \mathrm{kV}}{2.26\ \Omega} = 0.62\ \mathrm{kA}.$$

From the nominal active power we can derive the power factor

$$\cos(\varphi) = \frac{P}{3U_\mathrm{s}I_\mathrm{s}} = \frac{-4\ \mathrm{MW}}{3\cdot 2.24\ \mathrm{kV}\cdot 0.62\ \mathrm{kA}} = -0.96.$$

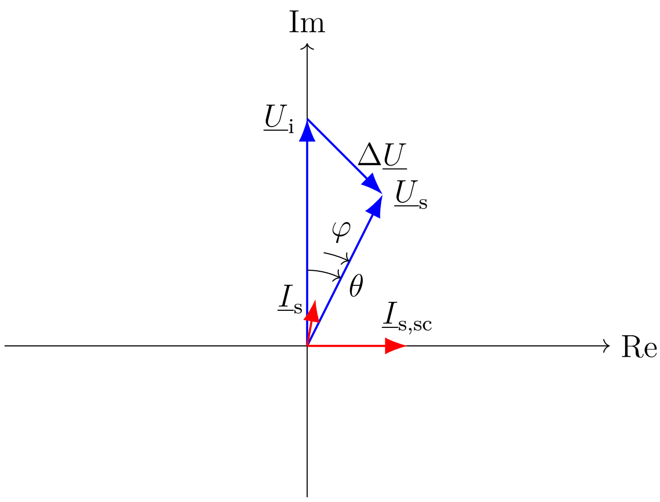
The angle φ is then

$$\varphi = -\arccos(0.96) = -16.25°.$$

The power factor angle is negative as the generator operates in an over-excited mode, i.e., its reactive power is negative. The resulting complex stator current is then

$$\underline{I}_\mathrm{s} = 0.62\ \mathrm{kA}\cdot e^{\mathrm{j}(16.25°+63.5°)} = 0.11\ \mathrm{kA} + \mathrm{j}0.61\ \mathrm{kA}.$$

Here, the angle 63.5° is the stator voltage angle counted from the real axis. The final solution phasor diagram is shown in Sol.-Fig. 1.



Solution Figure 1: Final phasor diagram

4.4 Determine the apparent power S and the reactive power Q. [2 Points]

<u>Answer:</u>
The apparent power is

$$S = 3U_\mathrm{s}I_\mathrm{s} = 3\cdot 2.24\ \mathrm{kV}\cdot 0.62\ \mathrm{kA} = 4.17\ \mathrm{MVA}.$$

The reactive power is

$$Q = 3U_\mathrm{s}I_\mathrm{s}\sin(\varphi) = 3\cdot 2.24\ \mathrm{kV}\cdot 0.62\ \mathrm{kA}\cdot\sin(-16.25°) = -1.17\ \mathrm{MVA}.$$

4.5 What torque T is associated with the above operating point for a pole pair number $p = 3$? What is the theoretical maximum torque T_{max} for the given stator voltage operating at a grid frequency of $f = 50$ Hz? [2 Points]

Answer:

The torque can be calculated from the active power as

$$T = \frac{P}{2\pi f/p} = \frac{3 \cdot -4 \text{ MW}}{2\pi \cdot 50 \text{ Hz}} = -38.2 \text{ kNm}.$$

Furthermore, the torque is also defined via

$$T = 3p\frac{U_\text{s}U_\text{i}}{\omega_\text{s}X_\text{s}}\sin(\theta) = T_\text{max}\sin(\theta)$$

where θ is the load angle. The maximum torque is then

$$T_\text{max} = \frac{T}{\sin(\theta)} = \frac{-38.2 \text{ kNm}}{\sin(-26.5°)} = 85.6 \text{ kNm}.$$

4.6 Determine a modified field excitation current I_f which delivers the same active power but reduces the reactive power to zero. Which load angle θ results? [2 Points]

Answer:

The described scenario renders $\varphi = 0$. The resulting stator current for this new scenario is

$$I_\text{s} = \frac{|P|}{3U_\text{s}} = \frac{4 \text{ MW}}{3 \cdot 2.24 \text{ kV}} = 0.60 \text{ kA}.$$

From that the new voltage difference is

$$\Delta U = X_\text{s}I_\text{s} = 2.26\ \Omega \cdot 0.6 \text{ kA} = 1.36 \text{ kV}$$

and the resulting induced voltage magnitude is

$$U_\text{i} = \sqrt{U_\text{s}^2 + \Delta U^2} = \sqrt{(2.24 \text{ kV})^2 + (1.36 \text{ kV})^2} = 2.62 \text{ kV}.$$

Since U_i is directly proportional to I_f, we can find

$$I_\text{f} = \frac{2.62 \text{ kV}}{3 \text{ kV}} \cdot 100 \text{ A} = 87.28 \text{ A}.$$

The new load angle θ is then

$$\theta = -\arccos\left(\frac{U_\text{s}}{U_\text{i}}\right) = -\arccos\left(\frac{2.24 \text{ kV}}{2.62 \text{ kV}}\right) = -31.24°.$$