Exercise 01: Fundamentals of the magnetic field

Acknowledgement: The following exercise is adapted from "Elektrische Maschinen und Antriebe Übungsbuch: Aufgaben mit Lösungsweg" by A. Binder, Springer, 2017

Task 1.1: Magnetic iron yoke

The iron core consists of thin single metal sheets with a cross-sectional area of $A = 900$ mm^2 and with an air gap of $\delta = 3$ mm. A simplified sketch is shown in Fig. 1.1.1. The material behavior of the selected iron is visualized in Fig. 1.1.2. The coil with N turns contains a direct current which results in a homogeneous magnetic flux density of $B_\delta = 1.8$ T in the air gap.

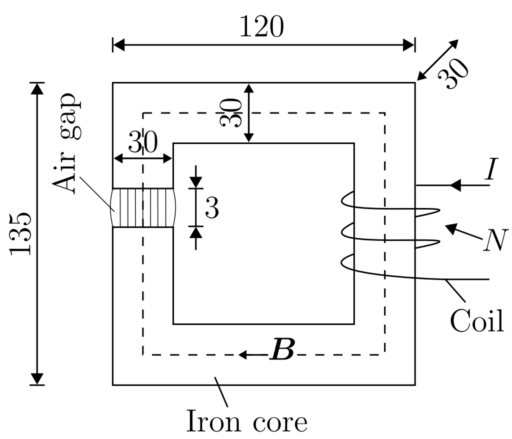


Figure 1.1.1: Simplified sketch of a magnetic iron core. All dimensions of the core are given in mm.

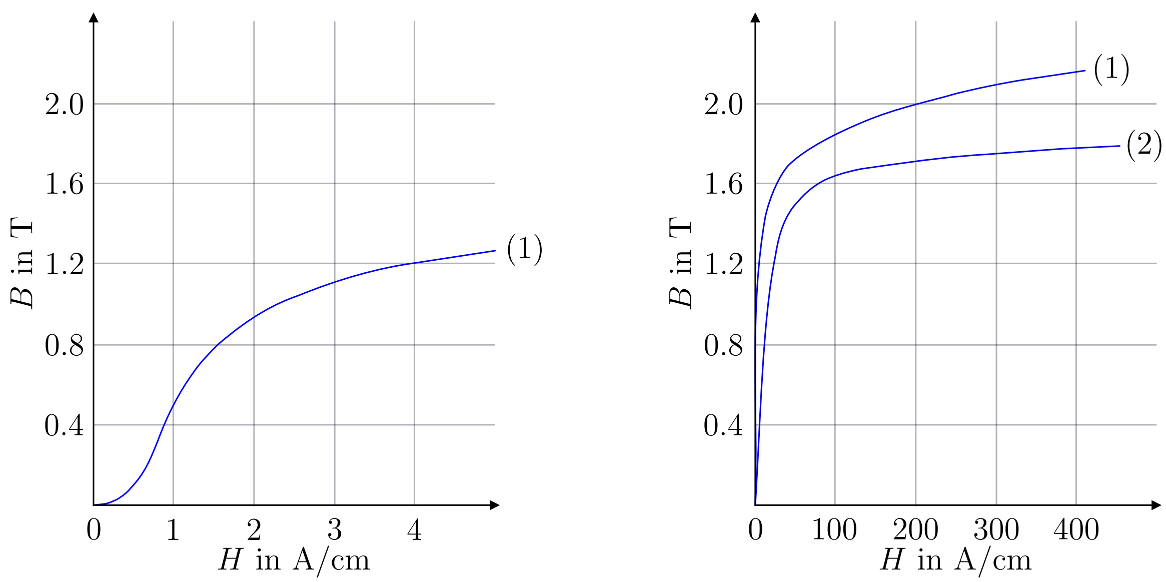


Figure 1.1.2: Direct current magnetization curves of electrical steel for (1) hot rolled processing with a thickness of 0.5 mm and (2) cold rolled processing with a thickness of 0.35 mm. The magnetization curve in figure on the left side is a zoomed version of material (1) for lower field strengths.

1.1.1 Calculate the magnetic flux ϕ_δ in the air gap.

1.1.2 How big is the magnetic flux density B_{Fe} in the iron core at the dotted line in Fig. 1.1.1? Neglect the leakage flux.

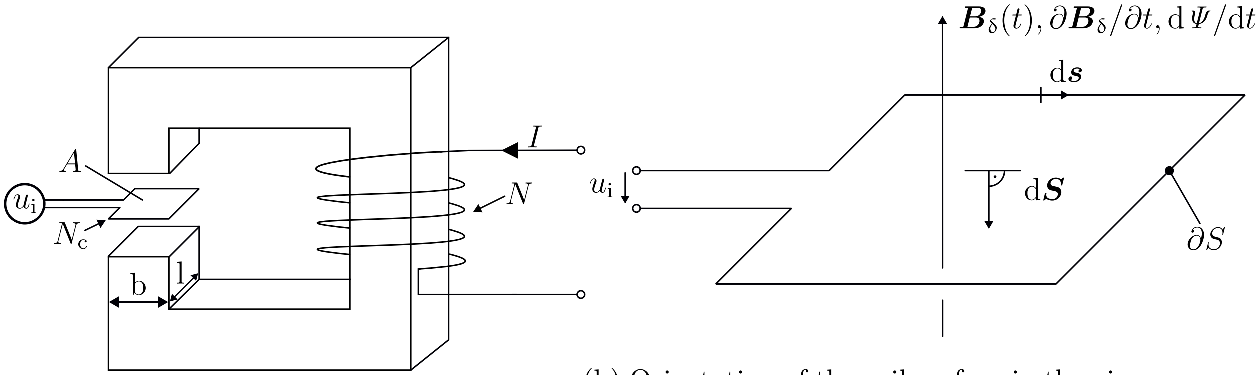
1.1.3 What is the value of the permeability μ and the magnetic field strengths H_δ and H_{Fe} in the air gap and in the iron for the given operating point?

1.1.4 What is the required magnetomotive force $\theta = N \cdot I$ in the excitation coil to excite the flux density $B_\delta = 1.8$ T?

1.1.5 What is the required current I if the coil has $N = 500$ turns?

Task 1.2: Electromagnetic induction

A magnetic circuit (cf. Fig. 1.2.1a) has the dimension of $\delta = 3$ mm, $b = l = 30$ mm. The excitation coil with $N = 500$ tuns id fed with an alternating current $i(t) = \hat{I}\sin(2\pi f t)$ with $f = 100$ Hz and $\hat{I} = 7.8$ A. The permeability of the iron μ_{Fe} is assumed to be infinite and the flux leakage can be neglected. A quadratic, non-moving coil with a side length of 30 mm and $N_{\mathrm{C}} = 10$ turns is within the air gap of the yoke. The orientation of the coil is shown in Fig. 1.2.1b.



(a) Iron yoke with a coil in the air gap.

(b) Orientation of the coil surface in the air gap of the given yoke.

Figure 1.2.1: Iron yoke and coil orientation.

1.2.1 Calculate the flux density $B_\delta(t)$ in the air gap. Sketch the trajectories of $i(t)$ and $B_\delta(t)$.

1.2.2 How large is the magnetic flux linkage $\Psi(t)$ of the magnetic field generated by the excitation coil to the coil in the air gap?

1.2.3 How large is the induced voltage $u_{\mathrm{i}}(t)$ of the coil in the air gap? Also, sketch the trajectory u_{i}.

1.2.4 Calculate the mutual inductance M between the excitation coil and the air gap coil.

Task 1.3: Moving current-carrying conductor in a magnetic field

An electrical conductor (length $l = 1$ m, resistance $R = 0.2\ \Omega$) is connected via two flexible supply lines from a battery (open circuit voltage $U_{\mathrm{B0}} = 12$ V, internal resistance $R_{\mathrm{Bi}} = 0.1\ \Omega$) and is excited with direct current I. The conductor is located in an air gap between two very long permanent magnets pole pieces, that are perpendicular to the conductor axis. The magnetic flux density directed downwards perpendicular to the conductor axis $B_\delta = 0.8$ T in the air gap. The self-inductance of the conductor and the wire connections is neglected.

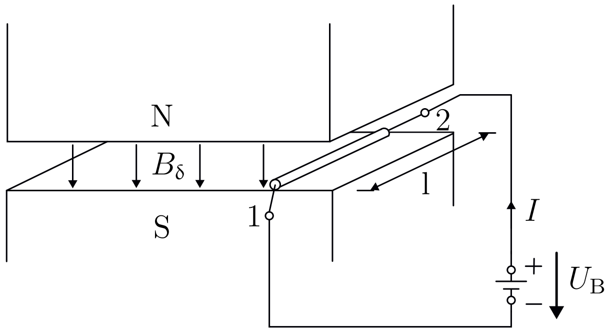


Figure 1.3.1: Current-carrying conductor in magnetic field.

1.3.1 Draw the electrical equivalent circuit diagram of the battery and resting conductor, and enter the direction of current flow I in the load convention style. How large is I?

1.3.2 In which direction shows the Lorentz force F on the conductor? How big is this force?

1.3.3 Draw the electrical equivalent circuit diagram for the combination of the moving conductor and feeding battery with the conditions at conductor side l. What additional electrical voltage occurs?

1.3.4 To what final velocity v_0 is the conductor in the air gap accelerated by F if no mechanical braking force acts on it and if one considers the air gap as arbitrary long? How is the current I in the conductor after reaching the final velocity?

1.3.5 Assume that the conductor experiences a braking force due to friction F_{R}= 10 N. To what final velocity v does the conductor now accelerate? What is the current I in the conductor?

1.3.6 What mechanical power P_{m} is required so that the conductor can move against the braking friction force $F_{\mathrm{R}} = 10$ N with the final velocity v determined in task 3.5. Sketch the curves $v(I)$ and $v(F)$ for a variable braking force F_{R} between v_0 and $v = 0$.

1.3.7 What is the electrical power drawn from the battery P_{el} for the operating point from task 3.5? What is the efficiency η and the power loss P_{l} when converting electrical power into mechanical power? How does the conductor act as an electromechanical energy converter?

Exercise 02: DC machines – design and conversion losses

Acknowledgement: The following exercise is adapted from "Elektrische Maschinen und Antriebe Übungsbuch: Aufgaben mit Lösungsweg" by A. Binder, Springer, 2017

Task 2.1: Six-pole loop winding

A six-pole DC machine with an axial laminated core length $l_\mathrm{z} = 120$ mm and a internal stator diameter $d_\mathrm{s} = 190$ mm has an ideal pole coverage $\alpha = 0.7$ and a maximum radial magnetic air gap flux density $\hat{B}_\delta = 0.85$ T at no-load. The armature of the machine is equipped with a two-layer lap winding with a coil winding number $N_\mathrm{c} = 20$ and $K = 31$ commutator segments. The machine has a interpole winding connected in series with the armature winding to improve commutation. The total resistance of the armature and interpole winding is $R_\mathrm{a} = 0.14\ \Omega$.

2.1.1 What is the total number z_a of armature conductors?

2.1.2 Calculate the induced voltage U_i at a rotational speed of $n = 4000$ 1/min.

2.1.3 The machine operates as a motor. For this purpose, a voltage of $U_\mathrm{a} = 600$ V is applied. How large is the armature current I_a?

2.1.4 Calculate the Lorentz force per conductor F_c and per pole F_pole. Calculate in addition the resulting average electromagnetic torque T. An air gap of $\delta = 1$ mm is assumed.

2.1.5 Calculate the motor losses. Assume that the iron losses and friction losses can be neglected as well as that the field excitation is produced via permanent magnets.

2.1.6 Calculate the efficiency η for the given operating point.

Task 2.2: Design parameters of a DC machine

The separately-excited four-pole DC machine with a two-layer lap winding has a stator with the diameter of $d_\mathrm{s} = 133$ mm and a length of $l_\mathrm{z} = 80$ mm. The armature has $Q = 30$ slots, $u = 3$ commutators per slot and layer as well as $N_\mathrm{c} = 9$ windings per armature coil. The maximum air gap flux density is $\hat{B}_\delta = 0.9$ T, the ideal pole coverage is $\alpha = 0.7$, and, the air gap width is $\delta = 1.5$ mm. The nominal speed of the machine is $n_\mathrm{n} = 1440\ \mathrm{min}^{-1}$, with an armature current of $I_\mathrm{a,n} = 22$ A. The excitation values are given with $I_\mathrm{f} = 0.5$ A and $U_\mathrm{f} = 230$ V.

2.2.1 What is the pole pitch τ_p and the flux per pole ϕ_δ?

2.2.2 What is the number of commutator elements K, the total number of armature conductors z_a and the number of parallel armature branches $2a$.

2.2.3 Determine the induced voltage U_i at nominal speed n_n and the electromagnetic torque T_n. What is the necessary armature voltage $U_\mathrm{a,n}$ during motor operation mode, when $R_\mathrm{a} = 1\ \Omega$? How large is the motor output power, neglecting the friction and the soft magnetic material losses (hysteresis + eddy current)? Determine the no-load rotational speed n_0 at the fixed flux ϕ_δ.

2.2.4 How many brush pairs does the machine have? How big is the current per brush? What is the circumferential speed of the armature under consideration of δ?

2.2.5 Determine the necessary excitation with ideal iron path ($\mu_\mathrm{r} \to \infty$) per pole. What is the number of necessary windings for each of the four coils of the excitation of the stator. Considering a real motor, is the excitation larger or smaller?

2.2.6 Determine the efficiency η_a of the armature and the resulting total efficiency η. Are these losses larger or smaller for real motors? Why? Give an explanation.

2.2.7 The motor should be operated with a constant armature voltage of $U_\mathrm{a,n} = 230$ V in the flux-weakening range with $T = 15$ Nm. What is the flux per pole ϕ_pole and the armature current I_a? How large is the resulting efficiency η, if the utilized iron shows no saturation and required field weakening is reached through reducing the field voltage U_f.

Task 2.3: Submarine with DC machine

A four-pole DC machine on the board of a submarine with $U_\mathrm{a,n} = 440$ V, $P_\mathrm{n} = 65$ kW, $n_\mathrm{n} = 1300\ \mathrm{min}^{-1}$ has an efficiency of $\eta_\mathrm{n} = 0.9$. The total losses P_l are separated in the ohmic losses $P_\mathrm{Cu,a}$ in the armature with 75 % and the soft magnetic material, friction, and additional losses ($P_\mathrm{Fe} + P_\mathrm{fr+add}$) with 25 % of the total losses P_l. The field excitation losses are neglected in the following.

2.3.1 Calculate the armature current $I_\mathrm{a,n}$, the torque T_n, the losses $P_\mathrm{Cu,a}$ and $P_\mathrm{fr+add}$, the armature winding resistance R_a and the no-load speed n_0.

2.3.2 Determine the value of the additional starter resistor R_d, such that the start-up torque T_1 at $U_\mathrm{a} = U_\mathrm{a,n}$ is limited to 150 % of the nominal torque.

2.3.3 With a power electronic converter, the armature voltage U_a is reduced to 0.8 $U_\mathrm{a,n}$. How large is the rotational speed n with the fixed flux ϕ_δ at the torque $T = 0.5\ T_\mathrm{n}$?

2.3.4 After the submarine surfaced, the machine is now used as a generator to charge the batteries powering with marine diesel at the speed $n_\mathrm{n} = 1530\ \mathrm{min}^{-1}$. What is the no-load induced voltage? How big is the armature voltage U_a at $I_\mathrm{a} = I_\mathrm{a,n}$ ($\phi = \phi_\delta$)?

2.3.5 How large is the voltage U_a at $\phi = 0.7\ \phi_\delta$ and $I_\mathrm{a} = I_\mathrm{a,n}/2$?

2.3.6 What is the rotational speed n such that the generator with the nominal flux ϕ_δ and the nominal current $I_\mathrm{a,n}$ induces an armature voltage of U_0?

Exercise 03: DC machines – operation behavior

Acknowledgement: The following exercise is adapted from "Grundlagen der Elektrotechnik Teil B" by J. Böcker, Paderborn University, 2020

Task 3.1: Series DC machine with DC and AC voltage supply

In this task, a ten-pole series DC machine is given. The supply voltage is $U_{\mathrm{DC}} = 325$ V with an electrical input power of $P_{\mathrm{el}} = 500$ W at a nominal speed of 1600 min^{-1}.

3.1.1 Calculate the nominal torque T_{n} and the nominal armature current $I_{\mathrm{a,n}}$ for an effective field inductance of $L_{\mathrm{f}}' = 1.24$ H.

3.1.2 Determine the efficiency for the given operating point.

3.1.3 The machine is manufactured with a lap winding, which contains $N_{\mathrm{a}} = 40$ armature windings. The number of field windings $N_{\mathrm{f}} = 10$ is given too. Calculate the field inductance L_{f}.

3.1.4 Calculate the peak and the average torque of the machine for an alternating voltage supply with $U = 230$ V and a frequency $f = 50$ Hz. Assume, that the armature inductivity is given as $L_{\mathrm{a}} = L_{\mathrm{f}}\frac{N_{\mathrm{a}}}{N_{\mathrm{f}}}$. Interpret the results.

Task 3.2: Shunt DC machine drive of a hand-guided grinder

3.2.1 Explain why a series machine is not suitable for a hand-guided grinder application.

3.2.2 Derive the steady-state torque-speed characteristic of a shunt DC machine and sketch it to highlight the usability of this DC machine configuration for a hand-guided grinder. Assume a constant voltage supply at the motor terminals.

3.2.3 Considering the motor parameters from Tab. 3.2.1, calculate the no-load speed ω_0 and the breakaway torque T_0.

Table 3.2.1: Parameters of an exemplary shunt DC machine.

Parameter	Description	Value
R_{a}	Armature winding resistance	8 Ω
R_{f}	Field winding resistance	150 Ω
L_{f}'	Effective field inductance	150 mH
U	Supply voltage	310 V

3.2.4 At which angular frequency does the grinder reaches its maximum mechanical output power? What is the maximum mechanical output power?

3.2.5 Calculate the efficiency of the shunt DC machine at the maximum mechanical output power.

3.2.6 How can the torque of a shunt DC machine be reversed for a given terminal voltage and speed?

Task 3.3: Unknown permanent magnet DC machine

An old, unknown permanent magnet DC machine is found. The only available information is the speed-torque characteristic shown in Fig. 3.3.1 which was retrieved from a partial data sheet remnant. Additionally, you measure the armature winding resistance with a multimeter and find $R_\mathrm{a} = 1.5\ \Omega$.

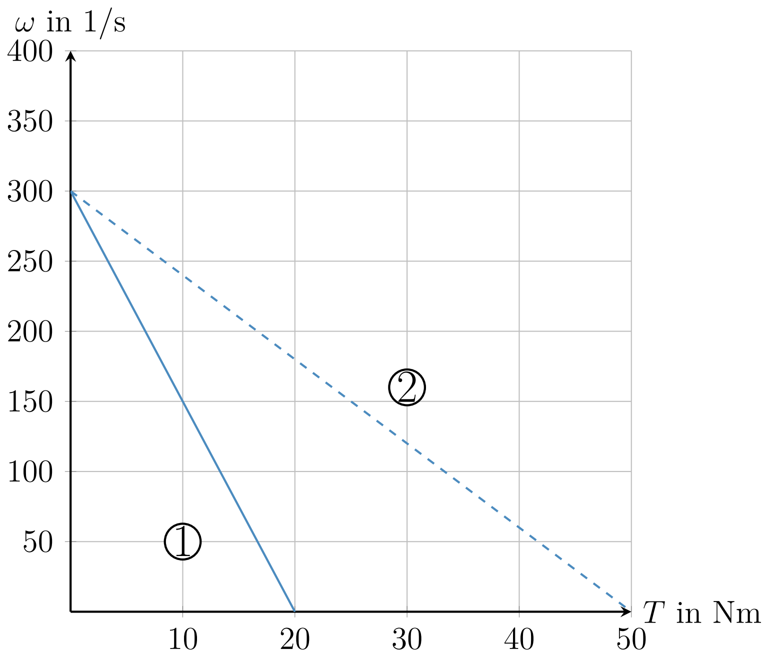


Figure 3.3.1: Torque-speed characteristic of the unknown permanent magnet DC machine (at nominal supply voltages)

3.3.1 It is known that the two characteristic curves in Fig. 3.3.1 were measured at the same supply voltage, but one was measured with and one without a dropping resistor in the armature circuit. Which curve is which? Explain your reasoning.

3.3.2 Determine the effective flux linkage ψ_f' and the nominal armature voltage.

3.3.3 Calculate the armature start-up current $I_{\mathrm{a},0}$.

3.3.4 You know that the start-up current with the active dropping resistor (slope 1) was limited to twice the nominal armature current. Calculate the nominal armature current I_a and the resistance of the dropping resistor R_d.

3.3.5 What is the machine's nominal operating point in terms of torque and angular frequency?

3.3.6 What is the machine's efficiency at the nominal operating point?

Exercise 04: Transformers

Acknowledgement: The following exercise is adapted from "Grundlagen der Elektrotechnik Teil B" by J. Böcker, Paderborn University, 2020

Task 4.1: Ideal transformer

Given is an ideal (no losses, no flux leakage) single-phase transformer with an apparent power of 5 kVA. The voltage U_1 on the primary side is 230 V and on the secondary side $U_2 = 110$ V. The transformer is operated at a frequency of 50 Hz.

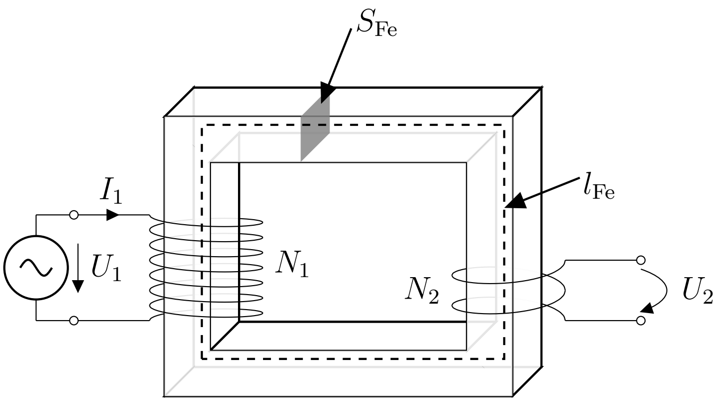
4.1.1 Determine the turn ratio $\ddot{u}$ of the transformer.

4.1.2 The transformer is operated at its rated operating point. Calculate the current through the primary and secondary winding.

4.1.3 Now, the transformer is operated on the secondary side with the rated terminal voltage and delivers an active power of $P_2 = 3.2$ kW with a power factor of $\cos\varphi = 0.8$ (inductive). Calculate the current of the primary and secondary winding.

Task 4.2: Magnetization current

A transformer has a primary voltage of $U_1 = 230$ V and a secondary voltage of $U_2 = 48$ V with a frequency of $f = 50$ Hz. The area of the iron core is $S_{\mathrm{Fe}} = 6$ cm^2. The average length of the iron core is given with $l_{\mathrm{Fe}} = 30$ cm. A simplified sketch of the transformer is shown on the left side in Fig. 4.2.1 below. On the right side, the magnetization curve of the utilized iron is visualized.



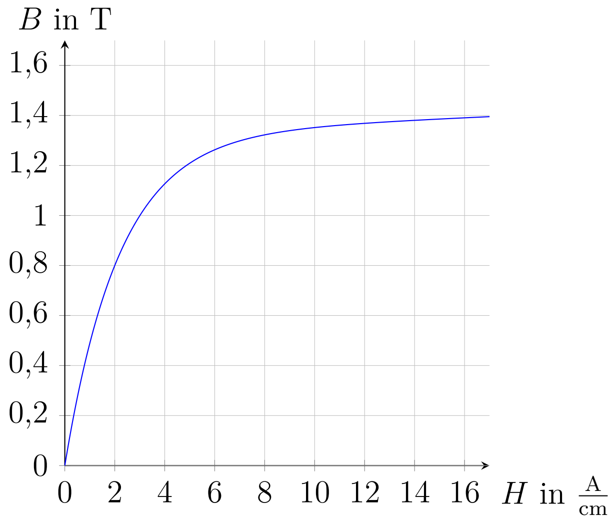


Figure 4.2.1: The left side shows a simplified sketch of the transformer given in the task. On the right hand side a magnetization curve is given.

4.2.1 How many winding turns are necessary, such that the maximal flux denxity in the iron is $\hat{B} = 0.8$ T? Hint: For the calculation of the winding turns, the magnetic coupling of the coils is assumed to be ideal. In addition, winding resistances are also neglected. Draw the T-equivalent circuit diagram for the given assumptions.

4.2.2 Which magnetization current I'_m consumed the transformer in the no-load operating mode? Assume that the iron losses are neglected.

4.2.3 Calculate the factor between the peak magnetization current $\hat{i}'_{\mathrm{m},2}$, when the applied voltage is risen from $U_1 = 230$ V to $U_1 = 400$ V.

Task 4.3: Parameter identification via no-load test

Given is a 50 Hz, 6 MVA single-phase transformer with $U_1 = 5$ kV and $U_2 = 100$ kV. The effective area of the core is $S_\mathrm{Fe} = 0.187$ m^2 and a maximum flux density of $\hat{B} = 1.5$ T. During the no-load operation, the primary voltage is $U_{1,\mathrm{o}} = 5$ kV with a no-load electrical power $P_\mathrm{o} = 8.8$ kW and the no-load current $I_\mathrm{o} = 2.6$ A.

4.3.1 Calculate the nominal currents and the transformer ratio.

4.3.2 Calculate the number of winding turns N_1 for the primary and N_2 for the secondary side.

4.3.3 Determine the iron losses and the apparent power S_o for no-load operation. Give the values for the magnetization current I'_m, the iron loss current I_c and the mutual inductance M'. Assume for the calculation that $R_1 << R_\mathrm{c}$ and $L_{1,\sigma} << M'$. In addition, draw the equivalent circuit diagram with the given assumptions.

Task 4.4: Inrush current

Given is a single-phase transformer in no-load operation (i.e., open circuit secondary side) which initial primary current is zero. A the time point $t = 0$, the voltage $u_\mathrm{n}(t) = \hat{u}_\mathrm{n}\sin(\omega t + \alpha)$ is applied. Remanence and iron losses are neglected. The self-inductance L_1, resistance R_1 and the number of winding turns N_1 are given.

4.4.1 Calculate the inrush current trajectory $i_1(t)$ and the trajectory of the flux $\phi(t)$.

4.4.2 Assume that $R_1 << \omega L_1$. Discuss the result for $i_1(t)$, when the voltage is applied at zero crossing with an angle $\alpha = 0$ and for an angle of $\alpha = \frac{\pi}{2}$.

Exercise 05: Rotating field theory and winding factor

Acknowledgement: The following exercise is adapted from "Geregelte Drehstromantriebe / Controlled AC Drives" by J. Böcker, Paderborn University, 2021 and "Elektrische Maschinen und Antriebe Übungsbuch: Aufgaben mit Lösungsweg" by A. Binder, Springer, 2017

Task 5.1: Fourier analysis of a field distribution of a three-phase winding

The winding of a three-phase and four-pole machine is described with the following parameters. The number of notches is given with $q = 2$ and the winding chording is $y/\rho_\mathrm{p} = 5/6$. The number of windings per coil is given with $N_\mathrm{c} = 5$ with $a = 1$. The air gap is given with $\delta = 1$ mm and the inner stator diameter is $d_\mathrm{s} = 80$ mm. The phase current root mean square value is given with $I_\mathrm{s} = 30$ A.

5.1.1 Calculate the number of slots per pole pair. In addition, determine the pole pitch ρ_p as an angular information and calculate the pole pitch τ_p as a distance in m. Furthermore, determine the number of conductors per phase.

5.1.2 Calculate the amplitude of the flux density fundamental in the air gap assuming an ideal homogenous and block-shaped flux distribution along the stator circumference.

5.1.3 Determine the amplitudes of the resulting flux density for the fundamental wave and the following six harmonic orders. Give the results as relative fractions to the amplitude of the fundamental wave. In addition, calculate the distribution, pitch and winding factor for the given harmonics and the fundamental wave.

5.1.4 Sketch the flux density of the fundamental wave in the air gap of phase a as a function of the electrical stator circumference ϑ_el. Assume $i_a(t) = I_s \cdot \sqrt{2}$, i.e., the phase a is at its current peak. In addition, draw the flux density of the 11^th harmonic.

Task 5.2: Distributed windings

Consider a 4-pole three-phase motor with 15 stator slots. A simplified sketch of this motor with the winding scheme of phase a is shown in Fig. 5.2.1.

5.2.1 Determine the pole pitch ρ_p and the number of notches. Is it an integral-slot or a fractional-slot winding?

5.2.2 Complete the winding scheme of the given machine in Tab. 5.2.1 for the phases b and c. Sketch the resulting winding scheme into Fig. 5.2.1.

5.2.3 How many layers are there in this winding scheme?

5.2.4 Calculate the complex winding factors $\underline{\xi}_{\mathrm{a},k}$ for the fundamental as well as for the 5^th and 7^th harmonic.

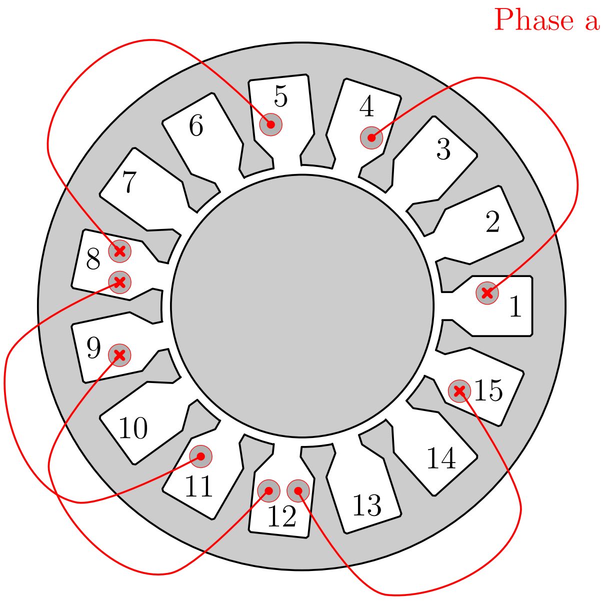


Figure 5.2.1: Simplified sketch of a distributed winding scheme. Only phase a is shown.

Table 5.2.1: Winding scheme of the distributed winding from Fig. 5.2.1.

Coil nr.	Phase a		Phase b		Phase c	
	In	Out	In	Out	In	Out
1	1	4				
2	8	5				
3	8	11				
4	9	12				
5	15	12				

5.2.5 Assume a block-shaped distribution of the flux density with a maximal value of $\hat{B} = 1$ T and a number of winding turns $N'_\mathrm{a} = 30$ (i.e., there are more turns per coil as indicated within Fig. 5.2.1). The axial length of the machine is $l = 0.35$ m and the diameter is $d_\mathrm{s} = 0.10$ m. The machine rotates with a mechanical speed of $n = 250\ \frac{1}{\mathrm{min}}$. Calculate the induced voltage of phase a for the fundamental wave as well as of the 5^th and 7^th harmonics.

5.2.6 Write a Python script to calcualte the complex winding factor $\underline{\xi}_{\mathrm{a},k}$ up to various harmonics.

Task 5.3: Concentrated windings

Consider the shown 10-pole 3-phase motor with 12 stator slots in Fig. 5.3.1. The winding scheme of phase a is shown in the figure.

5.3.1 Determine the pole pitch and und the number of notches. Is it an integral-slot or a fractional-slot winding?

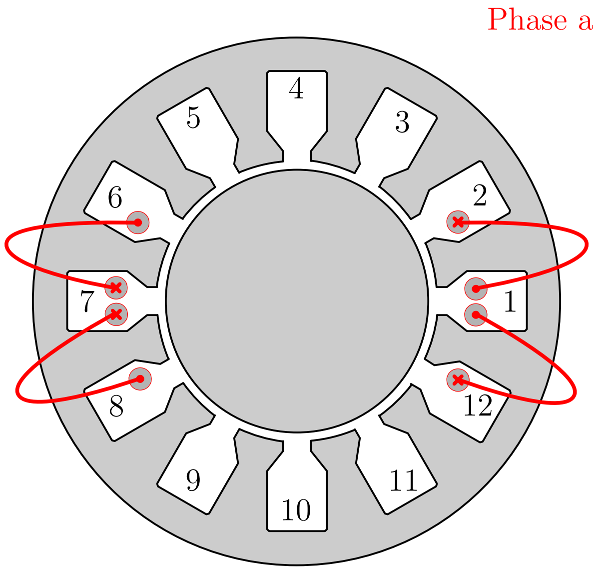


Figure 5.3.1: Simplified sketch of a concentrated winding. Only phase a is shown.

5.3.2 Complete Tab. 5.3.1 with the winding scheme for phases b and c. Sketch the resulting winding scheme in Fig. 5.3.1.

Table 5.3.1: Winding scheme of a concentrated winding from Fig. 5.3.1.

Coil nr.	Phase a		Phase b		Phase c	
	In	Out	In	Out	In	Out
1	2	1				
2	12	1				
3	7	6				
4	7	8				

5.3.3 How many layers are in this winding scheme?

5.3.4 Calculate the complex winding factors $\underline{\xi}_{\mathrm{a},k}$ for the fundamental as well as for the 5$^{\mathrm{th}}$ and 7$^{\mathrm{th}}$ harmonic.

5.3.5 Assume a block-shaped distribution of the flux density with a maximal value of $\hat{B} = 1$ T and a number of winding turns $N'_{\mathrm{a}} = 137$. The axial length of the machine is $l = 0.70$ m and the diameter is $d = 0.45$ m. The machine rotates with a mechanical speed of $n = 50\ \frac{1}{\mathrm{min}}$. Calculate the induced voltage of phase a for the fundamental wave. What are the 5$^{\mathrm{th}}$ and 7$^{\mathrm{th}}$ harmonics of the induced voltage in phase a?

5.3.6 Calcualte with a Python script the complex winding factor $\underline{\xi}_{\mathrm{a},k}$ up to various harmonics.

Exercise 06: Induction machines

Acknowledgement: The following exercise is adapted from "Geregelte Drehstromantriebe / Controlled AC Drives" by J. Böcker, Paderborn University, 2021

Task 6.1: Transient simulation of an induction machine

Given is a squirrel cage motor with the characteristics in Tab. 6.1.1. The motor is operated with the stator frequency $f_\mathrm{s} = 50$ Hz and a line-to-line voltage of $U_\mathrm{ll} = 400$ V, that is, connected to grid.

Table 6.1.1: Characteristics of the induction machine.

Symbol	Description	Value
T_n	Nominal torque	4.7 Nm
I_n	Nominal phase current	3.9 A
P_n	Nominal power	1.5 kW
n_n	Nominal speed	$3000\ \frac{1}{\mathrm{min}}$

6.1.1 Determine the equations of the induction machine in the rotor flux oriented coordinate system. Therefore, only the stator current $i_\mathrm{s,dq}(t)$ and the rotor flux $\psi_\mathrm{r,d}(t)$ should occur as state variables.

6.1.2 Write a Jupyter notebook to solve the derived equations in the dq coordinate system aligned to the rotor flux from the previous task. First, simulate the machine with no load. Analyze the transient motor response starting from the initial state $i_\mathrm{d}(t=0) = i_\mathrm{q}(t=0) = \psi_\mathrm{d}(t=0) = \omega_\mathrm{el}(t=0) = \epsilon_\mathrm{el}(t=0) = 0$ when excited with the above-mentioned stator voltage. Simulate as well as visualize the torque, speed, flux and current responses and plot the latter two in abc, $\alpha\beta$ and dq coordinates.

6.1.3 Add a speed dependent load with the following equation $T_\mathrm{l} = 0.00004 * \omega_\mathrm{r,el}^2$ to the machine model. Repeat the simulation from the previous task. How does the currents, torque and flux change? In addition, how changes the rotational speed of the machine between these two operating points?

Task 6.2: Steady-state operation of an induction machine

An induction machine with the characteristics in Tab. 6.2.1 is given.

6.2.1 Determine the amplitude of the complex AC voltage $\hat{U}$ and current phasor $\hat{I}$.

6.2.2 Determine the number of pole pairs p.

6.2.3 Calculate the rated slip s_n. Which slip occurs at an ideal no-load operation (no firction)?

6.2.4 Calculate the apparent, the reactive and the electrical power. In addition, determine the efficiency η_n for the rated operating point.

Table 6.2.1: Characteristics of the given induction machine.

Symbol	Description	Values
U_n	Nominal voltage	380 V
I_n	Nominal phase current	54 A
$f_\mathrm{s,n}$	Nominal frequency	50 Hz
P_n	Nominal power	25 kW
n_n	Nominal speed	1465 $\frac{1}{\mathrm{min}}$
$\cos(\varphi)$	Power factor	0.77
R_s	Stator resistance	0.48 Ω
R_r'	Rotor resistance	85 mΩ
M	Mutual inductance	100 mH
$L_{\sigma,\mathrm{s}}$	Stator leakage inductance	2 mH
$L_{\sigma,\mathrm{r}}'$	Rotor leakage inductance	2 mH

6.2.5 Determine the nominal torque generated by the induction machine.

6.2.6 Calculate the starting T_0 and the maximum torque T_max. For the latter determine first the slip s_max at the operating point with the maximum torque.

6.2.7 Determine the stator currents $I_\mathrm{s,dq}$, and, in addition, the induced rotor currents $I_\mathrm{r,dq}$.

6.2.8 Calculate the losses in the stator winding and in the rotor.

6.2.9 Draw the space vector diagram for the operation under rated conditions including the vectors $\underline{u}_\mathrm{s}$, $\underline{i}_\mathrm{s}$, $\underline{\psi}_\mathrm{s}$ and $\frac{\mathrm{d}}{\mathrm{d}t}\underline{\psi}_\mathrm{s}$. Assume that the current vector $\underline{i}_\mathrm{s}$ is oriented along the x-axis of the Cartesian coordinate system.

Exercise 07: Synchronous machines

Acknowledgement: Parts of the following exercise are adapted from "Elektrische Antriebstechnik" by J. Böcker, Paderborn University, 2020

Task 7.1: Transient simulation of a salient pole synchronous machine
Given is a salient pole synchronous machine with the parameters in Tab. 7.1.1.

Table 7.1.1: Parameters of the salient synchronous machine.

Symbol	Description	Value
R_s	Stator resistance	0.0196 Ω
R_f	Field winding resistance	54.7 Ω
$L_\mathrm{s,d}$	Stator inductance d-axis	2.7 mH
$L_\mathrm{s,q}$	Stator inductance q-axis	1.3 mH
L_f	Self field inductance	20.3 mH
M_fs	Mutual inductance	92.8 mH
I_n	Nominal stator current	450 A
$I_\mathrm{f,n}$	Nominal field current	7.854 A

7.1.1 Calculate the transient response for i_dq and i_f if a short circuit occurs at the running machine. Assume $i_\mathrm{d0} = i_\mathrm{q0} = 0$, $i_\mathrm{f0} = i_\mathrm{f,n}$ for $t = 0$ and a fixed rotational speed $\omega_\mathrm{r,el}$. Assume further that the ohmic stator resistance can be neglected for the short-term transient response.

7.1.2 Determine the maximum current $|i_\mathrm{s,dq}|$ for this scenario.

7.1.3 Write a Jupyter notebook to simulate the short transient response of the machine under the same initial conditions using an ODE solver. Compare the result including the impact of the stator resistance with the simplified analytical solution from the previous task. The simulation configuration is given in Tab. 7.1.2.

Table 7.1.2: Configuration of the simulation.

Symbol	Description	Value
T_sim	Simulation time	0.05 s
$\varepsilon_\mathrm{m,0}$	Start angle of the rotor	0°
$\omega_\mathrm{m,0}$	Start speed of the rotor	628.3 $\frac{1}{\mathrm{s}}$

7.1.4 Add a load with the following characteristic $T_\mathrm{l}(t) = 0.0001 \cdot \omega_\mathrm{r}^2(t)$ to the existing simulation. Repeat the simulation with the same initial conditions as in the task before. Extend the simulation time to $T_\mathrm{sim} = 0.5$ s and plot the currents $i_\mathrm{d}(t)$, $i_\mathrm{f}(t)$, the angular frequency of the rotor $\omega_\mathrm{r,el}(t)$ and the torque $T(t)$.

7.1.5 Now consider an additional damper winding with the parameters from Tab. 7.1.3 in the simulation. Compare the resulting current signalforms, the speed and the generated torque to the previous tasks with the identical simulation configuration as in the previous tasks. Consider the damper winding to not carry any current for $t = 0$.

Table 7.1.3: Parameters of the damper winding.

Symbol	Description	Value
M_{dD}	Mutual ind. rotor d-axis	2 mH
M_{qQ}	Mutual ind. rotor q-axis	1 mH
M_{fr}	Mutual ind. field-rotor	50 mH
L_{D}	Self ind. rotor d-axis	3 mH
L_{Q}	Self ind. rotor q-axis	2 mH
R_{rD}	Rotor winding res. d-axis	50 mΩ
R_{rQ}	Rotor winding res. q-axis	30 mΩ

Task 7.2: Cylinderical synchronous machine

In a thermal power station, a cylinderical synchronous generator is used which has the rated data in Tab. 7.2.1 and is connected in star. The generator is driven by a turbine. The converted energy is fed into the 50 Hz national grid via a transformer. Saturation effects and losses in the machine can be neglected.

Table 7.2.1: Parameters of the synchronous machine.

Symbol	Description	Value
$U_{\mathrm{star,n}}$	Star voltage	$\frac{10}{\sqrt{3}}$ kV
$I_{\mathrm{star,n}}$	Star current	6500 A
f_{n}	Nominal frequency	50 Hz
$\cos(\varphi_{\mathrm{n}})$	Power factor	0.8 (capacitive)
n_{n}	Nominal speed	1500 $\frac{1}{\mathrm{min}}$
$I_{\mathrm{f,n}}$	Field current	150 A
$I_{\mathrm{s,sc0}}$	Short circuit current	7800 A

7.2.1 Synchronization must be carried out to connect the synchronous generator to the electricity grid. Why is this process necessary? Name the synchronization conditions that must be checked before the generator can be connected to the grid.

7.2.2 How can the amount of active power output be influenced during generator operation? On the other hand, how can the inductive reactive power delivered or absorbed be influenced?

7.2.3 What is meant by synchronous condenser operation?

7.2.4 Calculate with the given data the pole pair number p, the synchronous reactance X_s, the apparent power S, the phase shift angle φ, the inner voltage U_i and the load angle θ at the rated conditions.

7.2.5 The generator should only take reactive power from the grid. How large is the stator current I_s and the field current I_f at this operating point, when a reactive power of 120 MVA is extracted from the grid? How large is the phase shift angle φ and the load angle θ? Draw the phasor diagram for this operating point.

7.2.6 In contrast to the previous subtask, the generator should provide 80 MW in addition to the reactive power now. How large is the stator current I_s, the phase shift angle φ and the load angle θ? Draw also for this operation point the phasor diagram.

7.2.7 Now, the generator should provide a reactive power of 60 MVA into the grid. How large is the generated torque T, that the generator operates within the rated apparent power? To which value must the field current I_f change, such that the rated current is reached in the stator winding?

Task 7.3: Steady-state operation of a synchronous machine

Consider a loss-free synchronous machine with the parameters from Tab. 7.3.1. First, the synchronous machine is operated as a a motor. Using the data given above, determine the following characteristic variables of the synchronous machine at rated operation. An example phasor diagram for this operation is shown in Fig. 7.3.1.

Table 7.3.1: Parameters of the synchronous machine.

Symbol	Description	Value
$U_\mathrm{star,n}$	Star voltage	$\frac{6}{\sqrt{3}}$ kV
$I_\mathrm{star,n}$	Star current	96 A
f_n	Nominal frequency	50 Hz
p	Pole pair number	2
$\cos(\varphi_\mathrm{n})$	Power factor	0.9 (capacitive)
T_n	Nominal torque	$\frac{T_\mathrm{max}}{2}$

7.3.1 How large is the maximum torque T_max?

7.3.2 Which load angle θ is set at the nominal operating point?

7.3.3 Determine the synchronous reactance X_s value.

7.3.4 Calculate the inner voltage U_i at the nominal operating point.

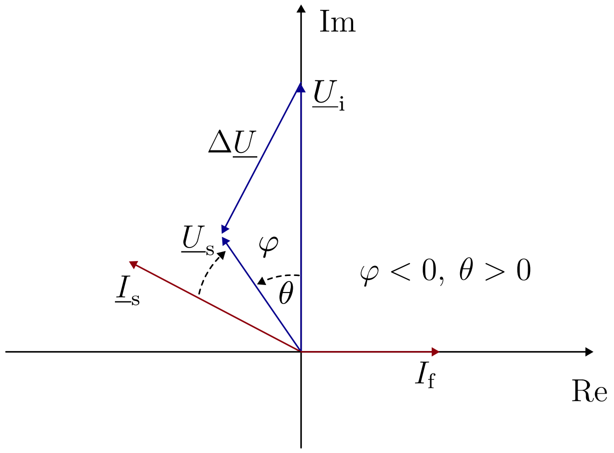


Figure 7.3.1: Phasor diagram of a over excited synchronous machine in motor mode.

7.3.5 In the next task, the machine operates as generator and the power is 500 kW. The phasor diagram for this operation is shown in Fig. 7.3.2. Which load angle θ is set at the new operating point?

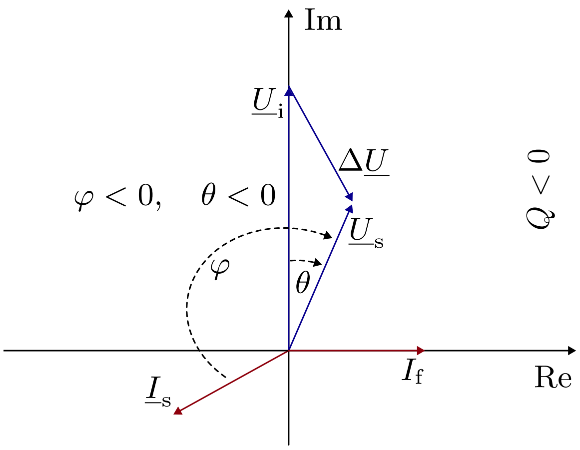


Figure 7.3.2: Phasor diagram of an over excited synchronous machine in generator mode.

7.3.6 Calculate the stator current I_s.

7.3.7 How large is the power factor $\cos(\varphi)$ at this operating point? In addition, calculate the angle φ.