Exercise 01: Safety in Automation and Static Characteristics of Measurements

Task 1.1: HARA for a 4-Corner Air Suspension System (4CAS)

A 4CAS system is used to maintain/control a vehicle's height during different operational conditions. These conditions gives rise to a number of hazards that could possibly lead to fatal results. Perform a Hazard Analysis and Risk Assessment for this mechatronic system and design system-level safety goals.

System Description:

- 4 pneumatic air springs, one at each corner of the vehicle.
- Each corner has a height sensor and a valve.
- ECU processes height data and control valves.
- Driver can select ride modes (Comfort, Eco, Sport, Off-road).
- ECU maintains level height during all conditions.

1.1.1 Identify different Operational Situations considering the common operating modes given in the system description.

Answer:

Considering common operating modes:

ID	Operational Situation	Description
OS1	Vehicle at standstill	Ignition on, suspension adjusting height
OS2	Normal driving	Vehicle moving at speed 0–120 km/h
OS3	Off-road mode	Uneven terrain, higher suspension level
OS4	Emergency braking	Sudden deceleration while suspension is adjusting

1.1.2 Identify the potential hazards that could arise from pneumatic, mechanical or E/E subsystems.

Answer:

Hazard ID	Description	Type
H1	Loss of air pressure in one spring → uneven height	Pneumatic / mechanical
H2	Sensor gives wrong height reading → incorrect leveling	E/E
H3	Valve fault (eg. Stuck open) → continuous height change	Electro-mechanical
H4	ECU fault → uncontrolled suspension movement	Software / control
H5	Power failure → suspension collapses at standstill	Electrical

1.1.3 Combine the hazards and operational situations to define the Hazardous Events.

Answer:

Event ID	OS ID	Hazard	Potential Effect
HE1	OS2	H1	Sudden tilt, loss of stability → possible loss of control
HE2	OS3	H2	Incorrect leveling → reduced ground clearance, bottoming out
HE3	OS2	H3	Continuous height oscillation → handling instability
HE4	OS4	H4	Unintended height adjustment during braking → change in braking dynamics
HE5	OS1	H5	Vehicle collapses on suspension → risk of injury when entering/exiting

1.1.4 Perform a Risk Assessment based on the Severity (S), Exposure (E) and Controllability (C) using ISO 26262 Framework. Also derive ASIL (Automotive Safety Integrity Level).

Answer:

ISO 26262 is the international standard for functional safety of electrical and electronic (E/E) systems in production automobiles.

It provides a framework and set of processes to ensure that automotive systems are designed, developed, and validated so that they do not cause unreasonable risk to human life, even if hardware or software faults occur.

It ensures safety by design.

The standard is particularly important for systems like Advanced Driver Assistance Systems (ADAS) and autonomous driving technologies.

ASIL (Automotive Safety Integrity Level)

A measure of required risk reduction — how "safe" a function must be.

Event ID	OS ID	Hazard
A	Low	Seat heating malfunction
B	Moderate	Wrong tire pressure reading
C	High	Loss of steering assist
D	Very High	Unintended acceleration
QM	Quality Management	Non-safety-critical (e.g., radio)

* Note: These scores may change depending upon the application and intuition.

Level	Description	Typical consequence	Example
S1	Light to moderate injuries	Minor discomfort, property damage	Slight height deviation at low speed
S2	Severe injuries (survivable)	Broken bones, major bruising	Sudden vehicle tilt at low speed
S3	Life-threatening or fatal injuries	Could lead to death or serious injury	Loss of vehicle control at high speed

Table 1.1.1: S - Potential extent of harm to a person if the hazardous event takes place.

Level	Description	Frequency of Situation	Example
E1	Very low	Happens rarely	Off-road mode used only occasionally
E2	Medium	Occurs sometimes	City driving (occasional highway)
E3	High	Happens frequently	Normal driving on road (daily)
E4	Very High	Almost always	Engine running, normal driving

Table 1.1.2: E - Probability of a vehicle being in a particular operational situation.

Level	Description	Meaning	Example
C1	Easily controllable	Driver can react safely	Height change is slow, driver adjusts
C2	Normally controllable	Average driver can manage	Noticeable tilt, needs careful steering
C3	Difficult or uncontrollable	Driver cannot avoid accident	Sudden suspension collapse at speed

Table 1.1.3: C - Likelihood that the driver can avoid harm after the event starts.

Event ID	S*	E*	C*	ASIL*
HE1	3	3	2	C
HE2	2	2	2	B
HE3	3	2	2	B
HE4	3	2	1	B
HE5	2	1	3	QM (Quality Management)

Table 1.1.4: ASIL Score is then determined by ASIL Determination Matrix.

1.1.5 Derive Mechatronics Safety Goals.

Answer:

Safety Goal ID	Description	Related Hazard(s)	ASIL
SG1	Prevent uncontrolled or asymmetric height changes during driving.	H1, H3, H4	C
SG2	Detect and isolate faulty height sensors to prevent wrong adjustments.	H2	B
SG3	Maintain stable vehicle height even under partial system failure.	H1, H5	B
SG4	Ensure safe stop and stable height on power loss.	H5	QM

Task 1.2: Measurement Error and Accuracy

A pressure gauge with a measurement range of 0 - 10 bar has a quoted inaccuracy of 1.0% of the full-scale reading.

1.2.1 What is the maximum measurement error expected for this instrument?

Answer:

The maximum error expected in any measurement reading is 1.0% of the full-scale reading, which is 10 bar for this particular instrument. Hence, the maximum likely error is:

$$\text{Maximum error} = 1.0\% \times 10 \text{ bar} = 0.1 \text{ bar}$$

1.2.2 What is the likely measurement error expressed as a percentage of the output reading if this pressure gauge is measuring a pressure of 1 bar?

Answer:

The maximum measurement error is a constant value related to the full-scale reading of the instrument, irrespective of the magnitude of the measured quantity.

Actual Pressure	**Max Error (bar)**	**Error as % of Reading**
10 bar (full scale)	0.1	1.0%
5 bar	0.1	2.0%
1 bar	0.1	10.0%
0.5 bar	0.1	20.0%

Table 1.2.1: Relative measurement errors at different pressures

At lower readings, the relative error becomes much larger. Therefore, if we are measuring pressures between 0 and 1 bar, we would not use an instrument with a measurement range of 0 - 10 bar.

Task 1.3: Measurement Precision

The width of a room is measured 10 times by an ultrasonic sensor and the following measurements are obtained (in meters):

$$3.521, 3.539, 3.538, 3.536, 3.525, 3.522, 3.534, 3.523, 3.527, 3.529$$

The width of the same room is then measured by a calibrated steel tape that gives a reading of 3.520 m, which can be taken as the correct value for the width of the room.

1.3.1 What is the measurement precision of the ultrasonic sensor?

Answer:

The mean (average) value of the 10 measurements made with the ultrasonic sensor is:

$$\bar{x} = 3.529 \text{ m}$$

The maximum deviation below this mean value is -0.008 m and the maximum deviation above the mean value is 0.01 m. Thus, the precision of the ultrasonic rule can be expressed as max (-0.008, 0.01) = 0.01 m.

$$\text{Precision} = 0.01 \text{ m}$$

1.3.2 What is the maximum measurement inaccuracy of the ultrasonic sensor?

Answer:

The correct value of the room width has been measured as 3.520 by the calibrated steel rule. All ultrasonic sensor measurements are above this, with the largest value being 3.539 m. The maximum measurement error can be calculated as:

$$\text{Maximum error} = 3.539 - 3.520 = 0.019 \text{ m (19 mm)}$$

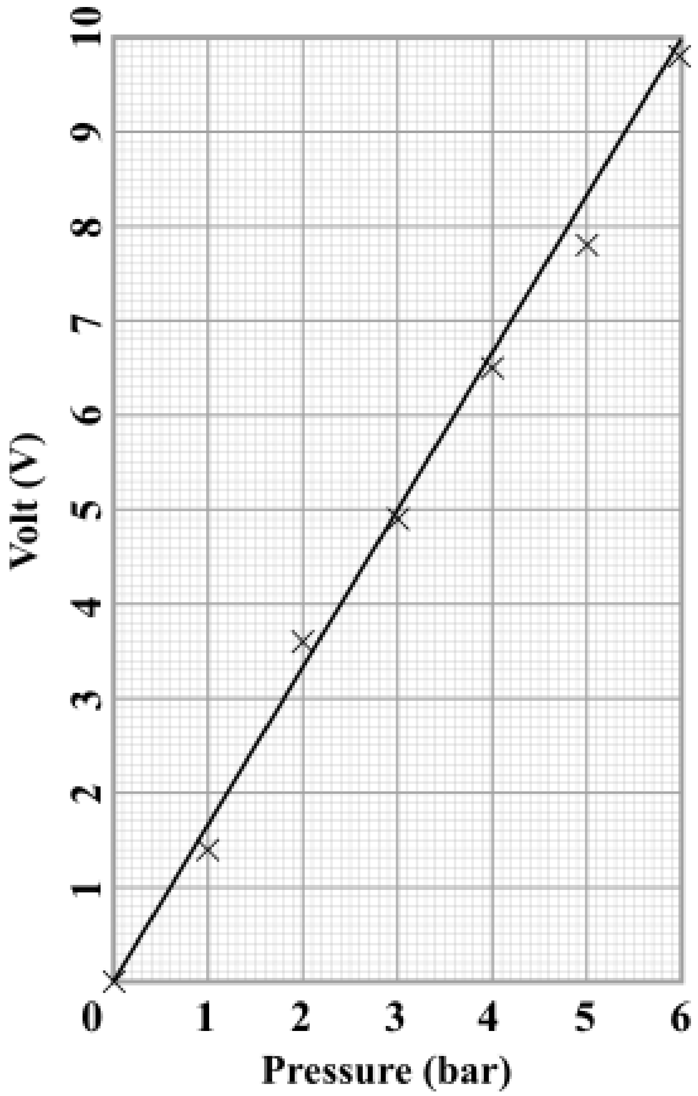
Thus, the maximum measurement inaccuracy can be expressed as:

$$\text{Inaccuracy} = 19 \text{ mm}$$

Task 1.4: Linearity

Suppose the instrument characteristic shown in figure below is that of a pressure sensor, in which the input units are pressure in bars from 1 to 6 bar, and the output units are expressed in volts from 1 to 10 V.

1.4.1 What is the maximum nonlinearity expressed as a percentage of the full-scale deflection?


Volt (V)
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
Pressure (bar)

Answer:

The maximum nonlinearity is the maximum deviation of any data point on the figure away from a straight line drawn through the data points. This deviation is indicated by the thick vertical line in the figure. The length of this line is 0.5 V, while the full-scale deflection (FSD) is 10.0 V.

Thus, the maximum nonlinearity is calculated as:

$$\text{Nonlinearity} = \frac{0.5}{10} \times 100 = 5\%. \tag{1.4.1}$$

Therefore, the maximum nonlinearity is 5 % of the full-scale deflection.

1.4.2 What is the resolution of the sensor as determined by the instrument characteristic given?

Answer:

The resolution of the sensor, as determined from the graph, is the smallest change in input pressure that can be detected. For the given graph, the naked eye cannot reliably distinguish any smaller change than one small square on the graph paper, corresponding to one-tenth of a unit (0.1 bar).

Hence, the resolution of the pressure sensor is:

$$\text{Resolution} = 0.1 \text{ bar}. \tag{1.4.2}$$

Task 1.5: Sensitivity

The following resistance values of a platinum resistance thermometer were measured at a range of temperatures. Determine the measurement sensitivity of the instrument in ohms per degree Celsius ($\frac{\Omega}{°\text{C}}$).

Table 1.5.1: Measured resistance values of a platinum resistance thermometer.

Resistance (Ω)	**Temperature (°C)**
307	200
314	230
321	260
328	290

1.5.1 Determine the measurement sensitivity of the instrument.

Answer:

From the data in Tab. 1.5.1, it can be observed that the relationship between resistance and temperature is approximately linear. For a temperature change of $\Delta T = 30$ °C, the corresponding change in resistance is $\Delta R = 7\ \Omega$.

Hence, the measurement sensitivity of the instrument is:

$$S = \frac{\Delta R}{\Delta T} = \frac{7}{30} = 0.233\ \frac{\Omega}{°\text{C}}. \tag{1.5.1}$$

Therefore, the sensitivity of the platinum resistance thermometer is 0.233 $\frac{\Omega}{^\circ\text{C}}$.

Task 1.6: Zero Drift

This table shows the output measurements of a voltmeter under two sets of conditions:

1. use in an environment kept at 20 °C, which is the temperature at which it was calibrated; and
2. use in an environment at 50 °C.

Table 1.6.1: Voltmeter readings at different ambient temperatures.

Input	**Reading at** 20 °C	**Reading at** 50 °C
1	10.2	10.5
2	20.3	20.6
3	30.7	31.0
4	40.8	41.1

1.6.1 Determine the zero drift and the zero drift coefficient.

Answer:

From Tab. 1.6.1, it is observed that each corresponding reading at 50 °C is consistently 0.3 V higher than the reading at 20 °C.

Hence, the **zero drift** is:

$$\Delta V = 0.3 \text{ V}. \tag{1.6.1}$$

The **zero drift coefficient** is the drift magnitude divided by the temperature change that caused it:

$$k = \frac{\Delta V}{\Delta T} = \frac{0.3}{50 - 20} = \frac{0.3}{30} = 0.01 \text{ V}/^\circ\text{C}. \tag{1.6.2}$$

Therefore, the zero drift coefficient of the voltmeter is 0.01 V/°C.

Exercise 02: Sensors

Task 2.1: Piezoelectric accelerometer under shock + drift

You're designing a 10 g full-scale accelerometer for impact testing using a PZT stack in the longitudinal mode (use d_{33} = 593 pC/N). The proof mass is m = 5 g. The shock is a half-sine a(t) = a_{pk} $\sin(\pi t/T)$ with a_{pk} = 500 m/s^2, T = 4 ms. The ceramic disk has active area A = 100 mm^2, thickness t = 0.5 mm, relative permittivity ε_r = 1500. Assume quasi-static force transfer F(t) = m a(t) and a charge amplifier with C_f = 100 pF, ideal op-amp.

2.1.1 Compute the sensor charge $Q(t)$ and amplifier output $v_o(t)$.

Answer:

Force:

$$F(t) = m\,a(t) = m\,a_{\text{pk}}\sin\left(\frac{\pi t}{T}\right).$$

Piezoelectric charge (direct effect, quasi-static transfer):

$$Q(t) = d_{33}\,F(t) = d_{33}\,m\,a_{\text{pk}}\sin\left(\frac{\pi t}{T}\right).$$

Charge amplifier ideal output:

$$v_o(t) = -\frac{Q(t)}{C_f} = -\frac{d_{33}\,m\,a_{\text{pk}}}{C_f}\sin\left(\frac{\pi t}{T}\right).$$

Calculating using the given values:-

$$F_{\text{peak}} = m\,a_{\text{pk}} = 0.005 \times 500 = 2.5\text{ N},$$

$$Q_{\text{peak}} = d_{33}\,F_{\text{peak}} = 593 \times 10^{-12} \times 2.5 = 1.4825 \times 10^{-9}\text{ C} = 1482.5\text{ pC},$$

$$v_{o,\text{peak}} = -\frac{Q_{\text{peak}}}{C_f} = -\frac{1.4825 \times 10^{-9}}{100 \times 10^{-12}} = -14.825\text{ V}.$$

Therefore, the time-domain results are:

$$Q(t) = 1.4825 \times 10^{-9}\sin\left(\frac{\pi t}{4\text{ ms}}\right)\ \text{C},$$

$$v_o(t) = -14.825\sin\left(\frac{\pi t}{4\text{ ms}}\right)\ \text{V},$$

for $0 \le t \le 4$ ms (and zero outside the half-sine).

Note: The output $v_o(t)$ follows a negative half-sine waveform with a peak amplitude of approximately -14.825 V, due to the inverting nature of the charge amplifier.

2.1.2 Estimate the sensor capacitance C_s and discuss how C_s versus C_f affects the scale factor and low-frequency droop.

Answer:

Ideal charge amplifier: For an ideal op-amp, the output depends only on the generated charge and the feedback capacitor:

$$v_o(t) = -\frac{Q(t)}{C_f}$$

so ideally the sensor capacitance C_s does not affect the scale factor, which is set by $1/C_f$.

$$C_s = \frac{\varepsilon_0 \varepsilon_r A}{t} \approx 2.7 \text{ nF}, \quad \text{with } C_s \gg C_f,$$

$$\text{the op-amp forces the signal charge onto } C_f: \quad |v_o| = \frac{Q}{C_f}.$$

Smaller C_f increases sensitivity but also raises noise and saturation risk.

2.1.3 If the device warms by +40°C, qualitatively discuss the expected sensitivity shift and explain why a constant force cannot be measured indefinitely with a piezoelectric sensor.

Answer:

Piezo sensors' sensitivity depends on d_{33} and ε_r, which decrease with temperature, while leakage and dielectric loss increase, causing faster signal droop. They generate charge under stress but cannot hold a DC output because leakage and dielectric absorption make the signal decay. Therefore, piezo sensors are best for measuring dynamic or transient forces, not constant ones.

Task 2.2: Strain-gauge bridge

A steel cantilever ($E = 210$ GPa) carries a tip load that produces surface strain $\varepsilon = 800\ \mu\varepsilon$ at the gauge location. Four identical 120 Ω foil strain gauges (gauge factor $k \approx 2$) are arranged in a full bridge with 5.0 V excitation. All four gauges experience the same temperature rise of +30 °C.

2.2.1 Derive the bridge output U_A with two active gauges in tension and two in compression.

Answer:

Take the standard Wheatstone full-bridge. Label resistances clockwise: R_1 (top left), R_2 (bottom left), R_3 (top right), R_4 (bottom right).

Choosing the following arrangement.

$$R_1 = R + \Delta R \text{ (tension)}, R_2 = R - \Delta R \text{ (compression)}, R_3 = R - \Delta R \text{ (compression)}, R_4 = R + \Delta R \text{ (tension)}$$

Node voltages (top node = U_{exc}, bottom node = 0):

$$V_A = U_{\text{exc}} \frac{R_2}{R_1 + R_2}, \qquad V_B = U_{\text{exc}} \frac{R_4}{R_3 + R_4}$$

Bridge output:

$$U_A = V_A - V_B$$

Substitute the resistances and linearize for $\Delta R \ll R$:

$$U_A \approx -U_{\text{exc}} \frac{\Delta R}{R} = -U_{\text{exc}}\, k\, \varepsilon$$

(Sign depends on which midpoint is subtracted from which; magnitude is $U_{\text{exc}} k \varepsilon$.)
Numerical value:

$$U_A = U_{\text{exc}}\, k\, \varepsilon = 5.0 \times 2 \times 800 \times 10^{-6} = 0.008 \text{ V} = 8.0 \text{ mV}$$

Thus, the full-bridge gives an 8 mV output for the stated strain.

2.2.2 Show why a full bridge largely cancels temperature effects.
Answer:
Temperature Compensation in a Full Bridge:
If all four gauges experience the same temperature change, adding ΔR_T to each arm, the common-mode shifts cancel out. The bridge output $V_A - V_B$ depends only on strain.
Practical note: small errors can occur due to TCR mismatches, uneven mounting, or self-heating, but a full bridge greatly reduces temperature effects compared to a single gauge or half-bridge.

2.2.3 If the adhesive allows only $I_{\text{exc}} \leq 15$ mA to avoid self-heating, check compliance.
Answer:
For a full bridge with $U_{\text{exc}} = 5$ V and two series legs ($\approx 2R$) in parallel:

$$R_{\text{eq}} = \frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)} \approx R$$

$$I_{\text{exc}} = \frac{U_{\text{exc}}}{R_{\text{eq}}} \approx \frac{5}{120} = 41.7 \text{ mA} > 15 \text{ mA (too high)}$$

Maximum excitation for $I_{\text{exc}} \leq 15$ mA:

$$U_{\text{exc,max}} = I_{\max} R = 0.015 \times 120 = 1.8 \text{ V}$$

Power at $U_{\text{exc}} = 5$ V:

$$P_{\text{tot}} = \frac{U_{\text{exc}}^2}{R_{\text{eq}}} = 0.208 \text{ W}, \quad P_{\text{gauge}} \approx 52 \text{ mW}$$

At $U_{\text{exc}} = 1.8$ V, $P_{\text{gauge}} \approx 6.75$ mW (safe).

Task 2.3: Silicon piezoresistive membrane pressure sensor: bias-point design

A square silicon diaphragm carries four diffused piezoresistors in a full bridge. At rated pressure, the longitudinal surface strain is $+500\ \mu\varepsilon$ at two opposite edges and $-500\ \mu\varepsilon$ at the other two. Each resistor is nominally 3 kΩ, with piezoresistive gauge factor $k_\pi = 80$ (n-type).

2.3.1 Find the small-signal bridge sensitivity $S = dU_A/dp$

Answer:

For each resistor, $\Delta R/R = k_\pi \varepsilon$. With two $+\varepsilon$ and two $-\varepsilon$:

$$\frac{U_A}{U_{\text{exc}}} \approx \frac{k_\pi \varepsilon}{2}$$

With $k_\pi = 80$ and $\varepsilon = 500 \times 10^{-6}$:

$$U_A \approx 0.02\, U_{\text{exc}}\ (20\ \text{mV/V})$$

Bridge sensitivity formula:

$$S = \frac{dU_A}{dp} = U_{\text{exc}}\, k_\pi\, \frac{d\varepsilon}{dp}$$

2.3.2 Discuss why silicon (vs. metal SG) achieves much higher sensitivity and how temperature-dependent ρ enters

Answer:

Silicon's large piezoresistive effect ($k_\pi \approx 80$) gives much higher sensitivity than metal gauges ($k \approx 2$). Temperature affects resistivity $\rho(T)$, adding a TCR term to $\Delta R/R$.

$$\frac{\Delta R}{R} = k_\pi\, \varepsilon + \alpha_T\, \Delta T$$

The bridge cancels common temperature shifts, but additional compensation is usually required.

Task 2.4: Hall vs field-plate (Gaussian/MR) for isolated current sensing

Measure up to 200A DC through a busbar with 1 mm air gap using either:
A. a Hall plate ($d = 10\ \mu$m, $b = 200\ \mu$m, $I_x = 5$ mA); or
B. a field-plate MR element ($R_0 = 1$ kΩ, $\alpha \approx -0.004\ \text{K}^{-1}$).
Flux density at full scale: $B = 80$ mT.

2.4.1 For A, estimate U_H using $U_H = (A_H B/d) I_x$ with a typical semiconductor A_H.

Answer:

$$U_H = \frac{A_H B}{d} I_x$$

With $A_H = 5 \times 10^{-4}$ m^3/C, $B = 0.08$ T, $d = 10^{-5}$ m, $I_x = 5$ mA:

$$U_H = \frac{5 \times 10^{-4} \cdot 0.08}{1 \times 10^{-5}} \cdot 0.005 = 0.02 \text{ V} = 20 \text{ mV}$$

Typical Hall output = 4–40 mV for A_H between 10^{-4} and 10^{-3} m^3/C

2.4.2 For B, use 5 mA constant-current bias and the $R(B)$ characteristic to estimate ΔV.
Answer:
With MR ratio $\Delta R/R_0 \approx 0.02$ at $B = 80$ mT:

$$\Delta V = I\, R_0 \frac{\Delta R}{R_0} = 0.005 \times 1000 \times 0.02 = 0.1 \text{ V} = 100 \text{ mV}$$

So MR yields = 100 mV output, higher than the Hall signal.

2.4.3 Compare temperature robustness and reasons to pick A vs. B.
Answer:
Temperature sensitivity: MR has $\alpha \approx -0.004$ K^{-1}, giving:

$$\left.\frac{\Delta R}{R}\right|_T = \alpha \Delta T, \quad \Delta V_T = I\, R_0\, \alpha \Delta T \approx -20 \text{ mV/K}$$

⇒ strong temperature drift, requires compensation.
Hall devices also vary with temperature, but typically only a few % over tens of °C due to changes in A_H and mobility — smaller drift and often compensated in integrated Hall ICs.
Choice: Hall → simple, linear DC response, moderate sensitivity (20 mV), better thermal stability. MR → higher sensitivity (100 mV) but large $|\alpha|$, needs calibration or temperature control.

Task 2.5: Magnetostrictive position sensor: time-of-flight and linearity
A magnetostrictive rod (wave speed $v = 2850$ m/s) is used in a non-contact absolute position transducer. A torsional pulse is launched at $t = 0$ and detected after interacting with a magnet placed at unknown position x along a 0.6 m rod. The measured time is $t = 152$ µs.

2.5.1 Estimate x.
Answer:
Assuming the pulse travels one way (no reflection path),

$$x = vt = 2850 \times 152 \times 10^{-6} = 0.4332 \text{ m} \approx 433 \text{ mm}.$$

2.5.2 Correct for the electronics delay of 3.0 µs.
Answer:
Corrected travel time:

$$t_{\text{true}} = t - 3.0\ \mu\text{s} = 152 - 3 = 149\ \mu\text{s}.$$

Then,

$$x_{\text{corr}} = vt_{\text{true}} = 2850 \times 149 \times 10^{-6} = 0.4247 \text{ m} \approx 425 \text{ mm}.$$

The delay caused an overestimate of ≈ 8.5 mm.

2.5.3 Temperature effect on position error.

Answer:

Given $\frac{\Delta v}{v} = -0.02\%/\text{K}$, for a +20 K change:

$$\frac{\Delta v}{v} = -0.0002 \times 20 = -0.004.$$

Since $x \propto v$,

$$\Delta x = -0.004 \times 0.4247 = -1.70 \times 10^{-3} \text{ m} \approx -1.7 \text{ mm}.$$

Thus, a +20 K temperature rise (uncorrected) causes ≈ -1.7 mm error.

2.5.4 Principle of operation (Villari and Wiedemann effects).

Answer:

Wiedemann effect (launch): A current pulse through the rod interacts with a magnetic bias, producing a torsional strain wave.

Villari effect (detection): The torsional stress under the external magnet alters the local magnetization, inducing a voltage in a pickup coil.

The time-of-flight of this torsional wave gives the absolute position of the magnet.

Task 2.6: Eddy-current proximity sensor: oscillator detuning and metal target

An inductive proximity sensor uses an LC oscillator with $L = 120\ \mu$H, $C = 680$ pF. A conductive aluminium target causes an inductance drop $\Delta L = -12\%$ at the switch point due to eddy currents.

2.6.1 Find the free-run frequency and the switch-point frequency.

Answer:

The free-run oscillator frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{120 \times 10^{-6} \times 680 \times 10^{-12}}} \approx 557 \text{ kHz}.$$

At the switch point, $L' = 0.88L$:

$$f_s = \frac{1}{2\pi\sqrt{L'C}} = \frac{f_0}{\sqrt{0.88}} \approx 594 \text{ kHz}.$$

2.6.2 Explain why conductive non-magnetic targets reduce L and how coil Q plays in detection margin.

Answer:

Conductive targets (e.g., aluminium) support eddy currents that *oppose* the coil's magnetic flux (Lenz's law), thus reducing effective L and increasing oscillator frequency.
Coil quality factor:

$$Q = \frac{\omega L}{R} = \frac{2\pi f_0 L}{R}.$$

High Q means a sharp, strong oscillation and higher sensitivity to ΔL; low Q broadens resonance and reduces detection margin.
Example: for $L = 120\ \mu\text{H}$, $R = 10\ \Omega$, $f_0 = 557$ kHz,

$$Q = \frac{2\pi(5.57 \times 10^5)(1.2 \times 10^{-4})}{10} \approx 4.2,$$

a typical moderate value for practical proximity sensors.
Eddy currents $\Rightarrow$ lower L, higher f.
High $Q \Rightarrow$ sharper frequency or amplitude change, improving switch detection.

Task 2.7: Capacitive thickness gauge with fringing

A non-contact thickness sensor measures a plastic sheet ($\varepsilon_r = 3.0$) with parallel electrodes $A = 4\ \text{cm}^2$ and nominal gap $d = 0.50$ mm. Sheet thickness variations Δd are to be resolved.

2.7.1 Compute nominal capacitance C_0.
Answer:

$$C_0 = \varepsilon_0 \varepsilon_r \frac{A}{d} = \frac{8.854 \times 10^{-12} \cdot 3.0 \cdot 4.0 \times 10^{-4}}{5.0 \times 10^{-4}} = 2.12 \times 10^{-11}\ \text{F} = 21.2\ \text{pF}.$$

2.7.2 If the sheet bows, causing a +5% fringing-equivalent increase in effective area, estimate the apparent thickness error if the parallel-plate model is used.
Answer:
If $A_{\text{eff}} = 1.05A$, then $C_{\text{meas}} = 1.05C_0$ while the model assumes A constant:

$$d_{\text{inf}} = \frac{\varepsilon_0 \varepsilon_r A}{C_{\text{meas}}} = \frac{d}{1.05} \Rightarrow \frac{\Delta d}{d} = \frac{1}{1.05} - 1 \approx -4.76\%.$$

Numerically, $\Delta d \approx -0.024\ \text{mm} = -23.8\ \mu\text{m}$. Thus, ignoring fringing makes the sheet appear ≈24 µm thinner.

Task 2.8: Field-plate (Gaussian effect) tacho vs toothed wheel: waveform and calibration

A field-plate bridge measures speed via a steel toothed wheel. Bridge excitation $U_{\text{exc}} = 3.3$ V, nominal arm $R_0 = 1.2\ \text{k}\Omega$. At the peak of a tooth the active element resistance increases by +8%.

2.8.1 Output waveform and shape
Answer:

As each tooth passes, the magnetic field B rises and falls, changing the resistance as $R = R_0(1 \pm \delta)$. The bridge output is:

$$U_A = U_{\text{exc}}\,\delta.$$

Because $R(B)$ is nonlinear and the field shape around the tooth is not perfectly sinusoidal, the output is a *quasi-sinusoidal but slightly distorted* waveform.

2.8.2 Estimate peak-to-peak output

Answer:

With opposite arms varying by $\pm 8\%$:

$$\Delta U_A \approx 2\,U_{\text{exc}}\,\delta_{\max} = 2 \times 3.3 \times 0.08 \approx 0.53\ \text{V}_{\text{p-p}}.$$

Hence about 0.26 V amplitude on each side of zero.

2.8.3 If the element self-heats by +15 K, estimate the DC drift using $\alpha \approx -0.004\ \text{K}^{-1}$ and discuss compensation.

Answer:

Self-heating by +15 K and $\alpha = -0.004\ \text{K}^{-1}$ gives:

$$\frac{\Delta R}{R} = \alpha \Delta T = -0.004 \times 15 = -0.06,$$

i.e. a -6% baseline drop, causing a DC offset in the bridge. Use ratiometric or differential sensing and temperature correction to cancel this drift.

Task 2.9: Thermistor/RTD Self-Heating: Measurement-Current Limit

An RTD with $R(25°\text{C}) = 100\ \Omega$ measures air temperature. The thermal resistance to ambient is $\Theta = 400$ K/W. The readout uses a constant current I. Allow at most +0.2 °C self-heating error.

2.9.1 Find the maximum measurement current $I_{\max}$.

Answer:

The self-heating temperature rise is

$$\Delta T = \Theta I^2 R.$$

Solving for $I_{\max}$:

$$I_{\max} = \sqrt{\frac{\Delta T}{\Theta R}} = \sqrt{\frac{0.2}{400 \cdot 100}} \approx 2.24\ \text{mA}.$$

2.9.2 If the response time is dominated by thermal capacitance $C_{\text{th}} = 30$ mJ/K, estimate the time constant τ and discuss the speed/accuracy trade-off.

Answer:

The thermal time constant is

$$\tau = \Theta \cdot C_{\text{th}} \approx 400 \cdot 0.03 = 12 \text{ s}.$$

Note: Increasing I improves the electrical signal-to-noise ratio (SNR), but it also increases self-heating ΔT. A compromise must be made: choose I such that the self-heating error is within the allowed budget while still meeting the required measurement bandwidth.

Task 2.10: Hall Current Sensor with Magnetic Circuit: Sizing and SNR

A C-core with a 1.0 mm gap surrounds a conductor carrying up to 150 A DC. The mean magnetic path length is $l_m = 120$ mm, $\mu_r = 2000$. A Hall plate with thickness $d = 20$ μm and bias current $I_x = 8$ mA sits in the gap.

2.10.1 Estimate the magnetic flux density B in the gap at full scale.
Answer:
The gap dominates the reluctance, so the magnetic flux density can be estimated as:

$$B \approx \frac{\mu_0 I}{g}$$

with $N = 1$ turn. Substituting values:

$$B \approx \frac{4\pi \times 10^{-7} \cdot 150}{1.0 \times 10^{-3}} \approx 0.19 \text{ T}.$$

2.10.2 Estimate the Hall voltage U_H for a semiconductor and a metal.
Answer:
The Hall voltage is:

$$U_H = \frac{A_H B}{d} I_x$$

- **Semiconductor:** A_H is large $\rightarrow$ U_H in practical range of tens of mV.
- **Metal:** A_H is small $\rightarrow$ U_H is tiny, often in μV range.

2.10.3 Give two reasons why a Hall sensor is preferred over an inductive pickup in this application.

Answer:

1. Hall sensors measure **DC and low-frequency currents directly**, while inductive pickups require changing magnetic flux (dB/dt) and cannot measure steady DC.
2. Hall sensors can provide a **compact, direct voltage output** proportional to current, simplifying signal processing, whereas inductive sensors need integration circuits for DC estimation.

Task 2.11: Light Barrier Sensor for Fast Conveyor

A sensor is needed for a "light barrier" application . The sensor must detect when a box on a fast-moving conveyor belt ($v = 2$ m/s) breaks a 1 cm wide light beam. The beam break time is:

$$t = \frac{0.01 \text{ m}}{2 \text{ m/s}} = 5 \text{ ms}.$$

The sensor's output must react within this time. Two components are considered (both based on the internal photoelectric effect):

1. Photoconductor (Photoresistor)
2. Silicon Photodiode

The text states that photoconductors are slow (response in seconds at low light) while photodiodes are fast (usable up to GHz).

2.11.1 Explain the physical reason for this major difference in response speed.

Answer:

Photoconductor: A photon excites an electron from the valence band to the conduction band, creating an electron-hole pair. These carriers increase the material's conductivity. The signal ends only when the carriers recombine, which is a slow, statistical process. At low light, the dark state is re-established very slowly (seconds).

Photodiode: Electron-hole pairs are generated within or near the depletion zone of a pn-junction. The built-in electric field quickly sweeps electrons to the n-side and holes to the p-side, creating a photocurrent. The signal ends immediately when photons stop, as there are no free carriers waiting to recombine. This drift-current mechanism is vastly faster than the slow recombination in photoconductors.

2.11.2 Which device is the only viable choice for this 5 ms application?

Answer:

The application requires a response time < 5 ms:

- The **Photoconductor** is not viable; its light-to-dark response is on the order of seconds.
- The **Photodiode** is suitable; its response time is limited by capacitance and is typically in the nanosecond-to-microsecond range, more than 1000 times faster than required.

Thus, the **Silicon Photodiode** is the correct choice.

Task 2.12: NTC Thermistor and Linearization

An NTC thermistor (Negative Temperature Coefficient resistor) is used to measure temperature. Its resistance follows:

$$R(T) = R_N \, e^{B\left(\frac{1}{T} - \frac{1}{T_N}\right)}$$

with the following parameters:

- Nominal resistance $R_N = 10$ kΩ at $T_N = 25$°C (298.15 K)
- Material constant $B = 3950$ K

2.12.1 Calculate the sensor resistance $R(T)$ at $T = 0°\text{C}$ and $T = 50°\text{C}$.
Answer:
At $T = 0°$C (273.15 K):

$$\begin{aligned} R(0°\text{C}) &= 10\ \text{k}\Omega \cdot e^{3950\left(\frac{1}{273.15} - \frac{1}{298.15}\right)} \\ &= 10\ \text{k}\Omega \cdot e^{3950(0.003661 - 0.003354)} \\ &= 10\ \text{k}\Omega \cdot e^{1.2126} \\ &\approx 33.6\ \text{k}\Omega \end{aligned}$$

At $T = 50°$C (323.15 K):

$$\begin{aligned} R(50°\text{C}) &= 10\ \text{k}\Omega \cdot e^{3950\left(\frac{1}{323.15} - \frac{1}{298.15}\right)} \\ &= 10\ \text{k}\Omega \cdot e^{3950(0.0030945 - 0.003354)} \\ &= 10\ \text{k}\Omega \cdot e^{-1.025} \\ &\approx 3.59\ \text{k}\Omega \end{aligned}$$

Self-check: $R(0°\text{C}) > R_N > R(50°\text{C})$, consistent with NTC behavior.

2.12.2 Explain qualitatively how adding a parallel resistor R_P helps linearize the sensor output.
Answer:
The $R_T(T)$ curve of the NTC is convex (steep at low temperatures, flatter at high temperatures). Adding a parallel resistor R_P gives an equivalent resistance:

$$R_{\text{eq}} = \frac{R_T \cdot R_P}{R_T + R_P}$$

- **At low temperatures (0°C):** $R_T = 33.6\ \text{k}\Omega \gg R_P$. Then $R_{\text{eq}} \approx R_P = 10\ \text{k}\Omega$, reducing the effective slope.
- **At high temperatures (50°C):** $R_T = 3.59\ \text{k}\Omega \ll R_P$. Then $R_{\text{eq}} \approx R_T = 3.59\ \text{k}\Omega$, leaving the slope almost unchanged.

The parallel resistor compresses the high-resistance (low-temperature) end of the curve more than the low-resistance (high-temperature) end, flattening the exponential behavior and making the response more linear over the 0–50°C range. This reduces sensitivity slightly but significantly improves linearity for simple analog voltage dividers.

Exercise 03: Signal Conditioning

Task 3.1: Band-Stop Filter

The signal conditioning section of an instrumentation system requires a tunable filter at its input in order to reject selected spot frequencies in the range 1 kHz to 3 kHz. The circuit chosen to undertake this filtering task is the *Wien frequency bridge*, as shown in Figure 3.1.1, having component values as follows:

$$C_1 = C_2 = 0.01\,\mu\text{F}, \qquad R_3 = 100\,\text{k}\Omega.$$

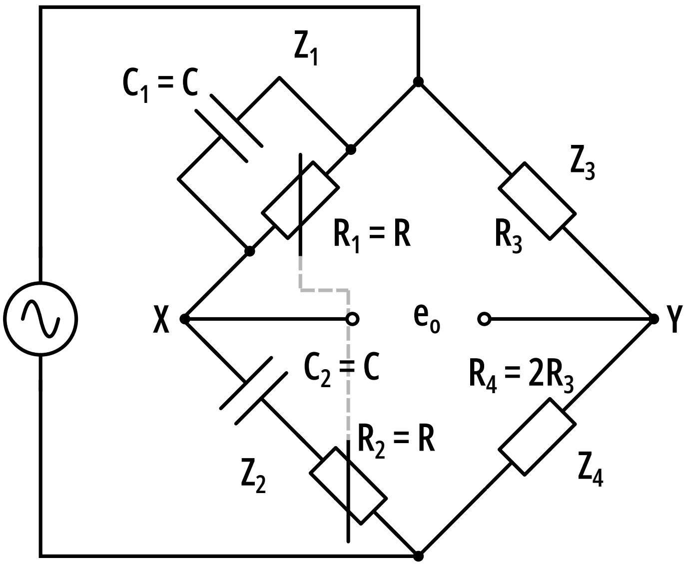


Figure 3.1.1: Wien frequency bridge circuit

3.1.1 State a suitable value for R_4.

Answer:

For the Wien frequency bridge used as a notch (band-stop) filter, the bridge null occurs when the following condition is satisfied:

$$\frac{R_4}{R_3} = 2.$$

Given:

$$R_3 = 100\ \text{k}\Omega,$$

the required value of R_4 is:

$$R_4 = 2R_3 = 200\ \text{k}\Omega.$$

Therefore, a suitable choice is:

$$\boxed{R_4 = 200\ \text{k}\Omega.}$$

3.1.2 Calculate the higher and lower values of R_1 and R_2 necessary to achieve the required frequency coverage.

Answer:

For the Wien frequency bridge (with $C_1 = C_2 = C$ and $R_1 = R_2 = R$ at balance), the bridge null (centre) frequency is

$$f_0 = \frac{1}{2\pi RC}.$$

Rearranging for R:

$$R = \frac{1}{2\pi f_0 C}.$$

Given:

$$C_1 = C_2 = C = 0.01\ \mu\text{F} = 0.01 \times 10^{-6}\ \text{F} = 1.0 \times 10^{-8}\ \text{F},$$

and the required frequency coverage $f_0 = 1$ kHz to 3 kHz, compute the corresponding resistor values.

For the lower frequency $f_0 = 1$ kHz:

$$R_{\text{high}} = \frac{1}{2\pi(1 \times 10^3)(1.0 \times 10^{-8})} = \frac{1}{2\pi \times 10^{-5}}$$
$$= 1.5915494309 \times 10^4\ \Omega \approx 15.915\ \text{k}\Omega.$$

For the higher frequency $f_0 = 3$ kHz:

$$R_{\text{low}} = \frac{1}{2\pi(3 \times 10^3)(1.0 \times 10^{-8})} = \frac{1}{6\pi \times 10^{-5}}$$
$$= 5.3051647697 \times 10^3\ \Omega \approx 5.305\ \text{k}\Omega.$$

Thus, to cover the band 1 kHz to 3 kHz you should make R_1 and R_2 equal and adjustable over the range:

$$\boxed{R_1 = R_2 \approx 5.305\ \text{k}\Omega\ \text{(for 3 kHz) to } 15.915\ \text{k}\Omega\ \text{(for 1 kHz).}}$$

Task 3.2: Shering Bridge

A transducer sensor having a passive impedance comprising a capacitance of 0.1 μF in series with a resistance of 500 Ω is interfaced to a signal conditioning system by a Schering bridge as shown in Figure 3.2.1. With no stimulus applied to the sensor the bridge is in balance and the output voltage, e_0, is zero. The bridge component values at balance are as follows:

$$C_1 = 0.025\ \mu\text{F}, \qquad C_3 = 0.1\ \mu\text{F}, \qquad R_1 = R_2 = 2\ \text{k}\Omega,$$

R_4 and C_4 together comprise the sensor, and the a.c. supply is 10 V at a frequency of 1 kHz. If the sensor is stimulated so causing its capacitance to increase by 10%, calculate the bridge output voltage.

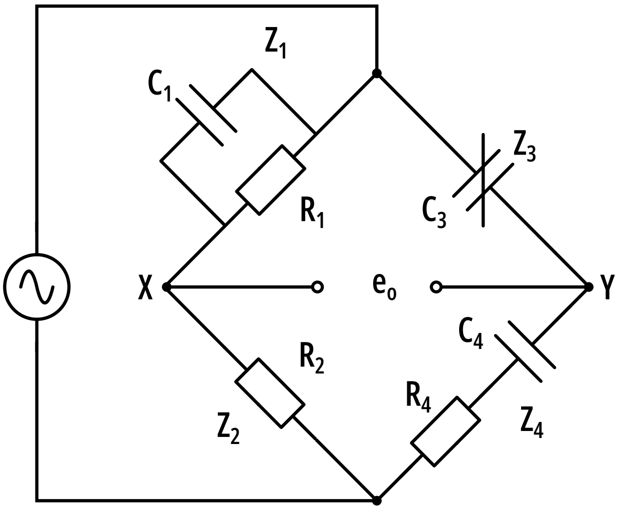


Figure 3.2.1: Figure 1.10 The Schering bridge.

Answer:

We use the Schering-bridge topology shown in Fig. 3.2.1. The four arms (measured from the top-left, top-right, bottom-left, bottom-right nodes) have the following impedances:

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}, \qquad Z_2 = R_2,$$
$$Z_3 = \frac{1}{j\omega C_3}, \qquad Z_4 = R_4 - \frac{j}{\omega C_4}.$$

Given (at balance):

$$R_1 = R_2 = 2\ \text{k}\Omega, \quad C_1 = 0.025\ \mu\text{F}, \quad C_3 = 0.1\ \mu\text{F},$$

$$R_4 = 500\ \Omega, \quad C_4 = 0.1\ \mu\text{F}, \quad f = 1\ \text{kHz}, \quad V_s = 10\ \text{V}.$$

Angular frequency:

$$\omega = 2\pi f = 2\pi(1000) = 6283.185\ \text{rad/s}.$$

At the left node (X) and right node (Y) the divider formulas give:

$$V_X = V_s \frac{Z_2}{Z_1 + Z_2}, \qquad V_Y = V_s \frac{Z_4}{Z_3 + Z_4}.$$

The bridge output is

$$e_0 = V_X - V_Y.$$

At initial balance $V_X = V_Y$ and $e_0 = 0$. If the sensor capacitance increases by 10%, then

$$C_4' = 1.1\, C_4 = 0.11\ \mu\text{F}.$$

We compute the new Z_4' and V_Y' and take V_X (unchanged).

Compute the relevant reactances and impedances:

$$X_{C_1} = \frac{1}{\omega C_1} = \frac{1}{6283.185 \times 0.025 \times 10^{-6}} \approx 6366.2\ \Omega,$$

so

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1} = \frac{2000}{1 + j\,(6283.185)(0.025 \times 10^{-6})(2000)} = \frac{2000}{1 + j\,0.31416}.$$

$$Z_1 \approx 1820.33 - j\,571.87\ \Omega.$$

Left-arm (lower) impedance:

$$Z_2 = R_2 = 2000\ \Omega.$$

Therefore the left-node divider gives

$$V_X = V_s \frac{Z_2}{Z_1 + Z_2} = 10 \cdot \frac{2000}{(1820.33 - j\,571.87) + 2000} = 10 \cdot \frac{2000}{3908.9 - j\,600.4}.$$

$$V_X \approx 5.116 + j\,0.786\ \text{V}.$$

Right-arm impedances before and after change:

$$Z_3 = \frac{1}{j\omega C_3} = -\frac{j}{\omega C_3} = -\frac{j}{6283.185 \times 0.1 \times 10^{-6}} = -j\,1591.55\ \Omega.$$

Sensor (original) $C_4 = 0.1\ \mu\text{F}$:

$$Z_4 = R_4 - \frac{j}{\omega C_4} = 500 - j\,1591.55\ \Omega.$$

Sensor (after +10%): $C_4' = 0.11\ \mu\text{F}$:

$$X_{C_4'} = \frac{1}{\omega C_4'} = \frac{1}{6283.185 \times 0.11 \times 10^{-6}} \approx 1446.86\ \Omega,$$

$$Z_4' = 500 - j\,1446.86\ \Omega.$$

Now compute the right-node divider with the updated Z_4':

$$V_Y' = V_s \frac{Z_4'}{Z_3 + Z_4'} = 10 \cdot \frac{500 - j\,1446.86}{(-j\,1591.55) + (500 - j\,1446.86)} = 10 \cdot \frac{500 - j\,1446.86}{500 - j\,(3038.41)}.$$

$$V_Y' \approx 4.9 + j\,0.839\ \text{V}.$$

Finally the bridge output (complex) is

$$e_0 = V_X - V_Y' \approx (5.116 + j\,0.786) - (4.9 + j\,0.839) = 0.216 - j\,0.07\ \text{V}.$$

The magnitude and phase are

$$|e_0| = \sqrt{0.216^2 + 0.07^2} \approx 0.232 \text{ V}, \qquad \angle e_0 = \tan^{-1}\left(\frac{0.07}{0.216}\right) \approx -18.27°.$$

$$\boxed{e_0 \approx 0.232 \text{ V at } -18.27°.}$$

Task 3.3: Operational Amplifiers output

For each of the circuits shown in Figure 3.3.1, calculate the output voltage e_0.

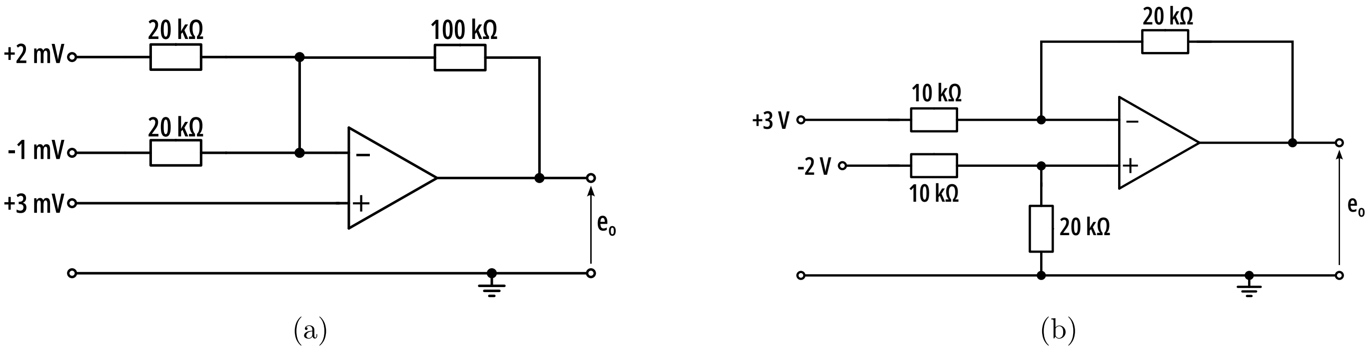


Figure 3.3.1: Circuits for task 3.3

3.3.1 (a) Summing Amplifier Circuit

Answer:

The circuit functions as a Superposition Amplifier. The total output e_0 is the sum of the contributions from the inverting inputs ($e_{0,\text{Inv}}$) and the non-inverting input ($e_{0,\text{NonInv}}$).

1. Contribution from Inverting Inputs (V_1, V_2): The circuit acts as an Inverting Summing Amplifier with $R_F = 100$ kΩ and $R_1 = R_2 = 20$ kΩ.

$$e_{0,\text{Inv}} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right)$$

$$e_{0,\text{Inv}} = -(5 \times (2 \text{ mV}) + 5 \times (-1 \text{ mV})) = -5 \text{ mV}$$

2. Contribution from Non-Inverting Input (V^+): The non-inverting input is $V^+ = +3$ mV. The equivalent input resistance R_{in} for the non-inverting stage is the parallel combination of R_1 and R_2:

$$R_{\text{in}} = R_1||R_2 = 20 \text{ k}\Omega||20 \text{ k}\Omega = 10 \text{ k}\Omega$$

The Non-Inverting Gain (A_{NonInv}) is:

$$A_{\text{NonInv}} = 1 + \frac{R_F}{R_{\text{in}}} = 1 + \frac{100 \text{ k}\Omega}{10 \text{ k}\Omega} = 11$$

$$e_{0,\text{NonInv}} = V^+ \times A_{\text{NonInv}} = 3 \text{ mV} \times 11 = 33 \text{ mV}$$

3. Total Output Voltage:

$$e_0 = e_{0,\text{Inv}} + e_{0,\text{NonInv}} = -5 \text{ mV} + 33 \text{ mV}$$

$$e_0 = +28 \text{ mV}$$

3.3.2 (b) Differential Amplifier Circuit

Answer:

The circuit is a balanced Differential Amplifier since $\frac{R_F}{R_{i1}} = \frac{R_g}{R_{i2}} = 2$. The output voltage e_0 is given by:

$$e_0 = \frac{R_F}{R_{i1}}(V_\text{P} - V_\text{N})$$

With $R_F = 20$ kΩ, $R_{i1} = 10$ kΩ, $V_\text{P} = -2$ V, and $V_\text{N} = +3$ V:

$$e_0 = \frac{20 \text{ k}\Omega}{10 \text{ k}\Omega} \times (-2 \text{ V} - 3 \text{ V})$$

$$e_0 = 2 \times (-5 \text{ V})$$

$$e_0 = -10 \text{ V}$$

Task 3.4: Inverting Op-Amp

An inverting operational amplifier circuit has an overall voltage signal gain of 19.5 dB. If the input resistor is 10 kΩ and the feedback resistor 100 kΩ. Assume a negligible signal source impedance.

3.4.1 Calcualte the feedback fraction, β

Answer:

The feedback fraction β is defined as the ratio of the fed-back voltage (V^-) to the source of that voltage (V_out):

$$\beta = \frac{V^-}{V_\text{out}}$$

Therefore, the feedback fraction (β) for an inverting operational amplifier is defined by the components connecting the inverting terminal to ground (R_1) and to the output (R_F):

$$\beta = \frac{R_1}{R_1 + R_F}$$

Substituting the values:

$$\beta = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 100 \text{ k}\Omega} = \frac{10}{110}$$

$$\beta = \frac{1}{11} \approx 0.0909$$

3.4.2 Calculate the amplifier open loop gain in decibels.

Answer:

Convert Closed-Loop Gain to Linear Ratio

$$A_{\mathrm{CL}} = 10^{\frac{A_{\mathrm{CL(dB)}}}{20}}$$

Substituting $A_{\mathrm{CL(dB)}} = 19.5$ dB:

$$A_{\mathrm{CL}} = 10^{\frac{19.5}{20}} = 10^{0.975}$$

$$A_{\mathrm{CL}} \approx 9.441$$

Use the Negative Feedback Equation
The closed-loop gain (A_{CL}) is related to the open-loop gain (A_{OL}) by:

$$A_{\mathrm{CL}} = \frac{A_{\mathrm{OL}}}{1 + \beta A_{\mathrm{OL}}}$$

Solving for A_{OL}:

$$A_{\mathrm{OL}} = \frac{A_{\mathrm{CL}}}{1 - A_{\mathrm{CL}}\beta}$$

Calculate A_{OL} (Linear Ratio)
Substituting $A_{\mathrm{CL}} \approx 9.441$ and $\beta = 1/11$:

$$A_{\mathrm{OL}} = \frac{9.441}{1 - 9.441 \times (1/11)}$$

$$A_{\mathrm{OL}} = \frac{9.441}{1 - 0.85827} \approx 66.61$$

Convert A_{OL} to Decibels

$$A_{\mathrm{OL(dB)}} = 20\log_{10}(A_{\mathrm{OL}})$$

$$A_{\mathrm{OL(dB)}} = 20\log_{10}(66.61)$$

$$A_{\mathrm{OL(dB)}} \approx 36.47 \text{ dB}$$

Task 3.5: Inverting Op-Amp with T-Resistance Network

For Figure 3.5.1 show that the gain of the circuit is approximately given by:

$$A_v \approx -\frac{R_f}{R_1}\left[1 + \frac{R_2}{R_3}\right]$$

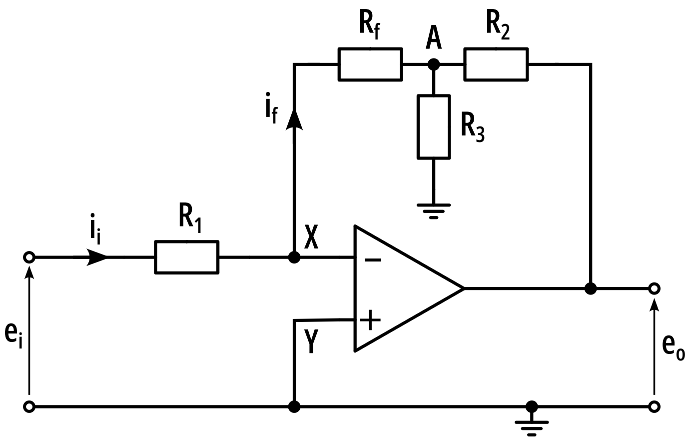


Figure 3.5.1: T-resistance network as feedback

Answer:

The circuit is analyzed using the ideal operational amplifier rules:

1. The voltage difference between the input terminals is zero (Virtual Short): $e_X = e_Y$.
2. No current flows into the input terminals: $i_X = i_Y = 0$.

Determine Voltages at Input Terminals

The non-inverting terminal Y is connected to ground, so:

$$e_Y = 0$$

Due to the virtual short (Rule 1):

$$e_X = e_Y = 0$$

Current at Node X (Inverting Terminal)

Apply Kirchhoff's Current Law (KCL) at node X: the input current (i_i) must equal the feedback current (i_f), since the current into the Op-Amp terminal (i_X) is zero (Rule 2).

$$i_i - i_f = 0 \quad \Rightarrow \quad i_i = i_f$$

Calculate Input Current (i_i)

The input current flows from e_i through R_1 to node X ($e_X = 0$):

$$i_i = \frac{e_i - e_X}{R_1} = \frac{e_i - 0}{R_1}$$

$$i_i = \frac{e_i}{R_1}$$

Calculate Feedback Current (i_f)

The feedback current i_f flows from node X ($e_X = 0$) through R_f to node A.

$$i_f = \frac{e_X - e_A}{R_f} = \frac{0 - e_A}{R_f}$$

$$i_f = -\frac{e_A}{R_f}$$

Therefore,

$$\frac{e_i}{R_1} = -\frac{e_A}{R_f}$$

Using the voltage divider rule at the output side:

$$e_A = e_0 \left(\frac{R_3}{R_2 + R_3} \right)$$

Substituting this in the previous equation:

$$\frac{e_i}{R_1} = -\frac{e_0 \left(\frac{R_3}{R_2+R_3} \right)}{R_f}$$

$$\frac{e_0}{e_i} = -\frac{R_f(R_2 + R_3)}{R_1 R_3}$$

$$\text{Gain} = -\frac{R_f R_2 + R_f R_3}{R_1 R_3}$$

The final desired result for the inverting amplifier is:

$$A_v = -\frac{R_f}{R_1} \left(1 + \frac{R_2}{R_3} \right)$$

Task 3.6: Programmable Gain Op-Amp

If in Figure 3.6.1 a logic 1 transistor-transistor logic (TTL) input closes the relevant switch, calculate the output voltage for the case where $E_i = 2$ mV and:

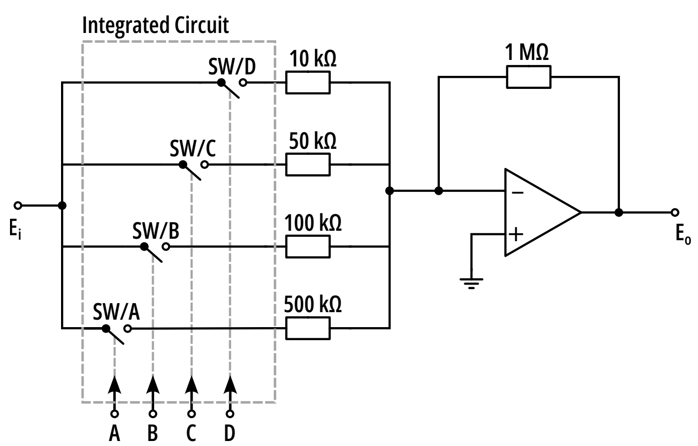


Figure 3.6.1: Programmable Gain Op-Amp

3.6.1 TTL digital input word is 1001.

Answer:

The circuit is an Inverting Summing Amplifier where the gain is controlled by the TTL digital word $DCBA$. A logic '1' closes the switch, connecting the corresponding resistor to the inverting terminal. The output voltage (E_o) is given by the formula:

$$E_o = -E_i \times R_F \times \left(\frac{D}{R_D} + \frac{C}{R_C} + \frac{B}{R_B} + \frac{A}{R_A}\right)$$

Where $D, C, B, A \in \{0, 1\}$ are the logic inputs.

Component Values:

- Input Voltage: $E_i = 2$ mV
- Feedback Resistor: $R_F = 1$ MΩ $= 1000$ kΩ
- Input Resistors: $R_D = 10$ kΩ, $R_C = 50$ kΩ, $R_B = 100$ kΩ, $R_A = 500$ kΩ

Digital Input Word $DCBA = 1001$

Inputs $D = 1$ and $A = 1$ are active.

$$E_o = -(2 \text{ mV}) \times (1000 \text{ k}\Omega) \times \left(\frac{1}{10 \text{ k}\Omega} + \frac{0}{50 \text{ k}\Omega} + \frac{0}{100 \text{ k}\Omega} + \frac{1}{500 \text{ k}\Omega}\right)$$

$$E_o = -(2 \text{ mV}) \times 1000 \text{ k}\Omega \times (0.1 \text{ mS} + 0.002 \text{ mS})$$

$$E_o = -(2 \text{ mV}) \times 1000 \text{ k}\Omega \times (0.102 \text{ mS})$$

Since $(1000 \text{ k}\Omega) \times (0.102 \text{ mS}) = 102$ (unitless gain):

$$E_o = -2 \text{ mV} \times 102$$

$$E_o = -204 \text{ mV}$$

3.6.2 TTL digital input word is 1001.

Answer:

Inputs $C = 1$ and $B = 1$ are active.

$$E_o = -(2 \text{ mV}) \times (1000 \text{ k}\Omega) \times \left(\frac{0}{10 \text{ k}\Omega} + \frac{1}{50 \text{ k}\Omega} + \frac{1}{100 \text{ k}\Omega} + \frac{0}{500 \text{ k}\Omega}\right)$$

$$E_o = -(2 \text{ mV}) \times 1000 \text{ k}\Omega \times (0.02 \text{ mS} + 0.01 \text{ mS})$$

$$E_o = -(2 \text{ mV}) \times 1000 \text{ k}\Omega \times (0.03 \text{ mS})$$

Since (1000 kΩ) × (0.03 mS) = 30 (unitless gain):

$$E_o = -2 \text{ mV} \times 30$$

$$E_o = -60 \text{ mV}$$

Task 3.7: Transducer and Op-Amp

A transducer bridge and an operational amplifier are connected as per Figure 3.7.1. $E = 10$ V, $R_f = 50$ kΩ and $R = 120$ Ω. The transducer has a passive resistance $R_x = 120$ Ω and is stimulated such that its resistance changes by 0.1%. Calculate the amplifier output voltage.

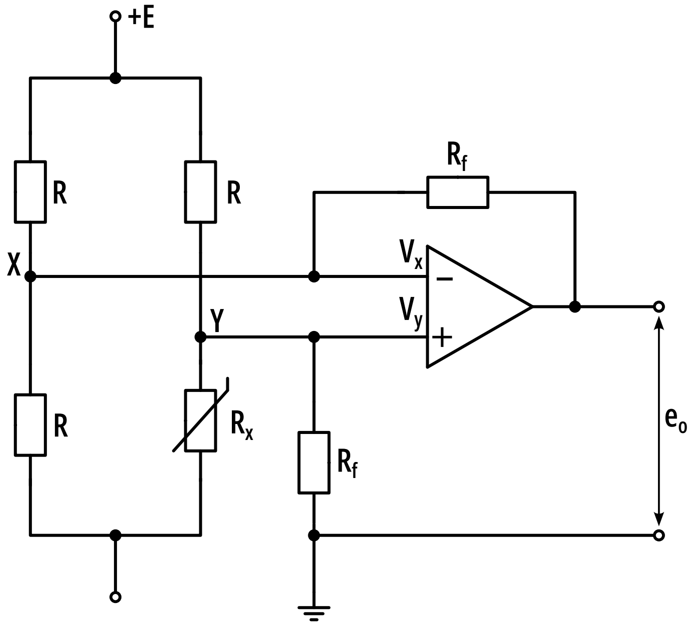


Figure 3.7.1: Transducer and Op-amp

Answer:

Equating currents at X and again assuming an ideal operational amplifier:

$$\frac{E - V_X}{R} = \frac{V_X - e_o}{R_f} + \frac{V_X - 0}{R}$$

Equating currents at Y and again assuming an ideal operational amplifier:

$$\frac{E - V_Y}{R} = \frac{V_Y - 0}{R(1+a)} + \frac{V_Y - 0}{R_f}$$

Because normal operational amplifier action makes $V_X = V_Y$, if we make both of these equal to V we can rewrite above equations to produce the following equation.

$$\frac{V - e_o}{R_f} + \frac{V}{R} = \frac{V}{R(1+a)} + \frac{V}{R_f}$$

Hence

$$e_o = \left[\frac{V}{R} - \frac{V}{R(1+a)}\right] R_f$$

But we can express V in terms of E and the circuit component values by examining the right limb of the transducer bridge:

$$V = V_Y = E\left[\frac{R(1+a) \parallel R_f}{R + R(1+a) \parallel R_f}\right]$$

whence

$$V = \frac{ER_f(1+a)}{R(1+a) + R_f + R_f(1+a)}$$

Substituting and rearranging we obtain:

$$e_o = \frac{ER_f a}{R} \frac{1}{(1+a)\left(\frac{R+R_f}{R_f}\right) + 1}$$

If a is small, the above equation is virtually linear.

Calculate the voltage V:

$$V = \frac{ER_f(1+a)}{R(1+a) + R_f + R_f(1+a)}$$

Substituting the values $E = 10$ V, $R = 120\ \Omega$, $R_f = 50000\ \Omega$, and $a = 0.001$:

$$V = \frac{10\text{ V} \cdot 50000\ \Omega \cdot (1.001)}{120\ \Omega \cdot (1.001) + 50000\ \Omega + 50000\ \Omega \cdot (1.001)}$$

$$V = \frac{500500}{120.12 + 50000 + 50050} = \frac{500500}{100170.12}$$
$$V \approx 4.9965\text{ V}$$

Calculate the output voltage e_o

$$e_o = \left[\frac{V}{R} - \frac{V}{R(1+a)}\right] R_f = \frac{VR_f}{R}\frac{a}{1+a}$$

Substituting the calculated V and component values:

$$e_o = \frac{4.9965\text{ V} \cdot 50000\ \Omega}{120\ \Omega} \cdot \frac{0.001}{1.001}$$

$$e_o \approx 4.9965\text{ V} \times 416.6667 \times 0.000999001$$

$$e_o \approx 2081.875 \times 0.000999$$

$$e_o \approx 2.080\text{ V}$$

Task 3.8: Capacitor Multiplier Circuit Analysis

A signal conditioning situation requires the use of a special capacitor having a value of 4780 μF. Unfortunately the most suitable capacitor available is only 1000 μF so it is decided to use the circuit shown in Figure 3.8.1 effectively to increase the value of the capacitor to 4780 μF.

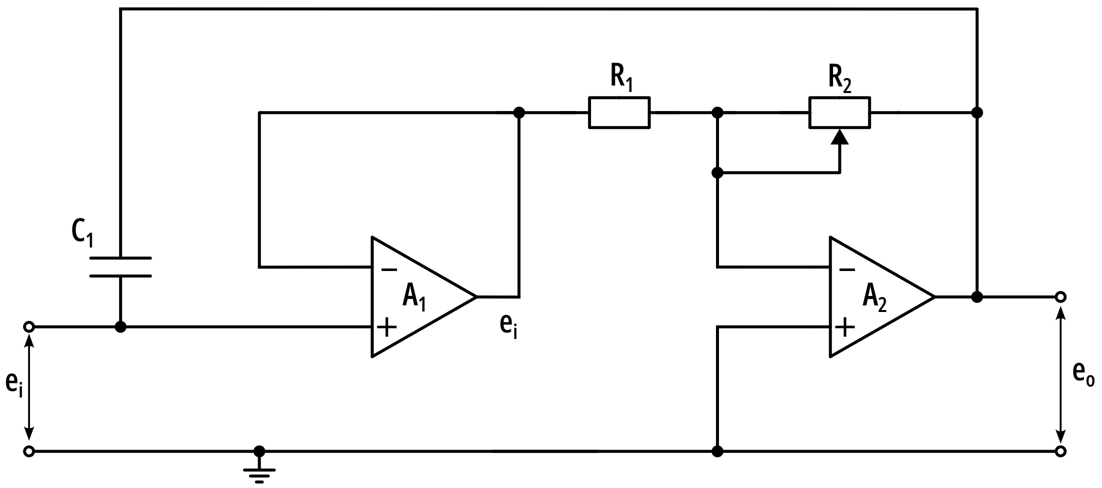


Figure 3.8.1: Capacitor Multiplier Circuit

3.8.1 Calculate the Required Gain (A_{V2})

Answer:

$$I_i = \frac{e_i - e_o}{\frac{1}{j\omega C_1}} = \frac{e_i - \left(-\frac{R_2}{R_1}e_i\right)}{\frac{1}{j\omega C_1}} = \frac{e_i\left(1 + \frac{R_2}{R_1}\right)}{\frac{1}{j\omega C_1}}$$

$$= \frac{e_i}{\frac{1}{j\omega C_{eff}}}, \quad \text{where } C_{eff} = C_1\left[1 + \frac{R_2}{R_1}\right]$$

The effective capacitance (C_{eq}) of the capacitance multiplier circuit shown is given by the formula:

$$C_{eq} = C_1\left(1 + \frac{R_2}{R_1}\right) = C_1 \cdot A_{V2}$$

where A_{V2} is the gain of the non-inverting amplifier A_2.

The required gain factor A_{V2} is:

$$A_{V2} = \frac{C_{eq}}{C_1}$$

Substituting the given values $C_{eq} = 4780$ μF and $C_1 = 1000$ μF:

$$A_{V2} = \frac{4780\ \mu\text{F}}{1000\ \mu\text{F}}$$

$$A_{V2} = 4.78$$

3.8.2 Suggest Suitable Resistor Values (R_1 and R_2)

Answer:

The gain of the non-inverting amplifier A_2 is $A_{V2} = 1 + \frac{R_2}{R_1}$. We need the ratio $\frac{R_2}{R_1}$:

$$\frac{R_2}{R_1} = A_{V2} - 1 = 4.78 - 1$$

$$\frac{R_2}{R_1} = 3.78$$

To suggest suitable values, we choose a standard resistor value for R_1 (e.g., 10 kΩ) and calculate R_2:

$$R_2 = 3.78 \cdot R_1$$

Let $R_1 = 10$ kΩ:

$$R_2 = 3.78 \cdot 10 \text{ k}\Omega = 37.8 \text{ k}\Omega$$

Suggested values:

- $R_1 = 10$ kΩ
- $R_2 = 37.8$ kΩ (A standard E96 value of 38.3 kΩ or 37.4 kΩ would be chosen in practice).

Task 3.9: Amplifier Noise and SNR Analysis

An antenna of noise temperature 160 K feeds a signal to an amplifier of noise temperature 260 K, bandwidth 2 MHz and power gain 90 dB. The SNR at the amplifier input is 30 dB. Assuming k to be 1.38×10^{-23} J/K, calculate the output signal power and the output SNR.

3.9.1 Convert Parameters to Linear Units

Answer:

- Boltzmann's Constant: $k = 1.38 \times 10^{-23}$ J/K
- Bandwidth: $B = 2$ MHz $= 2 \times 10^6$ Hz
- Linear Input SNR:
$$\text{SNR}_{\text{in}} = 10^{30/10} = 1000$$
- Linear Power Gain:
$$G = 10^{90/10} = 10^9$$

3.9.2 Calculate Reference Input Noise Power (N_A)

Answer:

The input noise power (N_A) used to define the SNR_{in} is the antenna noise power ($N_A = kT_AB$):

$$N_A = kT_AB = (1.38 \times 10^{-23} \text{ J/K}) \cdot (160 \text{ K}) \cdot (2 \times 10^6 \text{ Hz})$$

$$N_A = 4.416 \times 10^{-15}\ \text{W}$$

3.9.3 Calculate Output Signal Power (S_{out})

Answer:

First, find the input signal power (S_{in}) using N_A:

$$S_{\text{in}} = \text{SNR}_{\text{in}} \cdot N_A = 1000 \cdot (4.416 \times 10^{-15}\ \text{W})$$

$$S_{\text{in}} = 4.416 \times 10^{-12}\ \text{W}$$

The output signal power is $S_{\text{out}} = S_{\text{in}} \cdot G$:

$$S_{\text{out}} = (4.416 \times 10^{-12}\ \text{W}) \cdot (10^9)$$

$$S_{\text{out}} \approx 4.42 \times 10^{-3}\ \text{W}$$

3.9.4 Calculate Output SNR (SNR_{out})

Answer:

The output SNR is determined by the Noise Figure (NF) of the amplifier stage relative to the antenna temperature (T_A):

$$NF = 1 + \frac{T_E}{T_A} = 1 + \frac{260\ \text{K}}{160\ \text{K}} = 2.625$$

The output SNR is $\text{SNR}_{\text{out}} = \text{SNR}_{\text{in}}/NF$:

$$\text{SNR}_{\text{out}} = \frac{1000}{2.625}$$

$$\text{SNR}_{\text{out}} \approx 381$$

In dB units:

$$\text{SNR}_{\text{out}}(\text{dB}) = 10 \cdot \log_{10}(381)$$

$$\text{SNR}_{\text{out}} \approx 25.8\ \text{dB}$$

Task 3.10: Amplifier Noise and SNR Analysis

A 30 μW signal, having a SNR of 35 dB, is amplified by an amplifier of power gain 20 dB. The internal noise generated by the amplifier is equivalent to an additional noise power of 6 nW at its input. Calculate:

3.10.1 Parameters in Linear Units

Answer:

- Input Signal Power: $S_{\text{in}} = 30\ \mu\text{W} = 30 \times 10^{-6}\ \text{W}$

- Amplifier Added Noise: $N_A = 6\text{ nW} = 6 \times 10^{-9}\text{ W}$
- Linear Input SNR:

$$\text{SNR}_{\text{in}} = 10^{35/10} \approx 3162.28$$

- Linear Power Gain:

$$G = 10^{20/10} = 100$$

3.10.2 Input noise power caused by the 35 dB input SNR

Answer:

The input noise power N_{in} is derived from the definition of SNR:

$$N_{\text{in}} = \frac{S_{\text{in}}}{\text{SNR}_{\text{in}}} = \frac{30 \times 10^{-6}\text{ W}}{3162.27}$$

$$N_{\text{in}} \approx 9.4868 \times 10^{-9}\text{ W} \quad (\mathbf{9.49}\text{ nW})$$

3.10.3 Output Signal Power (S_{out})

Answer:

The output signal power is the input signal power multiplied by the linear power gain (G):

$$S_{\text{out}} = S_{\text{in}} \cdot G = (30 \times 10^{-6}\text{ W}) \cdot 100$$

$$S_{\text{out}} = 0.003\text{ W} \quad (\mathbf{3}\text{ mW})$$

3.10.4 Total output noise power

Answer:

The total noise power at the output is the amplified sum of the source noise (N_{in}) and the amplifier's added noise (N_A, referred to the input):

$$N_{\text{out, total}} = (N_{\text{in}} + N_A) \cdot G$$

$$N_{\text{out, total}} = (9.4868 \times 10^{-9}\text{ W} + 6 \times 10^{-9}\text{ W}) \cdot 100$$

$$N_{\text{out, total}} = (15.4868 \times 10^{-9}\text{ W}) \cdot 100$$

$$N_{\text{out, total}} \approx 1.5487 \times 10^{-6}\text{ W} \quad (\mathbf{1.55}\ \mu\text{W})$$

3.10.5 Calculate the Output SNR (SNR_{out})

Answer:

The output SNR is the ratio of S_{out} to $N_{\text{out, total}}$:

$$\text{SNR}_{\text{out}} = \frac{S_{\text{out}}}{N_{\text{out, total}}} = \frac{0.003 \text{ W}}{1.54868 \times 10^{-6} \text{ W}}$$

$$\text{SNR}_{\text{out}} \approx 1937.13$$

In dB units:

$$\text{SNR}_{\text{out}}(\text{dB}) = 10 \log_{10}(1937.13)$$

$$\text{SNR}_{\text{out}} \approx 32.87 \text{ dB}$$

Exercise 04: Analog and Digital Conversions

Task 4.1: Digital-to-Analog Converter (DAC) Output Calculation

A 7-bit DAC has the digital input word 1100111_2 and a reference voltage of 8 V.

Calculate:

4.1.1 Analog Output Voltage

Answer:

- Number of bits: $n = 7$
- Reference voltage: $V_{\text{ref}} = 8$ V

The given binary input is:

$$1100111_2$$

Converting to decimal:

$$1100111_2 = 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

$$= 64 + 32 + 0 + 0 + 4 + 2 + 1 = 103$$

For an ideal DAC, the output voltage is given by:

$$V_{\text{out}} = \frac{D}{2^n} V_{\text{ref}}$$

Substituting values:

$$V_{\text{out}} = \frac{103}{2^7} \times 8$$

$$V_{\text{out}} = \frac{103}{128} \times 8$$

$$V_{\text{out}} \approx 6.4375 \text{ V}$$

$$\boxed{V_{\text{out}} \approx 6.44 \text{ V}}$$

4.1.2 Conversion Resolution Voltage

Answer:

The resolution (voltage step size) of a DAC is defined as:

$$\Delta V = \frac{V_{\text{ref}}}{2^n}$$

Substituting the given values:

$$\Delta V = \frac{8}{2^7} = \frac{8}{128}$$

$$\Delta V = 0.0625 \text{ V}$$

$$\boxed{\Delta V = 62.5 \text{ mV}}$$

Task 4.2: Analog-to-Digital Converter (ADC) Quantization

An 8-bit ADC has a reference voltage of 10 V and an analogue input voltage of 6.875 V.

Calculate:

4.2.1 Digital Output Word

Answer:

- Number of bits: $n = 8$
- Reference voltage: $V_{\text{ref}} = 10$ V
- Input voltage: $V_{\text{in}} = 6.875$ V

The resolution (LSB size) of the ADC is:

$$\Delta V = \frac{V_{\text{ref}}}{2^n} = \frac{10}{2^8}$$

$$\Delta V = \frac{10}{256} = 0.0390 \text{ V}$$

The corresponding digital code is given by:

$$D = \frac{V_{\text{in}}}{\Delta V}$$

$$D = \frac{6.875}{0.0390} = 176$$

Converting the decimal value to binary:

$$176_{10} = 10110000_2$$

$$\boxed{\text{Digital Output Word} = 10110000_2}$$

4.2.2 Percentage Resolution

Answer:

$$\text{Percentage Resolution} = \frac{1}{2^n} \times 100\% = \frac{1}{256} \times 100\%$$

$$\boxed{\text{Percentage Resolution} \approx 0.39\%}$$

Task 4.3: DAC Reference Voltage Calculation

A 6-bit DAC is required to produce an output voltage of 9 V when all six input bits are at logic 1.

Calculate the required reference voltage for this DAC.

4.3.1 Reference Voltage

Answer:

- Number of bits: $n = 6$
- Full-scale digital input: $D_{\text{max}} = 2^6 - 1 = 63$
- Required output voltage: $V_{\text{out, max}} = 9$ V

For an ideal DAC, the output voltage is given by:

$$V_{\text{out}} = \frac{D}{2^n} V_{\text{ref}}$$

At full-scale input ($D = D_{\text{max}} = 63$):

$$9 = \frac{63}{64} V_{\text{ref}}$$

Solving for the reference voltage:

$$V_{\text{ref}} = \frac{9 \times 64}{63}$$

$$V_{\text{ref}} \approx 9.1429 \text{ V}$$

$$\boxed{V_{\text{ref}} \approx 9.14 \text{ V}}$$

Task 4.4: 4-bit Weighted DAC Error Analysis

A 4-bit weighted inverting DAC has the following parameters:

- Reference voltage: $V_R = 1$ V
- Feedback resistor: $R_f = 8R$
- Input resistors (MSB $\rightarrow$ LSB): R, $2R$, $4R$, $8R$

We calculate the output voltage V_{out} step by step using the current summation method.

4.4.1 Find currents through each resistor

Answer:

Each bit contributes a current through its resistor if the bit is logic 1:

$$I_n = \frac{V_R}{R_n}$$

Bit	Resistor	Current if bit = 1 (units of V_R/R)
b_3 (MSB)	R	$I_1 = 1$
b_2	$2R$	$I_2 = 0.5$
b_1	$4R$	$I_3 = 0.25$
b_0 (LSB)	$8R$	$I_4 = 0.125$

4.4.2 Total current and output voltage

Answer:

The total current entering the summing node:

$$I_f = I_1 + I_2 + I_3 + I_4$$

The inverting output voltage is:

$$V_{\text{out}} = -R_f \, I_f$$

Since $R_f = 8R$ and currents are in units of V_R/R, we have:

$$V_{\text{out}} = -8 \times (I_1 + I_2 + I_3 + I_4) \text{ V}$$

4.4.3 Compute V_{out} for all 16 inputs

Answer:

Binary Input	Sum of Currents I_f	V_{out} (V)
0000	0	0
0001	0.125	-1
0010	0.25	-2
0011	0.375	-3
0100	0.5	-4
0101	0.625	-5
0110	0.75	-6
0111	0.875	-7
1000	1	-8
1001	1.125	-9
1010	1.25	-10
1011	1.375	-11
1100	1.5	-12
1101	1.625	-13
1110	1.75	-14
1111	1.875	-15

4.4.4 Compare with given DAC outputs

Answer:

We now compare the calculated outputs (V_{out}) with the measured outputs:

NOTE: We compare the absolute vaules of the actual outputs since the negative signs defines the nature of inverting amplifier DAC.

Binary Input	**Calculated V_{out} (V)**	**Given V_{out} (V)**	**Comment**
0000	0	0	Correct
0001	1	1	Correct
0010	2	0	Incorrect
0011	3	1	Incorrect
0100	4	4	Correct
0101	5	5	Correct
0110	6	4	Incorrect
0111	7	5	Incorrect
1000	8	8	Correct
1001	9	9	Correct
1010	10	8	Incorrect
1011	11	9	Incorrect
1100	12	12	Correct
1101	13	13	Correct
1110	14	12	Incorrect
1111	15	13	Incorrect

4.4.5 Assuming the electronic switch is working correctly, state the most likely cause of the DAC functioning incorrectly.
Answer:

- All incorrect outputs occur when the **second least significant bit** (b_1, weight $4R$) is logic 1.
- Since the electronic switches are assumed working correctly, the most likely cause is:

$$\boxed{\text{Open-circuit or incorrect resistor in the } b_1 \text{ branch (weight } 4R)}.$$

- The DAC fails to add the b_1 contribution, producing the missing 2 V steps in the output.

Task 4.5: R-2R Ladder DAC Output Calculation

A 4-bit R-2R ladder DAC has the following parameters:

- Reference voltage: $V_{ref} = 10$ V
- Ladder resistor: $R = 20$ kΩ
- Feedback resistor: $R_f = 50$ kΩ
- Op-amp supply limits: ±18 V

4.5.1 Output Voltage for Input 1001

Answer:

For an inverting R-2R ladder DAC, the output voltage is given by:

$$V_{\text{out}} = -V_{\text{ref}} \frac{R_f}{R} \left(\frac{b_3}{2} + \frac{b_2}{4} + \frac{b_1}{8} + \frac{b_0}{16} \right)$$

Where b_3 is the MSB and b_0 is the LSB.

The input word is:

$$1001 \quad \Rightarrow \quad b_3 = 1,\ b_2 = 0,\ b_1 = 0,\ b_0 = 1$$

$$V_{\text{out}} = - - 10 \cdot \frac{50\text{k}\Omega}{20\text{k}\Omega} \left(\frac{1}{2} + 0 + 0 + \frac{1}{16} \right)$$

$$\boxed{V_{\text{out}} \approx -14.06\ \text{V}}$$

4.5.2 Maximum Digital Input Word That Can Be Converted

Answer:

The maximum output voltage of the DAC is limited by the op-amp supply: $|V_{\text{out}}| \leq 18$ V.

The full-scale output (all bits = 1) is:

$$V_{\text{FS}} = -V_{\text{ref}} \frac{R_f}{R} \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right)$$

$$V_{\text{FS}} = -10 \cdot 2.5 \cdot \frac{15}{16} = -25 \cdot 0.9375$$

$$V_{\text{FS}} = -23.4375\ \text{V}$$

Since $|V_{\text{FS}}| = 23.44$ V > 18 V, the op-amp saturates.

To find the maximum digital word that does not saturate, let D_{max} be the decimal equivalent of the digital input:

$$V_{\text{out}} = -V_{\text{ref}} \frac{R_f}{R} \frac{D_{\text{max}}}{2^N} \leq 18$$

$$25 \cdot \frac{D_{\text{max}}}{16} \leq 18$$

$$D_{\text{max}} \leq \frac{18 \cdot 16}{25} = 11.52$$

$$\Rightarrow D_{\text{max}} = 11 \quad \text{(round down to nearest integer)}$$

Convert to 4-bit binary:

$$D_{\text{max}} = 1011$$

$$\boxed{\text{Maximum input word that can be converted without saturation: 1011}}$$

Task 4.6: AD Conversion with successive approximation

A 12-bit converter with an input voltage range of 0-10V converts a voltage of 7.1875V.

4.6.1 Calculate the number of conversion steps (Clock cycles) necessary to get the exact result as a digital number.

Answer:

This converter needs a start bit. If the bit is set exactly one conversion is performed. In the case of the input voltage chosen, the DA converter output is exactly matching with it after 4 bits have been tested. Every additional voltage out of the DA converter is too much. That means the remaining bits will be all "0". The converter always tests all bits. That means the conversion doesn't stop if a match appear !

The conversion time is always the same.

For 12-bit resolution, it needs 12 cycles regardless of the input value.

$$V_{\text{out}} = V_{\text{ref}} \left(\frac{b_{N-1}}{2} + \frac{b_{N-2}}{4} + \cdots + \frac{b_1}{2^{N-1}} + \frac{b_0}{2^N} \right)$$

$$V_{\text{out}} = V_{\text{ref}} \left(2^{N-1} b_{N-1} + 2^{N-2} b_{N-2} + \cdots + 2b_1 + b_0 \right) / 2^N$$

$$V_{\text{out}} = V_{\text{ref}} D / 2^N$$

The resolution of an N-bit ADC is:

$$\Delta V = \frac{V_{\text{FS}}}{2^N - 1}$$

where V_{FS} is the full-scale voltage.

$$\Delta V = \frac{10}{2^{12} - 1} = \frac{10}{4095} \approx 0.002442 \text{ V per step}$$

The digital output code corresponding to V_{in} is:

$$D = \frac{V_{\text{in}}}{\Delta V}$$

$$D = \frac{7.1875}{0.002442} \approx 2943$$

For a successive approximation ADC, the number of steps to reach this output is equal to the digital code plus 1 (starting from zero):

$$\text{Clock cycles} = D + 1 = 2943 + 1 = 2944$$

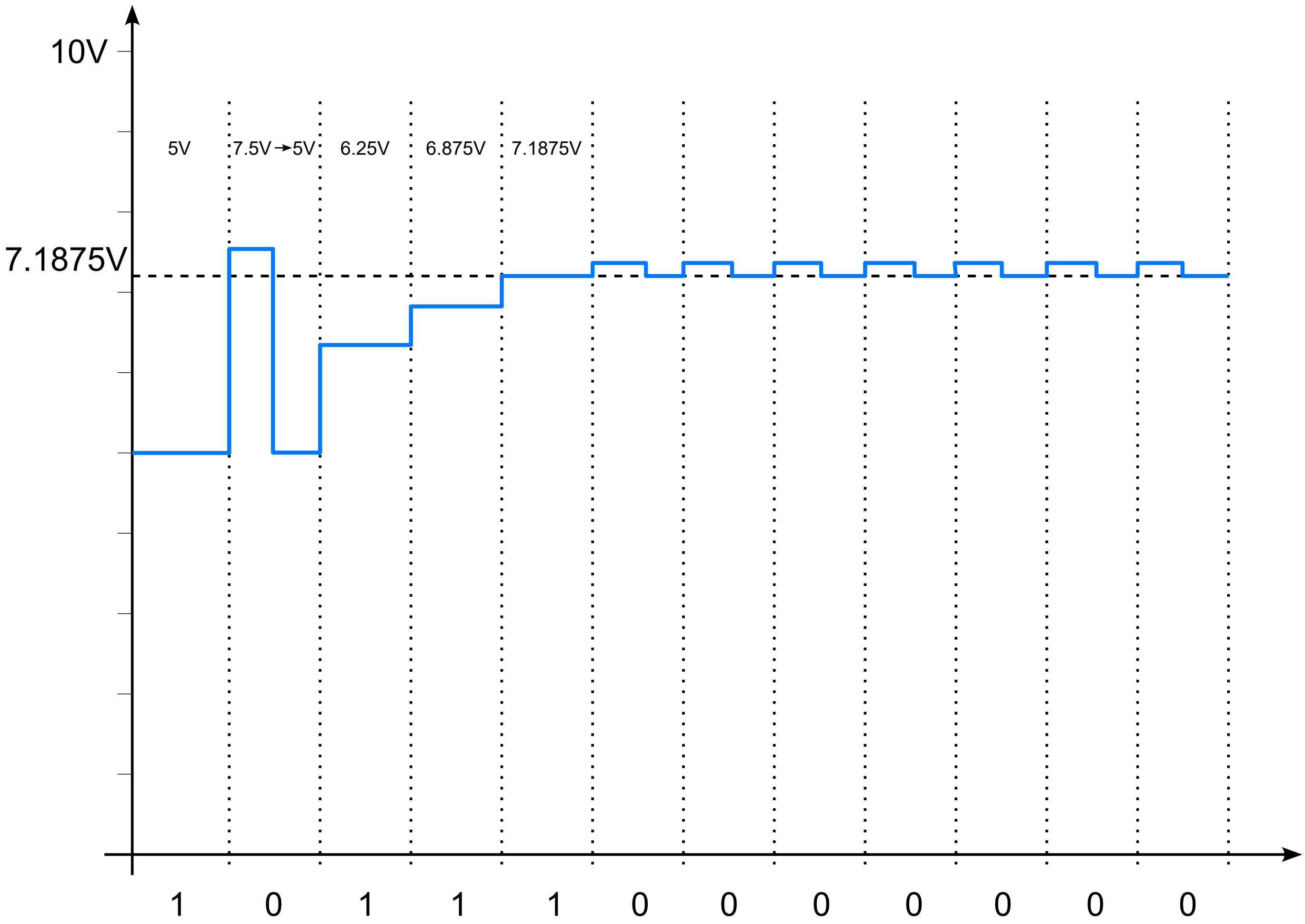
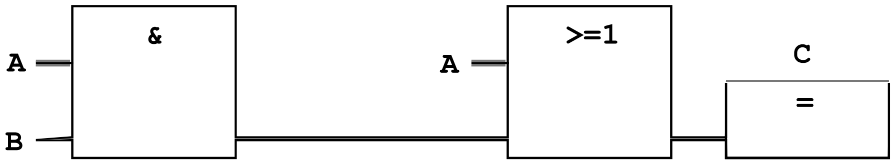


Figure 4.6.1: Successive Approximation ADC

Exercise 05: PLC

Task 5.1: Logic Analysis (1)

The logic below is given: Please analyse it and simplfy it !



5.1.1 Analog Output Voltage

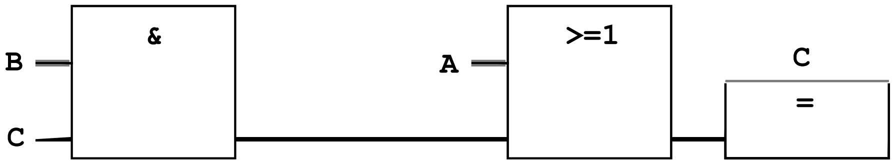
Answer:

A	B	C
0	0	0
0	1	0
1	0	1
1	1	1

The above table shows that the output solely depends on the input A. Therefore, the simplified logic is $C = A$.

Task 5.2: Logic Analysis (2)

Which function is fulfilled by the logic below ?



Answer:

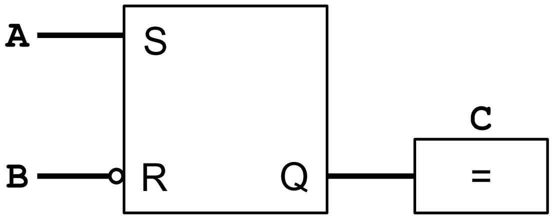
A	B	C	C_{n+1}
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

where C_{n+1} is the output for the next cycle.
We have a feedback mechanism. Output C is given as feedback. We can set $C = 1$ with A and the

to reset $C = 0$ we need A and B.

Once $C = 1$ is set, this ciruit holds itself.

It is a Self-holding ciruit. This results in the following simplified circuit.



Task 5.3: PLC Statement List (STL) Representation

5.3.1 What does the representation of

- the conjunction (AND function) of the inputs I0.0, NOT I0.1 and I0.2 and
- the assignment of the result of the logic operation RLO to the output Q0.0 in the form of a statement list (STL) of a PLC look like?

Answer:

The corresponding STL code is:

A	$I0.0$
AN	$I0.1$
A	$I0.2$
$=$	$Q0.0$

5.3.2 What does the representation of

- the disjunction (OR function) of the inputs I1.0, I1.1, NOT I1.2 and
- the assignment of the RLO to the output Q0.1 and to the Memory bit M 9.1 in the form of a statement list (STL) of a PLC look like?

Answer:

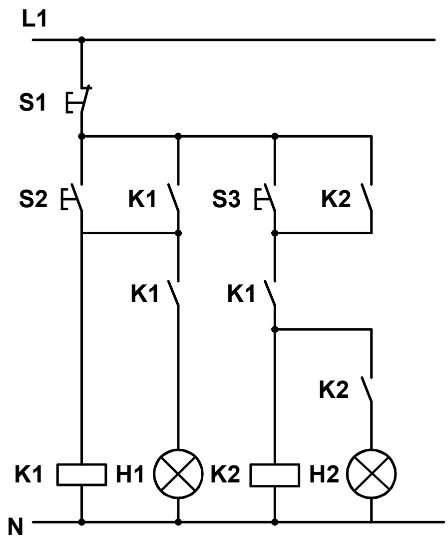
The corresponding STL code is:

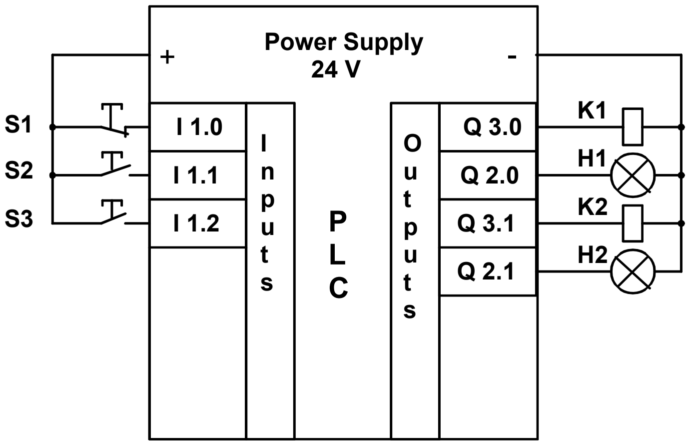
O	$I1.0$
O	$I1.1$
ON	$I1.2$
$=$	$Q0.1$
$=$	$M9.1$

Task 5.4: Contactor circuit

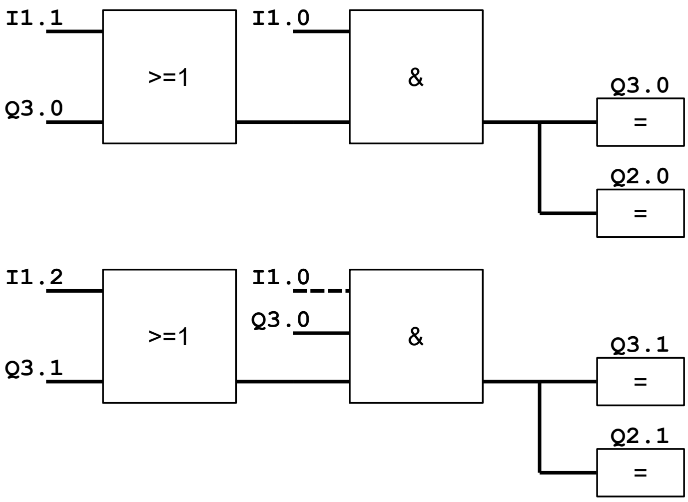
The contactor circuit is given below:

Please find the PLC internal circuit in the Function Block Diagram (FBD) which is equivalent to the contactor circuit !

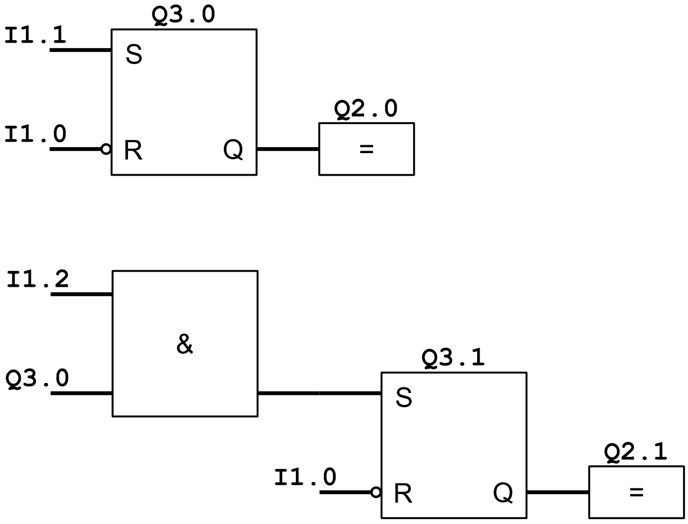




Answer:
The following FBD represents the equivalent contactor circuit:



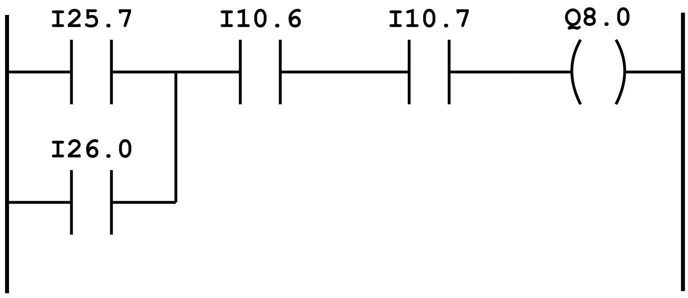
Alternative Solution using SR Flip-Flops:



Task 5.5: LAD Representation of a Two-Staged Boolean Operation

Please create the LAD representation of a two staged boolean operation which complies with the following description:
The result of the OR operation with the inputs I 25.7 and I 26.0 is to be linked into an AND operation with the inputs I 10.6 und I 10.7. The result is to be assigned to the output Q 8.0.

Answer:



Task 5.6: Emulation of the Antivalence (XOR) Operation in a PLC

How is the antivalence operation (XOR) to be emulated, if it is not contained in the basic operation pool of a certain PLC?

5.6.1 STL

Answer:

A	**B**	**A ⊕ B**
0	0	0
0	1	1
1	0	1
1	1	0

Table 5.6.1: XOR Gate Truth Table

$$XOR = A\overline{B} + \overline{A}B$$

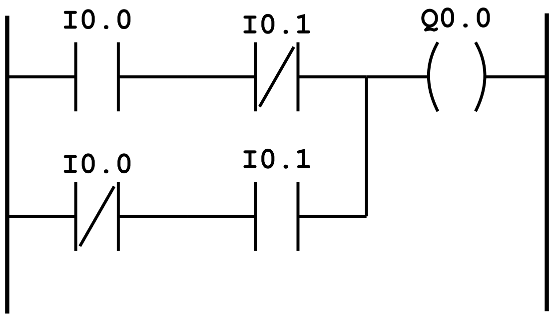
OR

$$XOR = (A + B)(\overline{A} + \overline{B})$$

O(	
A	I0.0
AN	I0.1
)	
O(	
AN	I0.0
A	I0.1
)	
=	Q0.0

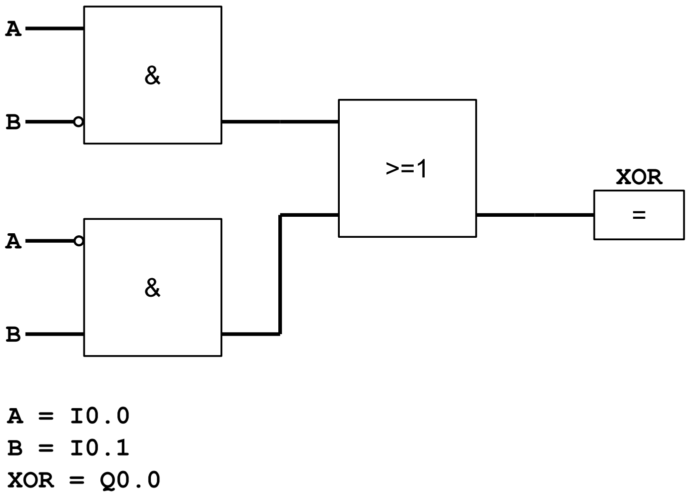
5.6.2 LAD

Answer:



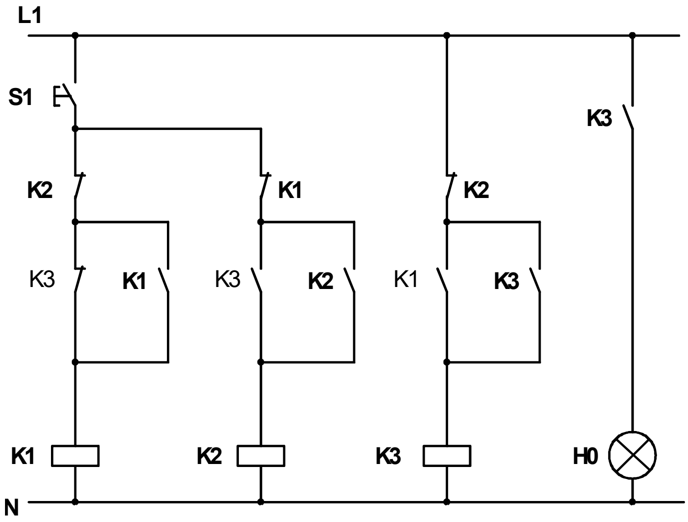
5.6.3 FBD

Answer:



Task 5.7: Pulse Switch

Problem: Substitution of a contactor circuit by a PLC



List of Variables

Symbol	Absolute	Comment
S1	I 1.0	Momentary-contact switch
H0	Q 2.2	Lamp
K1	M 0.0	Memory bit
K2	M 0.1	Memory bit
K3	M 0.2	Memory bit

Functional description: If the pushbutton S1 is pressed, the contactor K1 is switched on as long as the pushbutton is pressed. At the same time the contactor K3 is switched on and holds itself and the lamp H0 is on.

If the pushbutton S1 is pressed again, contactor K2 is switched on. Thereby contactor K3 drops and the lamp is off.

The procedure can be repeated arbitrarily.

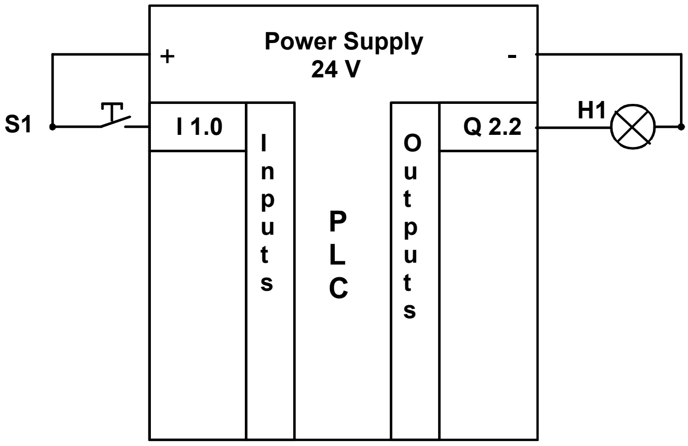
The pushbutton is wired to the input I 1.0. Contactors are to be emulated by memory bits. The output Q 2.2 is directly connected to the lamp.

Please create:

5.7.1 The hardware configuration

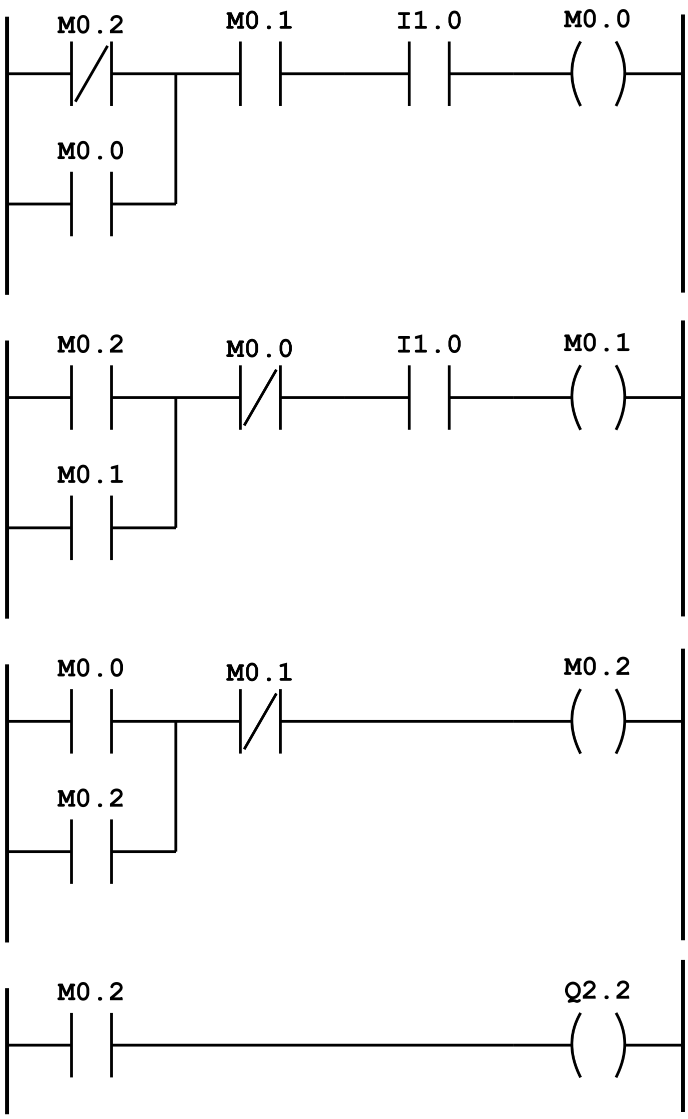
Answer:

When switch S1 is pressed, the lamp H1 turns ON and remains ON. When the S1 switch is again pressed, the lamp H1 turns OFF.



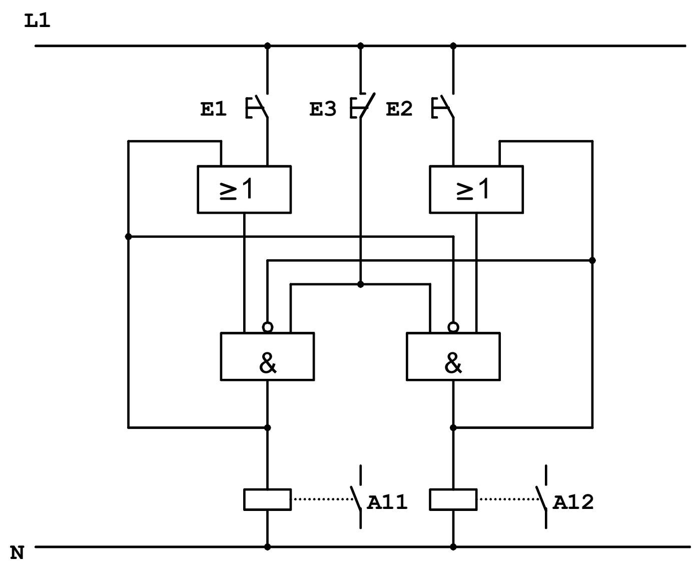
5.7.2 The LAD

Answer:



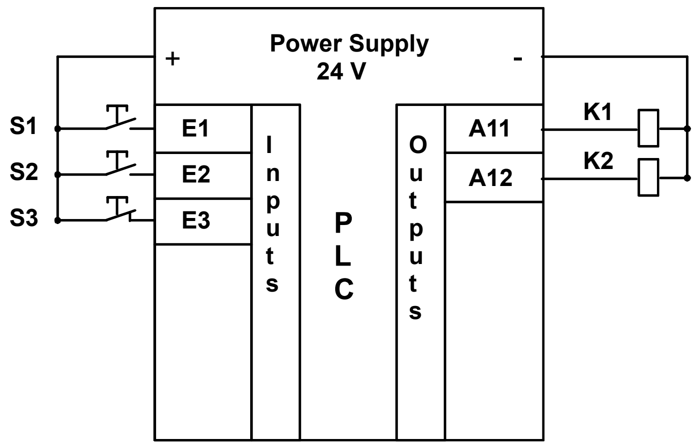
Task 5.8: Motor Control Circuit

A function diagram of a motor control circuit is given:



5.8.1 Please specify the hardware configuration when using a PLC!

Answer:



5.8.2 Please develop the statement list (STL)!

Answer:

A(	
O	A11
O	E1
)	
AN	A12
A	E3
=	A11

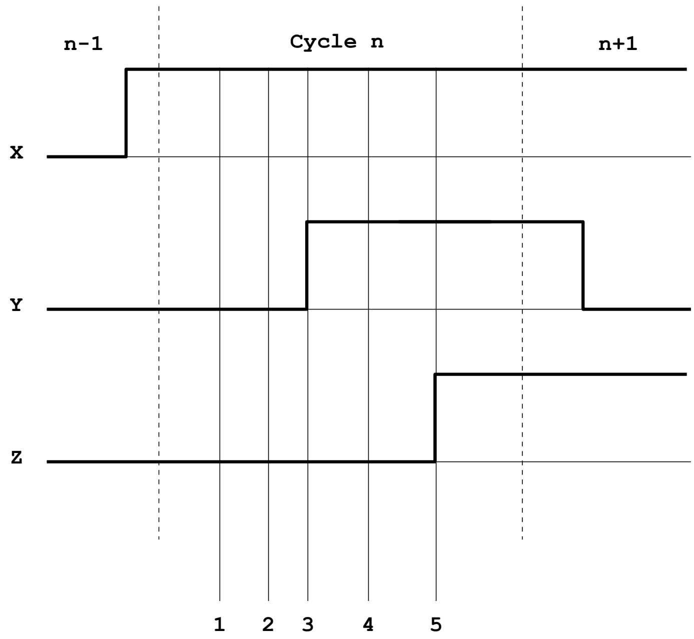
A(	
O	E2
O	A12
)	
AN	A11
A	E3
=	A12

Task 5.9: Momentary Pulse in STL

Please give a sequence of statements for a so called momentary pulse in STL.

(A momentary pulse is the logic "1"-state of a boolean variable for exactly one program cycle as a reaction to the rising edge of another boolean variable.)

Answer:



1	A	X
2	AN	Z
3	=	Y
4	A	X
5	=	Z

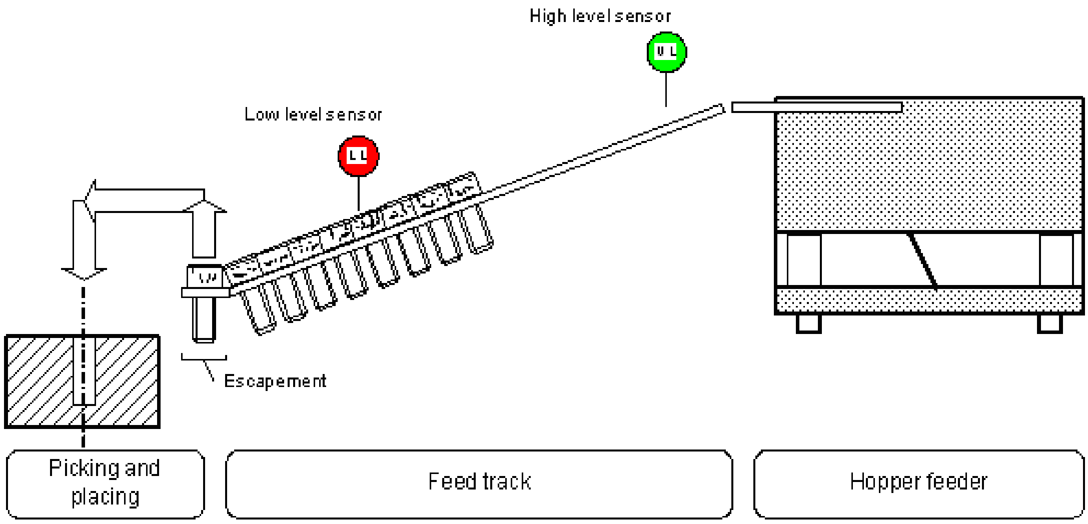
Note: in step 4, we are not using AND operation. We are just reading the value of input X and then in step 5 assigning it to Z.

Task 5.10: Feed Track Control in a Parts Delivery System

The feed track in the parts delivery system should always contain enough parts to make sure that the picking and placing process can operate continuously without waiting for parts.

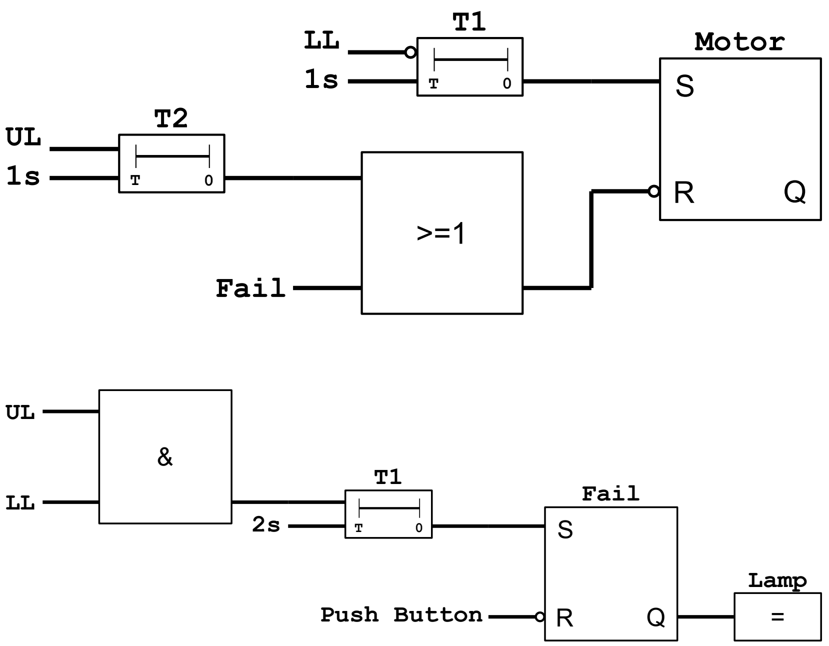
When the upper level switch sees a part for longer than 1 second, the motor of the hopper feeder should be switched off. It should be switched on if the lower level sensor sees no part for longer than one second.

A lamp indicates a failure state. This failure state is set in case of a congestion. It is reset by a pushbutton. The motor may not be on in case of a failure.



Wanted: Logic in FBD

Answer:

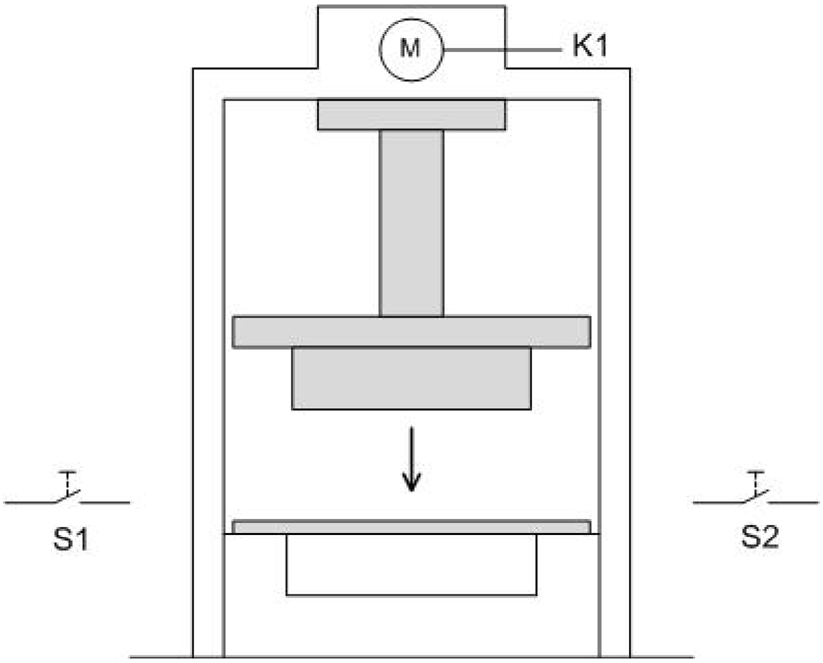


Task 5.11: Manually operated press

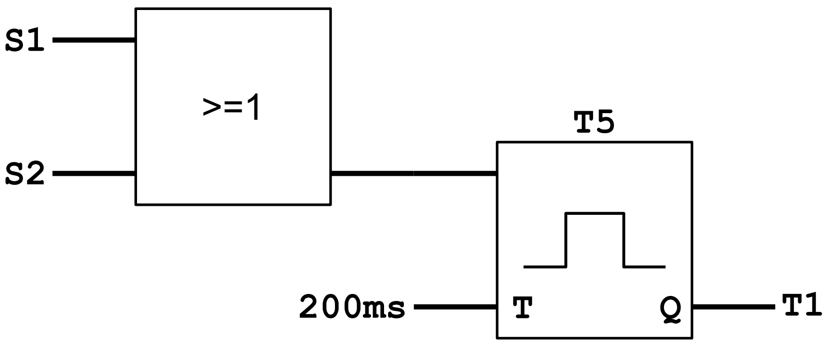
This method is usually applied to protect the hands of the operators who operate dangerous machines. The machine only starts if both activation push buttons are pressed nearly at the same time (within 0.2 seconds). This has to happen using the left and the right hand.

The machine stops immediately if one of the pushbuttons is released.

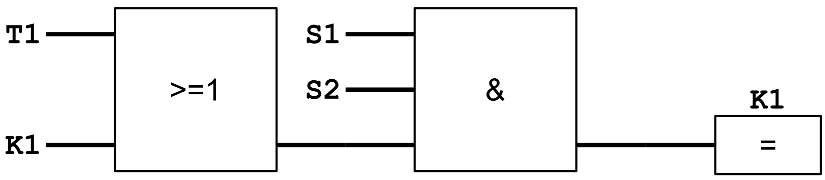
Please develop a control program to operate the press cylinder while meeting the above safety requirements.



Answer:



The timer T5 keeps the signal T1 ON untill 200ms expires.

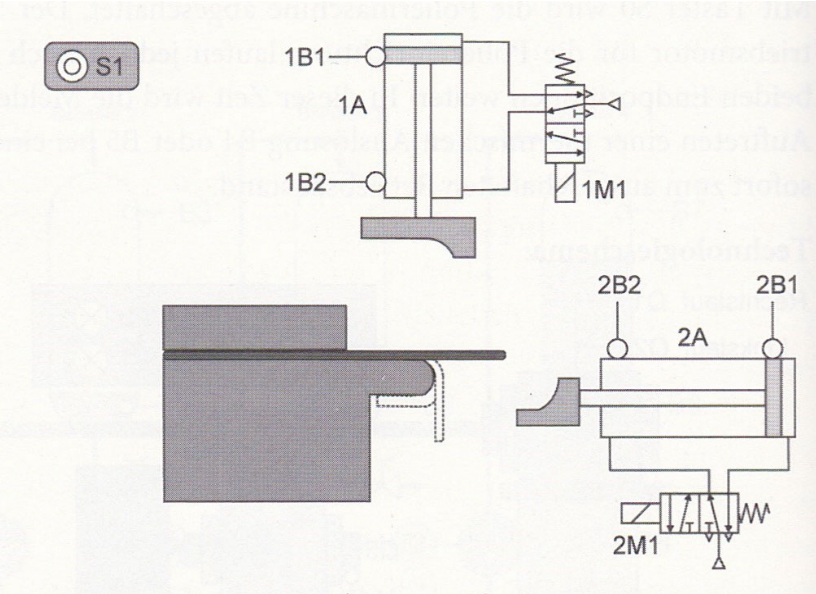


Note: The "brick on the button" trick won't work!
That is why edge triggered pulse is used.

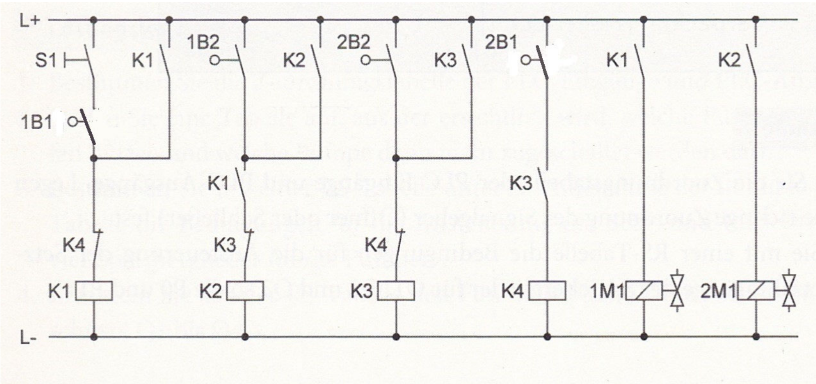
Task 5.12: Bending Tool

Metal sheets are bent to a certain shape using a bending tool. The bending sequence is as follows: After the pushbutton S1 was pressed, the piston of Cylinder 1A moves out. Thereby the sheet is clamped and pre-bent. When the piston of Cylinder 1A is in its outer end position, the piston of Cylinder 2A moves out, finishes the bending of the sheet and is retracted after it has reached its outer end position. When the inner end position of Cylinder 2A is reached, the piston of Cylinder 1A is also retracted.

The technology sketch below shows the arrangement of the cylinders.



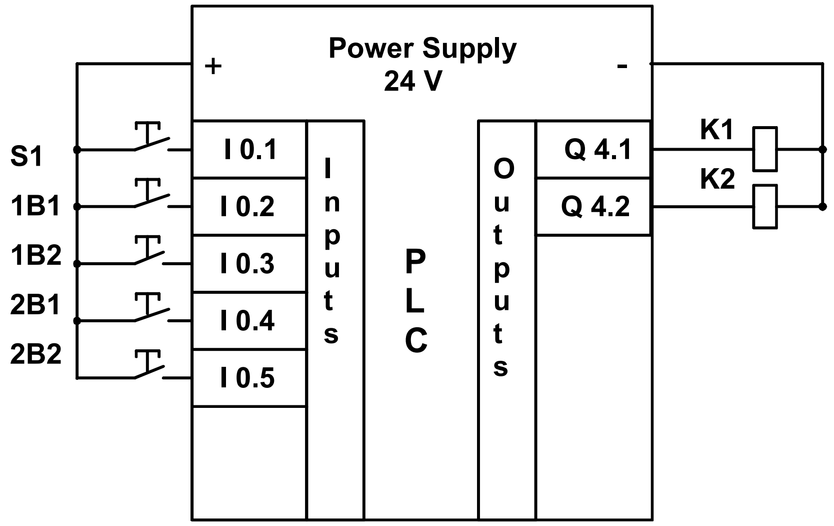
The excitation of the solenoid valves was implemented up to now in a way as shown in the following circuit diagram:



This contactor control shall be replaced by a software solution inside a PLC.

Answer:

Hardware configuration:

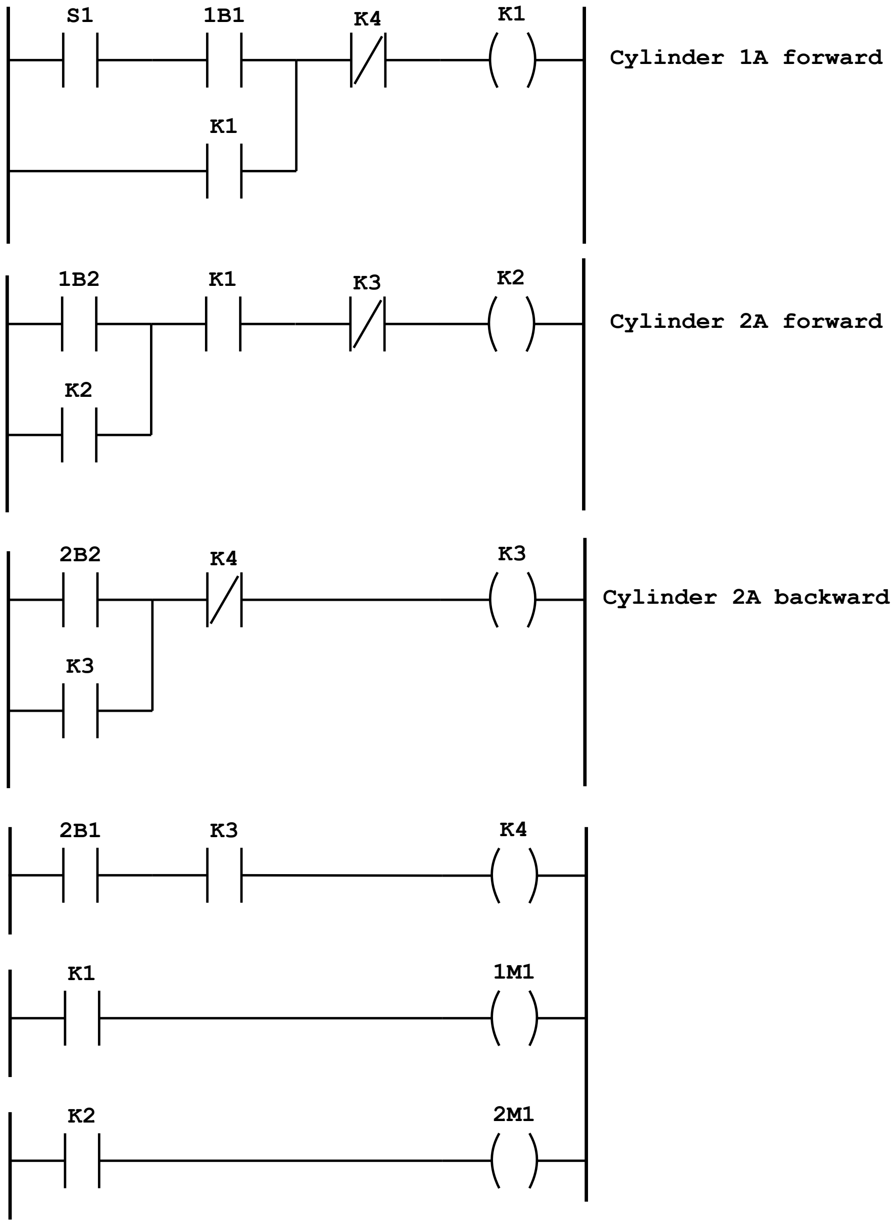


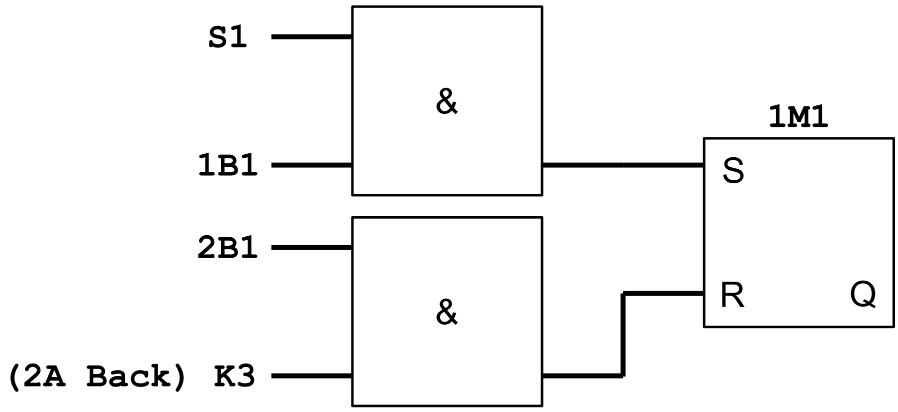
Symbol Table:

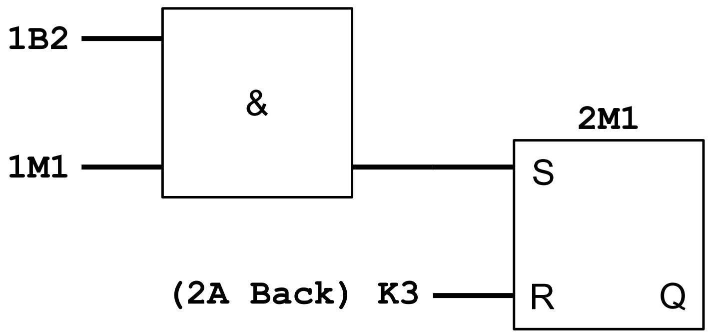
Address	Symbol	Comment
I0.1	S1	Push button: START
I0.2	1B1	Inner-end pos cylinder 1A
I0.3	1B2	Outer-end pos cylinder 1A
I0.4	2B1	Inner-end pos cylinder 2A
I0.5	2B2	Outer-end pos cylinder 2A
Q4.1	1M1	Solenoid coil cylinder 1A
Q4.2	2M1	Solenoid coil cylinder 2A

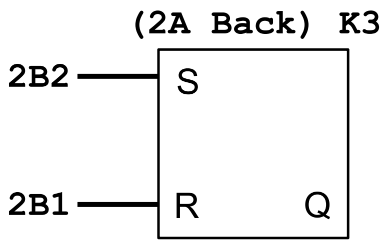
LAD:

FBD using Flip Flops:


S1
1B1
K4
K1
K1
Cylinder 1A forward
1B2
K1
K3
K2
K2
Cylinder 2A forward
2B2
K4
K3
K3
Cylinder 2A backward
2B1
K3
K4
K1
1M1
K2
2M1

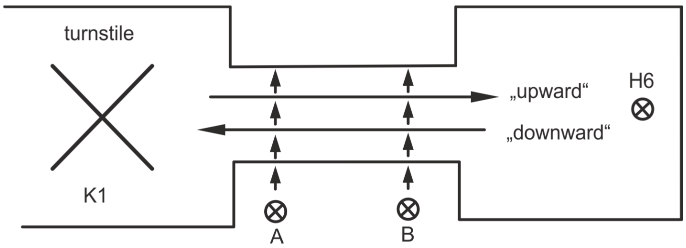

S1
1B1
2B1
(2A Back) K3
&
&
1M1
S
R
Q


1B2
1M1
&
2M1
S
R
Q
(2A Back) K3


(2A Back) K3
2B2
2B1
S
R
Q

Task 5.13: Person counter

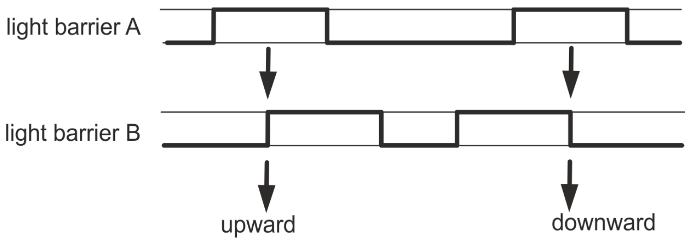
An Up-/Down-Counter is to be programmed. The persons in a room are to be counted. For this purpose, the entrance to the room is equipped with two light barriers which are installed in such a way that during the passage through the entrance first one and then both light barriers are interrupted. From that arrangement the counting signal is derived. The sketch shows the counting direction.



The light barriers provide a logic “1” signal if the beam is interrupted. The turnstile at the entrance is blocked if the room is occupied.

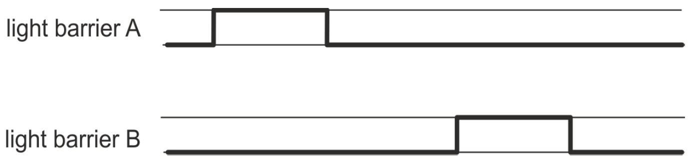
For resetting the counter, a pushbutton S3 is to be used. H6 is for room lighting. It is only on if there is someone in the room. K1 serves for the turnstile control. The maximum number of persons in the room is 10.

From the timing sequence of the interruption of the beams of light, the counting direction can be determined. If light barrier A is interrupted first, then the counting direction is upward. In case of counting direction “downward”, light barrier B is interrupted first.



Precondition for counting is that at least one light barrier (in this example light barrier A) is interrupted. If now the state of signal B changes from “0” to “1”, the counter counts upward. If B changes from “1” to “0”, the counter counts downward.

If only one light barrier is interrupted, counting is prohibited.



Here the precondition for counting is not fulfilled (signal state “1” for A and an edge at light barrier B).

Answer:

Symbol Table:

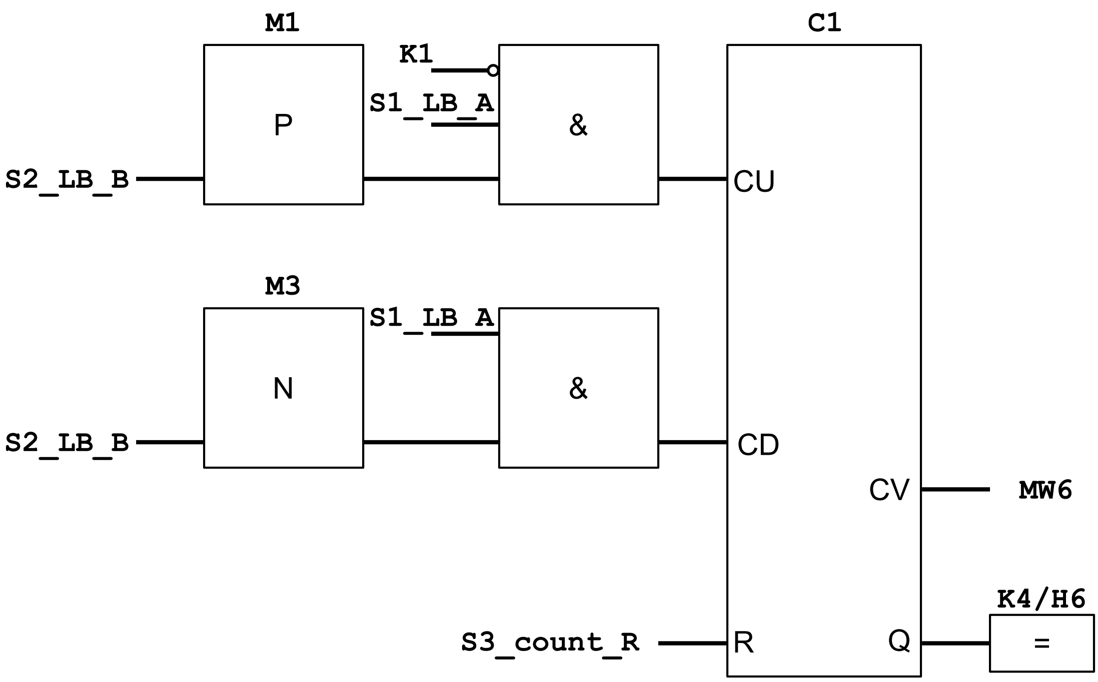
Address	Symbol	Comment
Q4.5	K1	Lock turnstile
M1.0	M1	Auxiliary bit for rising edge
M2.0	M3	Auxiliary bit for falling edge
I0.1	S1_LB_A	Light Barrier A
I0.2	S2_LB_B	Light Barrier B
I0.3	S3_count_R	Counter Reset
Q4.6	K4 or H6	Room Lighting
MW6	MW6	Count Value

Network 1: Counter 1
CU: Count Up when the
-room is not full ($\overline{K1}$)
-Light Barrier A is interrupted.
-Rising Edge of B is detected.

CD: Count Down when the
-Light Barrier A is interrupted.
-Falling Edge of B is detected.

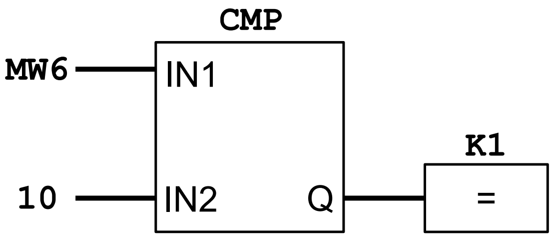
CV: Count Value

P block: Positive Edge Detection
N block: Negative Edge Detection



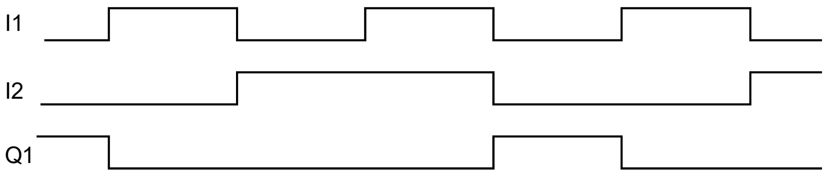
Network 2: Turnstile

$lock \rightarrow ON$ (1)
$unlock \rightarrow OFF$ (0)



Task 5.14: Generation of STL from a Timing Diagram
For the control function displayed in a timing diagram, the STL is to be generated.

5.14.1 a



Answer:

I1	I2	Q1
0	0	1
0	1	0
1	0	0
1	1	0

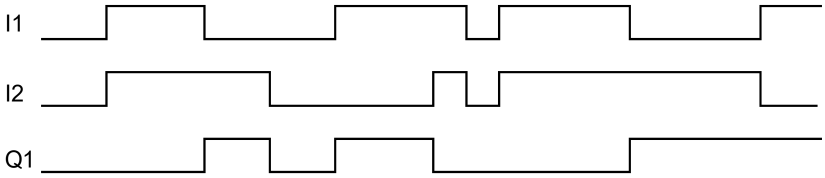
Table 5.14.1: Truth Table for Logic Function (a)

O	I1
O	I2
=N	Q1

OR

AN	I1
AN	I2
=	Q1

5.14.2 b



I1	**I2**	**Q1**
0	0	0
0	1	1
1	0	1
1	1	0

Table 5.14.2: Truth Table for Logic Function (b)

Answer:

O(	
A	I1
AN	I2
)	
O(	
AN	I1
A	I2
)	
=	Q1

Task 5.15: Analysis of a Statement List (STL)

The statement list below is given in STL:

```
A      I0.0
AN     M5.0
S      M5.0
AN     I0.0
R      M5.0
```

What happens here? Can this be done simpler?

Answer:

- If the input I0.0 is **TRUE** and the memory bit M5.0 is **FALSE**, then M5.0 is **set**.
- If the input I0.0 is **FALSE**, then M5.0 is **reset**.

Thus, the memory bit M5.0 follows the state of the input I0.0. M5.0 is set when I0.0 becomes TRUE and reset when I0.0 becomes FALSE.

This behavior corresponds to a simple assignment of the input I0.0 to the memory bit M5.0.

The same function can be implemented in a simpler and clearer way using a direct assignment:

A	I0.0
=	M5.0

Exercise 06: Digital Signal Processing

Task 6.1: Sampling

An analog signal composed of three sinusoidal components is given by

$$x(t) = 3\sin(7\Omega_0 t) + 5\sin(5\Omega_0 t) + 7\sin(11\Omega_0 t).$$

The signal is sampled to form a discrete-time signal.

6.1.1 What is the recommended sampling frequency for the given signal?

Answer:

The given signal contains the angular frequencies

$$5\Omega_0,\ 7\Omega_0,\ \text{and}\ 11\Omega_0.$$

The highest angular frequency component is $11\Omega_0$. According to the Nyquist sampling theorem, the sampling angular frequency must satisfy

$$\Omega_s \geq 2 \times f_{\max} = 2 \times 11\Omega_0 = 22\Omega_0.$$

Hence, the recommended sampling frequency is

$$\Omega_s = 22\Omega_0 \text{ or higher.}$$

6.1.2 Derive an expression for the discrete-time signal obtained after sampling.

Answer:

Sampling the signal with sampling period $T_s = \frac{2\pi}{\Omega_s}$ gives the discrete-time signal

$$x[n] = x(nT_s).$$

Substituting $t = nT_s$ into $x(t)$,

$$x[n] = 3\sin(7\Omega_0 nT_s) + 5\sin(5\Omega_0 nT_s) + 7\sin(11\Omega_0 nT_s).$$

Equivalently, in terms of discrete-time angular frequencies,

$$x[n] = 3\sin(\omega_1 n) + 5\sin(\omega_2 n) + 7\sin(\omega_3 n),$$

where

$$\omega_k = \Omega_k T_s.$$

6.1.3 Explain the consequence of sampling the signal at a frequency of $10\Omega_0$.

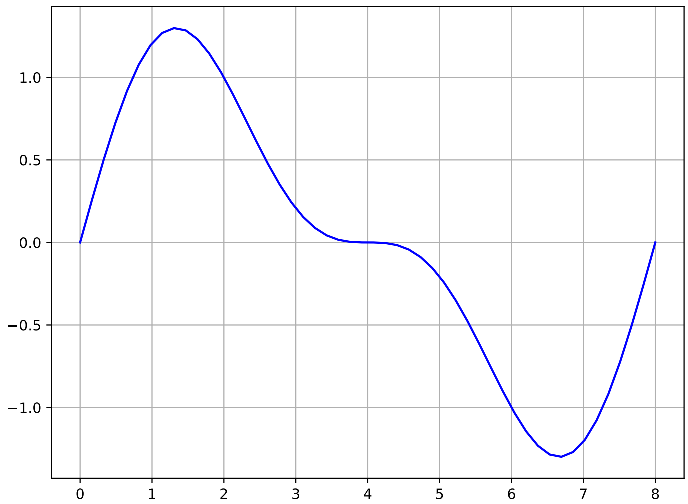
Answer:
If the signal is sampled at $\Omega_s = 10\Omega_0$, the sampling frequency is below the Nyquist rate since

$$10\Omega_0 < 22\Omega_0.$$

As a result, the frequency components above $\frac{\Omega_s}{2} = 5\Omega_0$ will alias into lower frequencies. In particular, the components at $7\Omega_0$ and $11\Omega_0$ will overlap with lower-frequency components in the discrete-time spectrum. This causes irreversible distortion, and the original analog signal cannot be perfectly reconstructed from the samples.

Task 6.2: DFT and FFT Frequency Analysis
Consider the discrete-time signal

$$x[n] = \sin\left(\frac{2\pi}{8}n\right) + 0.5\sin\left(\frac{4\pi}{8}n\right), \quad n = 0, 1, \ldots, 7$$



6.2.1 Compute the Discrete Fourier Transform (DFT) $X[k]$ of $x[n]$ for $k = 0, 1, \ldots, 7$, and identify the frequency components present in the signal.
Answer:
The DFT is defined as

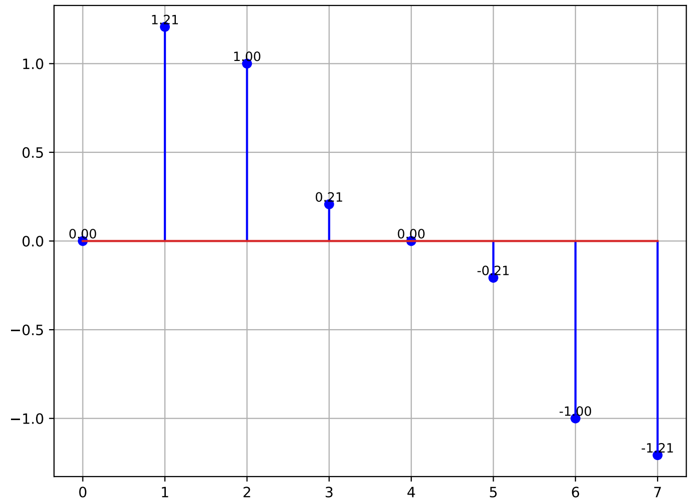
$$X[k] = \sum_{n=0}^{7} x[n] e^{-j\frac{2\pi}{8}kn}, \quad k = 0, 1, \ldots, 7$$

Step 1: Compute signal samples

$$x[n] = \sin\left(\frac{\pi}{4}n\right) + 0.5\sin\left(\frac{\pi}{2}n\right)$$

n	$\sin\left(\frac{\pi n}{4}\right)$	$0.5\sin\left(\frac{\pi n}{2}\right)$	$x[n]$
0	0	0	0
1	0.7071	0.5	1.2071
2	1	0	1
3	0.7071	-0.5	0.2071
4	0	0	0
5	-0.7071	0.5	-0.2071
6	-1	0	-1
7	-0.7071	-0.5	-1.2071

Table 6.2.1: Discrete-time signal components and sum



Step 2: Compute each DFT coefficient

$$X[0] = \sum_{n=0}^{7} x[n] = 0$$

$$X[1] \approx -4j$$

$$X[2] \approx -2.4142j$$

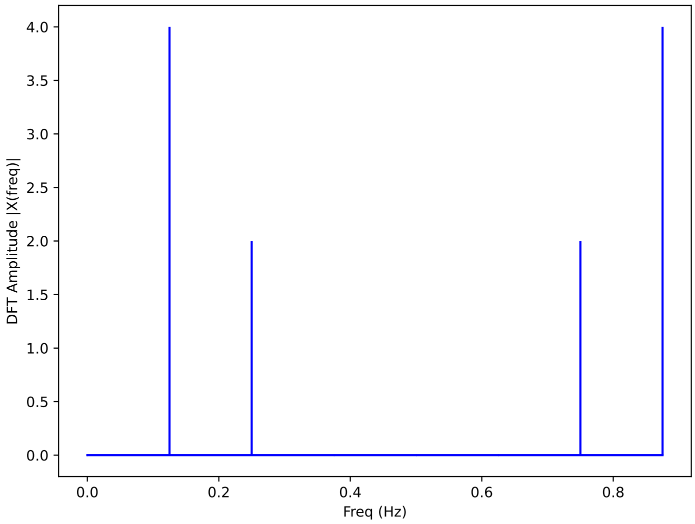
$$X[3] \approx 0$$

$$X[4] = 0$$

$$X[5] \approx 0$$

$$X[6] \approx 2.4142j$$

$$X[7] \approx 4j$$



Step 3: Identify frequencies

Each DFT index corresponds to the discrete-time frequency

$$\omega_k = \frac{2\pi}{N}k, \quad N = 8$$

k	$X[k]$	$\omega_k = \frac{2\pi k}{8}$	Interpretation
0	0	0	DC component
1	$-4j$	$\pi/4$	sinusoid at $\pi/4$ rad/sample ($2\pi/8$)
2	$-2.4142j$	$\pi/2$	sinusoid at $\pi/2$ rad/sample ($4\pi/8$)
3	~ 0	$3\pi/4$	negligible
4	0	π	Nyquist
5	0	$5\pi/4$	negligible
6	$2.4142j$	$3\pi/2$	negative frequency of $\pi/2$
7	$4j$	$7\pi/4$	negative frequency of $\pi/4$

Table 6.2.2: DFT of $x[n]$ and interpretation of frequency components

Note: The value of X[k] (e.g., -4j) gives amplitude and phase, while k determines the frequency.

6.2.2 FFT Computation of the 8-Point Signal

Answer:

$$x = [0, 1.2071, 1, 0.2071, 0, -0.2071, -1, -1.2071].$$

Step 1: Split into even and odd indices

$$x_{\text{even}} = [x[0], x[2], x[4], x[6]] = [0, 1, 0, -1]$$
$$x_{\text{odd}} = [x[1], x[3], x[5], x[7]] = [1.2071, 0.2071, -0.2071, -1.2071]$$

Step 2: Compute 4-point DFTs of even and odd parts

$$X_{\text{even}}[k] = \sum_{n=0}^{3} x_{\text{even}}[n] e^{-j2\pi kn/4}, \quad k = 0, 1, 2, 3$$

$$\begin{aligned}
X_{\text{even}}[0] &= 0 + 1 + 0 - 1 = 0 \\
X_{\text{even}}[1] &= -2j \\
X_{\text{even}}[2] &= -2 \\
X_{\text{even}}[3] &= 2j
\end{aligned}$$

$$X_{\text{odd}}[k] = \sum_{n=0}^{3} x_{\text{odd}}[n] e^{-j2\pi kn/4}, \quad k = 0, 1, 2, 3$$

$$\begin{aligned}
X_{\text{odd}}[0] &= 0 \\
X_{\text{odd}}[1] &\approx -2j \\
X_{\text{odd}}[2] &\approx 2 \\
X_{\text{odd}}[3] &\approx 2j
\end{aligned}$$

Step 3: Combine using twiddle factors

$$X[k] = X_{\text{even}}[k] + W_8^k X_{\text{odd}}[k], \quad k = 0, 1, 2, 3, \quad W_8 = e^{-j2\pi/8} = e^{-j\pi/4}$$

$$\begin{aligned}
X[0] &= 0 + 1 \cdot 0 = 0 \\
X[1] &\approx -2j + e^{-j\pi/8}(-2j) \approx -4j \\
X[2] &\approx -2 + (e^{-j\pi/4}) \cdot 2 = -4 \\
X[3] &\approx 2j + (e^{-j3\pi/8}) \cdot 2j \approx 0.2071 + \dots
\end{aligned}$$

Step 4: Use symmetry for k = 4..7

$$X[k+4] = X_{\text{even}}[k] - W_8^k X_{\text{odd}}[k], \quad k = 0, 1, 2, 3$$

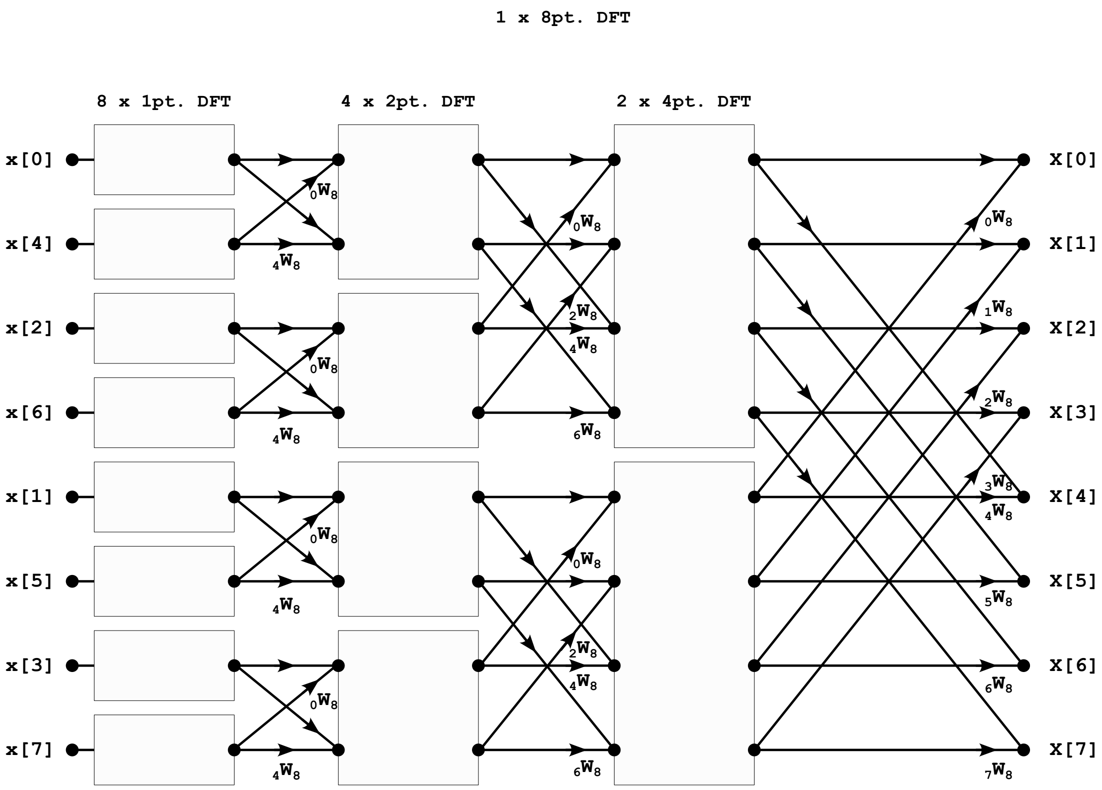
Step 5: Summary Table (Magnitude and Phase)

k	X[k]	$\|X[k]\|$	Phase (rad)
0	0	0	0
1	-4j	4	$-\pi/2$
2	-2.4142j	2.4142	$-\pi/2$
3	0	0	0
4	0	0	0
5	0	0	0
6	2.4142j	2.4142	$\pi/2$
7	4j	4	$\pi/2$

Observation: FFT efficiently computes the same DFT as direct computation using divide-and-conquer and twiddle factors, reducing operations from $N^2 = 64$ to $N \log_2 N = 24$.

6.2.3 Draw FFT Flow Graph.

Answer:



6.2.4 Explain how the Fast Fourier Transform (FFT) could be used to compute the same DFT more efficiently and compare the number of operations.

Answer:

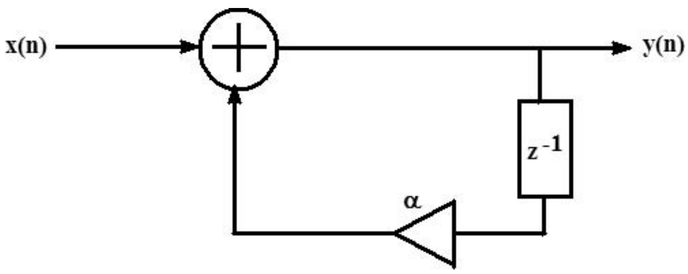
- Direct DFT computation requires $N^2 = 8^2 = 64$ complex multiplications.
- Using the FFT reduces the operations to $N \log_2 N = 8 \cdot 3 = 24$ complex multiplications.
- The FFT exploits symmetry and periodicity of the DFT to compute the same frequency components much faster.

The DFT identifies frequencies present in the signal by projecting onto complex exponentials, and the FFT allows this computation efficiently for larger signals.

Task 6.3: Discrete Time Systems

Develop an expression between the input and output of the following discrete-time systems. Also mention the type of system and its order.

6.3.1



Answer:

First-Order Feedback System:

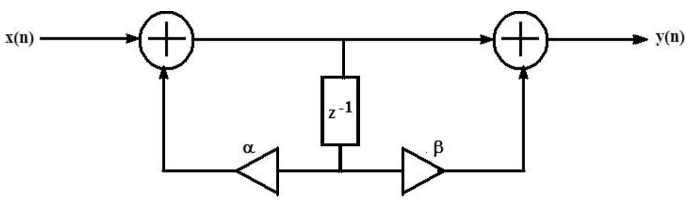
This is a recursive system where the output is fed back into the input.

- **Summing Junction**: The output $y[n]$ is the sum of the input $x[n]$ and the feedback signal.
- **Feedback Path**: The output $y[n]$ is delayed by one unit to produce $y[n-1]$, which is then multiplied by a gain a.

The resulting difference equation is

$$y[n] = x[n] + \alpha\, y[n-1].$$

6.3.2



Answer:

The system can be analyzed entirely in the time domain by tracing the signals through the block diagram.

Let $v[n]$ denote the signal immediately after the first summing junction.

Step 1: Internal Signal

The signal $v[n]$ is formed by summing the input and the feedback signal. The feedback signal is obtained by delaying $v[n]$ by one sample and multiplying by the gain a.

$$\boxed{v[n] = x[n] + \alpha\, v[n-1]}$$

(Equation A)

Step 2: Output Signal

The output $y[n]$ is formed by summing the current value of $v[n]$ and a feedforward term consisting of a delayed version of $v[n]$ multiplied by the gain b.

$$\boxed{y[n] = v[n] + \beta\, v[n-1]}$$

(Equation B)

Step 3: Elimination of the Internal Signal

To obtain a direct relationship between $y[n]$ and $x[n]$, we eliminate $v[n]$.
From Equation B, shift by one time step:

$$y[n-1] = v[n-1] + \beta\, v[n-2]$$

Multiplying by α:

$$\alpha\, y[n-1] = \alpha\, v[n-1] + \alpha\, \beta\, v[n-2]$$

Subtracting the above two equations we get:

$$y[n] - \alpha\, y[n-1] = v[n] + \beta\, v[n-1] - \alpha\, v[n-1] - \alpha\, \beta\, v[n-2]$$

Rearranging,

$$y[n] - \alpha\, y[n-1] = v[n] - \alpha\, v[n-1] + \beta\, (v[n-1] - \alpha\, v[n-2])$$

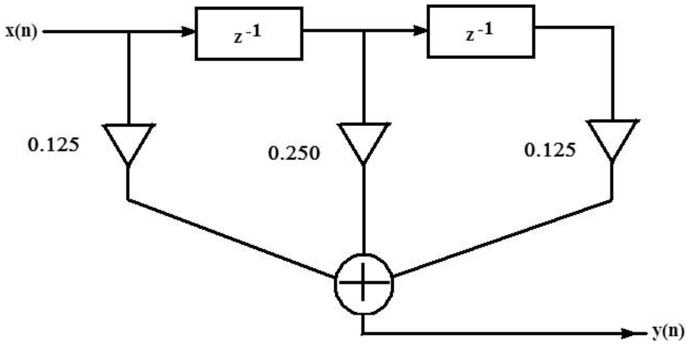
Using Equation A, the delayed internal signal is:

$$v[n-1] = x[n-1] + \alpha\, v[n-2]$$

Substituting and rearranging terms leads to the input–output difference equation:

$$\boxed{y[n] - \alpha\, y[n-1] = x[n] + \beta\, x[n-1]}$$

6.3.3



Answer:

Second Order FIR System.

This is a feedforward (Finite Impulse Response) system, where the output depends only on the current and past values of the input.

- **Path 1**: The current input $x[n]$ is multiplied by b_0.
- **Path 2**: The input is delayed once to obtain $x[n-1]$ and multiplied by b_1.
- **Path 3**: The input is delayed twice to obtain $x[n-2]$ and multiplied by b_2.

The resulting difference equation is

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2].$$

Task 6.4: Discrete-Time Fourier Transform (DTFT)

Compute the DTFTs of the following discrete-time signals using the Fourier transform analysis equation

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

6.4.1 $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$

Answer:

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^{n-1}, & n \geq 1 \\ 0, & n < 1 \end{cases}$$

Apply the DTFT definition:

$$X_a(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}.$$

Factor out $2 = \left(\frac{1}{2}\right)^{-1}$:

$$X_a(e^{j\omega}) = 2\sum_{n=1}^{\infty} \left(\frac{1}{2}e^{-j\omega}\right)^n.$$

Geometric Series:

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}, \quad |r| < 1, \quad r = \frac{1}{2}e^{-j\omega}.$$

Thus,

$$X_a(e^{j\omega}) = 2 \cdot \frac{\frac{1}{2}e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}} = \frac{e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}.$$

$$\boxed{X_a(e^{j\omega}) = \frac{e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}}$$

6.4.2 $x[n] = \left(\frac{1}{2}\right)^{|n|}$

Answer:

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 2^n, & n < 0 \end{cases}$$

Apply the DTFT definition:

$$X_b(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} + \sum_{n=-\infty}^{-1} 2^n e^{-j\omega n}.$$

Compute each sum.

Positive $n \geq 0$:

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}e^{-j\omega}\right)^n = \frac{1}{1-\frac{1}{2}e^{-j\omega}}, \quad \left|\frac{1}{2}e^{-j\omega}\right| < 1.$$

Negative $n < 0$, let $m = -n$:

$$\sum_{n=-\infty}^{-1} 2^n e^{-j\omega n} = \sum_{m=1}^{\infty} 2^{-m} e^{j\omega m} = \sum_{m=1}^{\infty} \left(\frac{1}{2}e^{j\omega}\right)^m = \frac{\frac{1}{2}e^{j\omega}}{1-\frac{1}{2}e^{j\omega}}.$$

Combine sums:

$$X_b(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{\frac{1}{2}e^{j\omega}}{1-\frac{1}{2}e^{j\omega}}.$$

$$\boxed{X_b(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{\frac{1}{2}e^{j\omega}}{1-\frac{1}{2}e^{j\omega}}}$$

Task 6.5: FIR Filter Design Analysis

A digital communication system requires a low-pass FIR filter to isolate a baseband signal. The design specifications are:

- **Passband Edge Frequency:** $f_p = 4$ kHz
- **Stopband Begin Frequency:** $f_c = 6$ kHz

- **Sampling Frequency:** $f_s = 20$ kHz
- **Maximum Passband Ripple:** $\delta_1 = 0.01$
- **Minimum Stopband Attenuation:** $\delta_2 = 0.001$

6.5.1 Transition Band Width

Answer:

The transition band is the frequency region between f_p and f_c.

- **Frequency width:**

$$\Delta f = f_c - f_p = 6 \text{ kHz} - 4 \text{ kHz} = 2 \text{ kHz}$$

- **Normalized width (radians/sample):**

$$\Delta\omega = 2\pi \frac{\Delta f}{f_s} = 2\pi \frac{2000}{20000} = 0.2\pi \text{ rad/sample}$$

6.5.2 Ripple Ratio Calculation

Answer:

The ripple ratio is determined from the maximum passband ripple δ_1 and the minimum stopband attenuation δ_2:

$$\text{Ripple Ratio} = \frac{\delta_2}{\delta_1} = \frac{0.001}{0.01} = 0.1$$

Exercise 07: Industrial Communication

Task 7.1: RS 232 Serial Link

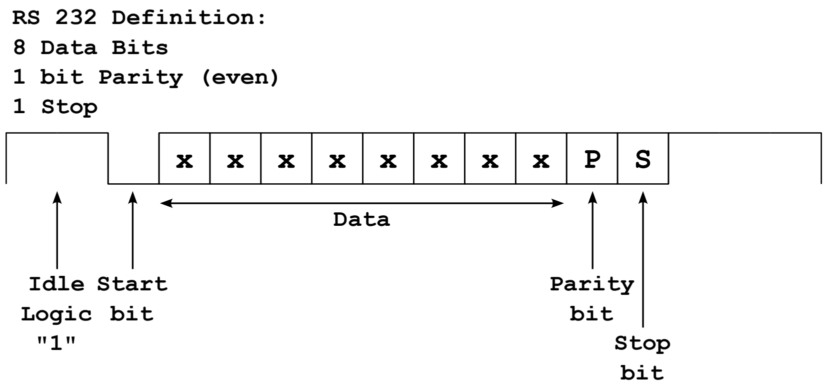
Two digital automation devices communicate via an asynchronous serial link according to RS 232 (CCITT V.24) with a transmission speed of 19.2 kBaud. The interface chips in both devices (UART = Universal Asynchronous Receiver/Transmitter) produce an interrupt to the CPU after receipt of a character (not of a bit!), so that the CPU can read the character from the receive buffer. Afterwards the chip receives the next character.

Data of the transmission link:

$$v_{\text{kom}} = 19,2 \text{ kBaud}; \quad \text{UART character format: 8 Data bits, Parity even, 1 Stop bit}$$

7.1.1 In which time interval is the CPU interrupted by the serial interface?

Answer:



Number of bits per character:

$$1 \text{ Start bit} + 8 \text{ Data bits} + 1 \text{ Parity bit} + 1 \text{ Stop bit} = 11 \text{ bits}$$

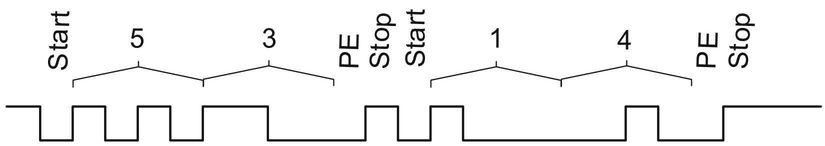
$$\text{Number of bits per character: 11 bits}$$

$$T_s = \frac{1}{v_{\text{kom}}} = \frac{1}{19,200} \approx 52.08\ \mu\text{s per bit}$$

$$\text{Time per character } = 11 \cdot T_s \approx 573\ \mu\text{s}$$

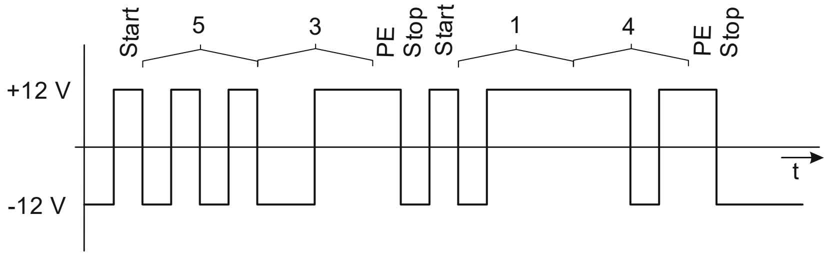
7.1.2 Please draw the logical state of the transmission channel as a function of time for the transmission of the characters "5" and "A" in ASCII representation (Hex 35 and 41).

Answer:



7.1.3 How does the voltage on the transmit line look like as a function of time in this data transmission?

Answer:



Task 7.2: RS485 Transmission

On a RS-485 transmission network the text string “Automation exercise run” is transmitted with a transmission speed of 500 kBaud asynchronously using UART characters.

Definition of the characters: 8 Data bits, parity even, 1 stop bit (PROFIBUS conform). Telegram propagation delay as well as software processing time are not to be considered.

7.2.1 No protocol overhead is to be assumed, i.e., exclusively user data are transmitted. How long does the transmission of a string last? How often can this text string be transmitted continuously in one second?

Answer:

Number of bits per character (UART format: 8 data bits, parity even, 1 stop bit):

11 bits per character.

The string “Automation exercise run” has 23 characters.

Transmission speed: $R = 500{,}000$ bits/sec.

Time per string:

$$T_s = \frac{\text{Total bits}}{R} = \frac{23 \cdot 11}{500{,}000} = \frac{253}{500{,}000} \approx 0.000506 \text{ sec} = 506\ \mu\text{s}.$$

Number of transmissions per second:

$$f_{\text{transmit}} = \frac{1}{T_s} \approx \frac{1}{0.000506} \approx 1976 \text{ times/sec}.$$

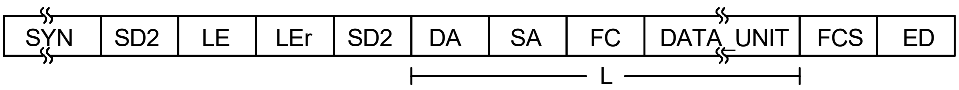
7.2.2 Now the PROFIBUS telegram SD2 is used. Before the send telegram the SYN time has to be awaited. As response a short acknowledge is to be taken into account. Per text string one such communication cycle is used. How many telegrams can be transmitted in one second?

Answer:

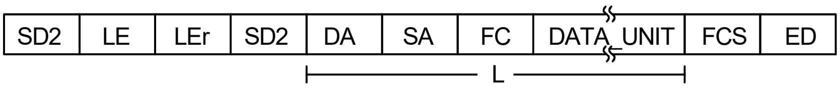
Formats with variable information field length

For a variable number of data bytes the actual length has to be transmitted in the telegram header. This length indication is located twice (because of transmission security) at the telegram beginning in a fixed telegram header.

A) Format of the request telegram (Send/request frame):



B) Format of the response telegram (Response frame):



SYN = Synchronisation bits, min. 33 bits idle state (Idle)
SD2 = Start byte 2 (Start delimiter), Code: 68H
LE = Length, Value: 4 to 249
LEr = Length byte repeated (repeat)
DA = Destination address
SA = Source address
FC = Frame Control
DATA_UNIT = Data field, variable Length (L-3), max 246 bytes
FCS = Frame Check Sequence
ED = End Delimiter, Code: 16H
L = Length of information field, variable number of bytes L = 4 bis 249

Fig. Frame format with variable length

We have: - 23 payload characters,
- 9 level 2 protocol characters,
- 33 bits for SYN,
- 1 short acknowledge character.

Total bits per transmission:

$$N_{\text{bits}} = (23 + 9) \cdot 11 + 33 + 11$$

Transmission time:

$$R_{SD2} = \frac{500,000 bits}{N_{\text{bits}}} = \frac{500{,}000}{(23+9) \cdot 11 + 33 + 11} \approx 1262 \text{ s}^{-1}.$$

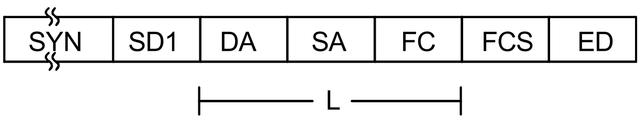
7.2.3 Now assume that a PROFIBUS master requests the telegram by polling with a SD1 telegram from a slave. The slave responds with a SD2 telegram. How many response telegrams can be transmitted in one second?

Telegram formats

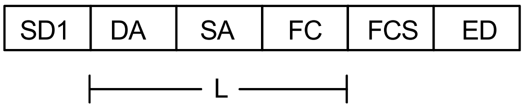
The figures contained in the following text don´t show sequences (Request ---> Acknowledge or response), but telegram formats of equal category (Hamming distance HD=4, fixed lenght with/without Data field and variable length), i.e. different acknowledge or response telegrams can follow the request telegrams (see Chapter 5).

Formats with fixed length of the information field without data field

A) Format of the request telegram (Request-Frame):



B) Format of the acknowledgement frame:



C) Format of the short acknowledgement

SYN = Synchronisation bits, min. 33 bit idle state
SD1 = Start byte 1 (Start Delimiter), Code: 10H
DA = Destination address
SA = Source address
FC = Frame Control
FCS = Frame Check Sequence
ED = End Delimiter, Code: 16H
L = Length of information field, fixed number of Bytes L = 3
SC = Single character, Code: E5H

Fig. Frame format with fixed legth without data field

Answer:

We have: - 33 bits SYN, - 611 bits for SD1+SD2 headers, - (23+9) characters payload.

Total bits per transmission:

$$N_{\text{bits}} = 33 + 6 \cdot 11 + (23 + 9) \cdot 11$$

Transmission time:

$$R = \frac{500,000 bits}{N_{\text{bits}}} \approx 1108.7\ \text{s}^{-1}.$$

7.2.4 How big is the protocol overhead in the cases 2 and 3?

Answer:

Case 2 (SD2 only):

$$\text{Overhead fraction} = \frac{9+3+1}{23+9+3+1} \approx 36.1\%$$

Case 3 (SD1 request + SD2 response):

$$\text{Overhead fraction} = \frac{3+6+9}{3+6+(23+9)} \approx 43.9\%$$

Task 7.3: Calculation of Delay and Offset for Synchronization of Real Time Clocks

A slave clock is 50 μs later compared to a master clock. When the master has $T_{M1} = 1051$ μs on his clock, he sends a Sync telegram to the slave. When the slave is reading $T_{S1} = 1002$ μs on his clock, he receives the Sync telegram. The master reports the exact sending time with a Follow Up telegram, namely $T_{M1} = 1051$ μs. The slave now sets his clock ahead by the difference $T_{M1} - T_{S1} = 49$ μs.

7.3.1 Are the clocks synchronized now or is there still an error existing?
Shortly later the master sends a new Sync telegram at $T_{M2} = 1063$ μs.

Answer:

No, clocks are not syncronized. Slave was originally behind the master clock by $50\mu s$ and it corrected $49\mu s$. There is still $1\mu s$ error left.

7.3.2 At which time on the slave clock T_{S2} is the slave receiving the telegram?
Now the slave is sending a Delay Request to the master. Sending time is $T_{S3} = 1080$ μs. Symmetry is assumed.

Answer:

Propogation delay: Time taken for Sync to travel from master to slave.

$$\text{delay} = T_{S1} - (T_{M1} - \text{Offset})$$

$$\text{delay} = 1\mu s$$

$$T_{S2} = T_{M2} + \text{delay} - \text{Corrected\_Offset}$$

$$T_{S2} = 1063 + 1 - 1$$

$$T_{S2} = 1063\mu s$$

7.3.3 At which time T_{M3} does the master receive the request?

Answer:

$$T_{S3} = T_{M2} + \text{delay} + \text{Corrected\_Offset}$$

$$T_{S3} = 1080 + 111$$

$$T_{S3} = 1082\mu s$$

7.3.4 What can be calculated by the slave after receiving the Delay Response?

Answer:
Slave calculates the network delay from the DelayReq. and DelayResp.

7.3.5 Why is no Follow Up telegram needed after the Delay Request?

Answer:
No Follow-Up needed after the delay request because after the DelayResp. clocks are synchronized as network delay is also corrected by the slave clock.